10.1 Wave propagation in a beam

Governing Equations for a Beam

Statics

$$\frac{\partial^2}{\partial x^2}\Big[\underbrace{EI\frac{\partial^2 v(x)}{\partial x^2}}_{M_z}\Big] = p(x)$$

Dynamics

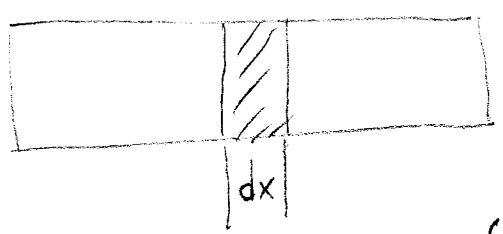
$$\frac{\partial^2}{\partial x^2}\Big[EI\frac{\partial^2 v(x,t)}{\partial x^2}\Big] = \overbrace{p(x,t) - m\frac{\partial^2 v(x,t)}{\partial t^2}}^{\bar{p}(x,t)}$$

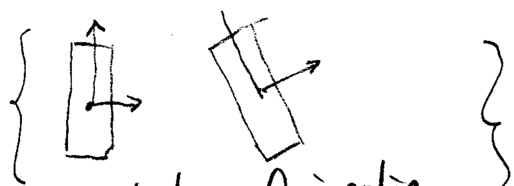
$$m = \rho A \qquad (10.1)$$

$$\bar{p}(x,t) = \underbrace{p(x,t)}_{\text{applied force}} + \underbrace{\Big[-m\frac{\partial^2 v(x,t)}{\partial t^2}\Big]}_{\text{inertia force}}$$

inertia force (it is a resisting force according to Newton)

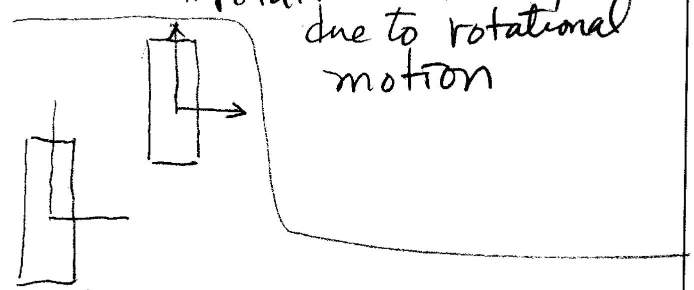
* Modification for rotatary inertia





* rotational inertia due to rotational motion

The term $-\left(-m\frac{\partial^2 v(x,t)}{\partial t^2}\right)$ represents only translational motion.



Equation (10.1) models only the translational inertia. Hence, the rotational inertia needs to be added to (10.1):

$$\text{rotational velocity: } \dot{\theta} = \frac{\partial}{\partial t}(\theta(x,t)) \doteq \frac{\partial}{\partial t}\Big[\frac{\partial v(x,t)}{\partial x}\Big]$$

So that we have:

$$\frac{\partial^2}{\partial x^2}\Big[EI\frac{\partial^2 v(x,t)}{\partial x^2}\Big] = p(x,t) - m\frac{\partial^2 v(x,t)}{\partial t^2} + \frac{\partial^2}{\partial x\partial t}\Big[I_p\frac{\partial^2 v(x,t)}{\partial x\partial t}\Big] \qquad (10.2)$$