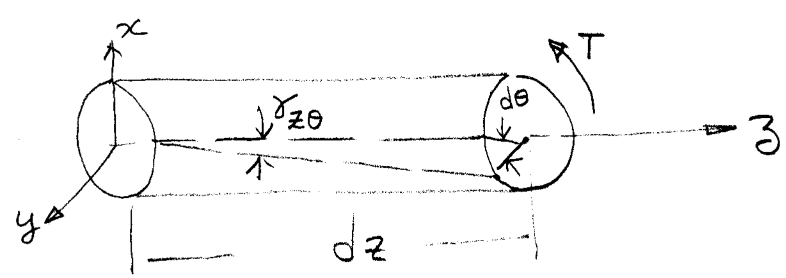
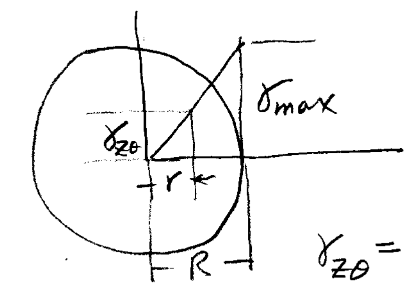
Lecture 06: Torsion

- Saint-Venant's Theory of Torsion (Note source: Theory of Elasticity by S. P. Timoshenko, chapter 10)
  - Application of elasticity theory
  - How to formulate the appropriate boundary conditions
  - Good solutions are often obtained by a combination of analytical and intuitively assumed solutions

* Elementary theory of torsion for a circular cylinder





$$\gamma_{z\theta} = \frac{\gamma_{max}\, r}{R}$$

$$\tau_{z\theta} = G\gamma_{z\theta} = Gr\frac{d\theta}{dz} \Rightarrow T = \int_A \left(Gr\frac{d\theta}{dz}\right) r\,dA = \frac{d\theta}{dz} G\int r^2 dA = GJ\frac{d\theta}{dz}$$

$$\tau_{z\theta} = Gr\cdot\frac{T}{GJ} = \frac{Tr}{J} \Rightarrow \frac{d\theta}{dz} = \frac{T}{GJ}, \quad J = \int r^2 dA$$

* Question: What about a non-circular cross section?

- it warps, that is, a plane cross section does not remain plane.
- Barre de Saint-Venant (1855) solved this problem.
- Approach: part analytical & part assumed (guessed) solutions