ASEN5012/MCEN5023 Mechanics of Solids - Fall 2007: Homework 5

Due at Start of Class on Wednesday, 03 October 2007

Problem 1. Derive the equations of equilibrium derived in the class for the rectangular Cartesian coordinates in terms of stress components expressed in the cylidrical coordinates, viz.,

$$\begin{aligned}
\mathbf{r} &= x\mathbf{i} + y\mathbf{j} + z\mathbf{k} = r\,\mathbf{e}_r + z\mathbf{k} \\
x &= r\cos\theta, \quad y = r\sin\theta \\
\mathbf{e}_r &= \cos\theta\mathbf{i} + \sin\theta\mathbf{j} \\
\mathbf{e}_\theta &= \mathbf{k}\times\mathbf{e}_r = -\sin\theta\mathbf{i} + \cos\theta\mathbf{j}
\end{aligned}$$

where the stress tesnor in the cylindrical coordinates is given by

$$\boldsymbol{\sigma}_{cyl} = \langle\, \mathbf{e}_r \quad \mathbf{e}_\theta \quad \mathbf{k}\,\rangle \begin{bmatrix} \sigma_{rr} & \sigma_{r\theta} & \sigma_{rz} \\ \sigma_{\theta r} & \sigma_{\theta\theta} & \sigma_{\theta z} \\ \sigma_{zr} & \sigma_{z\theta} & \sigma_{zz} \end{bmatrix} \begin{Bmatrix} \mathbf{e}_r \\ \mathbf{e}_\theta \\ \mathbf{k} \end{Bmatrix}$$

Hint: Express

$$\nabla_{Cart} = \frac{\partial}{\partial x}\mathbf{i} + \frac{\partial}{\partial y}\mathbf{j} + \frac{\partial}{\partial z}\mathbf{k}$$

in the form of

$$\nabla_{cyl} = f_r\mathbf{e}_r + f_\theta\mathbf{e}_\theta + f_z\mathbf{k}$$

where (f_r, f_θ, f_z) are operators associated with each of the directional derivatives in the cylindrical coordinates.

The equilibrium equations are obtained as

$$\nabla_{cyl}\cdot\boldsymbol{\sigma}_{cyl} + \mathbf{F}_{cyl} = \mathbf{0}$$

where $\mathbf{F}_{cyl}$ is the external forces expressed in the cylindrical coordinates:

$$\mathbf{F}_{cyl} = F_r\mathbf{e}_r + F_\theta\mathbf{e}_\theta + F_z\mathbf{k}$$

Problem 2. Transform the linear strain tensor given by

$$\varepsilon = \langle\, \mathbf{i} \quad \mathbf{j} \quad \mathbf{k}\,\rangle \begin{bmatrix} \epsilon_{xx} & \epsilon_{xy} & \epsilon_{xz} \\ \epsilon_{xy} & \epsilon_{yy} & \epsilon_{yz} \\ \epsilon_{xx} & \epsilon_{yz} & \epsilon_{zz} \end{bmatrix} \begin{Bmatrix} \mathbf{i} \\ \mathbf{j} \\ \mathbf{k} \end{Bmatrix}$$

into the strain tensor in the cylindrical coordinates in the form:

$$\varepsilon = \langle\, \mathbf{e}_r \quad \mathbf{e}_\theta \quad \mathbf{k}\,\rangle \begin{bmatrix} \epsilon_{rr} & \epsilon_{r\theta} & \epsilon_{rz} \\ \epsilon_{r\theta} & \epsilon_{\theta\theta} & \epsilon_{\theta z} \\ \epsilon_{xz} & \epsilon_{\theta z} & \epsilon_{zz} \end{bmatrix} \begin{Bmatrix} \mathbf{e}_r \\ \mathbf{e}_\theta \\ \mathbf{k} \end{Bmatrix}$$