

Solutions for Homework #4

Problem 1.

$$1.1 \quad \underline{\tau} = (\underline{i} \quad \underline{j} \quad \underline{k}) \begin{bmatrix} 5 & 2 & 0 \\ 2 & -3 & 0 \\ 0 & 0 & 4 \end{bmatrix} \begin{Bmatrix} \underline{i} \\ \underline{j} \\ \underline{k} \end{Bmatrix}$$

$$\underline{I} = \tau_1 \underline{i} + \tau_2 \underline{j} + \tau_3 \underline{k}$$

$$\tau_{12} = \tau_{12} \nu_1 + \tau_{22} \nu_2 + \tau_{32} \nu_3$$

$$\nu = \underline{i} : \quad \underline{I} = 5\underline{i} + 2\underline{j}$$

$$\nu = \underline{j} : \quad \underline{I} = 2\underline{i} - 3\underline{j}$$

$$\nu = \underline{k} : \quad \underline{I} = 4\underline{k}$$

$$1.2 \quad \underline{\nu} = \frac{1}{\sqrt{2}}(\underline{i} + \underline{j} + 0\underline{k}) \Rightarrow \nu_1 = \frac{1}{\sqrt{2}}, \nu_2 = \frac{1}{\sqrt{2}}, \nu_3 = 0$$

$$\tau_1 = \tau_{11} \nu_1 + \tau_{21} \nu_2 = \frac{1}{\sqrt{2}}(5\underline{i} + 2\underline{j})$$

$$\tau_2 = \tau_{12} \nu_1 + \tau_{22} \nu_2 = \frac{1}{\sqrt{2}}(2\underline{i} - 3\underline{j})$$

Alternative solution:

$$\underline{I} = \underline{\nu} \cdot \underline{\tau} = (\nu_1 \quad \nu_2 \quad \nu_3) [\underline{\tau}] \begin{Bmatrix} \underline{i} \\ \underline{j} \\ \underline{k} \end{Bmatrix}$$

$$= (\tau_{11} \nu_1 + \tau_{21} \nu_2 + \tau_{31} \nu_3) \underline{i} + (\tau_{12} \nu_1 + \tau_{22} \nu_2 + \tau_{32} \nu_3) \underline{j} \\ + (\tau_{13} \nu_1 + \tau_{23} \nu_2 + \tau_{33} \nu_3) \underline{k}$$

Problem 2.

$$2.1 \quad \underline{I}^n = \frac{1}{\sqrt{3}}(\underline{i} + \underline{j} + \underline{k}) \cdot \underline{\tau}$$

$$= \frac{1}{\sqrt{3}} [1 \quad 1 \quad 1] [\underline{\tau}] \begin{Bmatrix} \underline{i} \\ \underline{j} \\ \underline{k} \end{Bmatrix}, \quad [\underline{\tau}] = \begin{bmatrix} 3 & 1 & 4 \\ 1 & 2 & -5 \\ 4 & -5 & 0 \end{bmatrix}$$

$$= \frac{1}{\sqrt{3}} (8\underline{i} - 2\underline{j} - \underline{k})$$