Lecture 05 - Linear Elasticity

- Generalized Hooke's Law

$$\sigma^{ij} (= \tau_{ij}) = D^{ijkl} e^{*}_{kl}$$

$$e^{*}_{ij} = \begin{cases} \frac{1}{2}(u_{i,j} + u_{j,i}) & \text{if } u_i \text{ is infinitesimal,} \\ \text{the Almansi strain} & \text{if } u_i \text{ is finite.} \end{cases}$$

$$\begin{aligned} D^{ijkl} &= D^{jikl} && \text{since } \sigma^{ij} = \sigma^{ji} \\ &= D^{ijlk} && \text{since } e_{kl} = e_{lk} \\ &= D^{klji} && \text{if } W = \tfrac{1}{2} D^{ijkl} e_{ij} e_{kl} \text{ is symmetric} \end{aligned}$$

The result: we reduce elastic constants to 21.

Further if the material is elastically isotropic, the number of independent elastic constants becomes 2. isotropy - identical in all directions. Composites are not in general isotropic.

- Isotropic Elastic constitutive (stress-strain) relations

$$\sigma_{\alpha\alpha} = 3K e_{\alpha\alpha}$$

$$\sigma'_{ij} = 2G e'_{ij}$$

σ'_{ij} - stress deviation

e'_{ij} - strain deviation

K: the bulk modulus of the material

G: the modulus of elasticity in shear, shear modulus, the modulus of rigidity.

$$\sigma'_{ij} = \sigma_{ij} - \frac{1}{3}\sigma_{\alpha\alpha}\delta_{ij}$$

$$e'_{ij} = e_{ij} - \frac{1}{3} e_{\alpha\alpha}\delta_{ij}$$