Lecture 07: Plane Problems (Two-dimensional Problems)

Plane stress: $\sigma_{zz} = \sigma_{zx} = \sigma_{zy} = 0$ and for isotropic materials (P.1)

$$\begin{bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{xy} \end{bmatrix} = \frac{E}{1-\nu^2} \begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & (1-\nu) \end{bmatrix} \begin{bmatrix} e_{xx} \\ e_{yy} \\ e_{xy} \end{bmatrix}, \quad e_{xy} = \frac{1}{2}\gamma_{xy} \qquad (P.2)$$

$$\begin{bmatrix} G\nabla^2 + G\frac{1+\nu}{1-\nu}\frac{\partial}{\partial x^2} & G\frac{1+\nu}{1-\nu}\frac{\partial^2}{\partial x \partial y} \\ G\frac{1+\nu}{1-\nu}\frac{\partial^2}{\partial x \partial y} & G\nabla^2 + G\frac{1+\nu}{1-\nu}\frac{\partial^2}{\partial y^2} \end{bmatrix} \begin{Bmatrix} u \\ v \end{Bmatrix} = \begin{Bmatrix} -F_x \\ -F_y \end{Bmatrix}, \quad \nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \qquad (P.3)$$

Plane strains: $\varepsilon_{zz} = 0, \quad \frac{\partial u}{\partial z} = \frac{\partial v}{\partial z} = w = 0, \quad \sigma_{zz} = \nu(\sigma_{xx} + \sigma_{yy})$ (P.4)

$$\begin{bmatrix} G\nabla^2 + \frac{G}{1-2\nu}\frac{\partial^2}{\partial x^2} & \frac{G}{1-2\nu}\frac{\partial^2}{\partial x \partial y} \\ \frac{G}{1-2\nu}\frac{\partial^2}{\partial x \partial y} & G\nabla^2 + \frac{G}{1-2\nu}\frac{\partial^2}{\partial y^2} \end{bmatrix} \begin{Bmatrix} u \\ v \end{Bmatrix} = \begin{bmatrix} -F_x \\ -F_y \end{bmatrix} \qquad (P.5)$$

Observation: replace $\frac{1+\nu}{1-\nu}$ in (P.3) by $\frac{1}{1-2\nu}$ or $\nu \leftarrow \frac{\nu}{1+\nu}$, then (P.3) can be used for the solution of (P.5).