Lecture 08: Variational Principles in Solid Mechanics

8.1 Euler's equation via the stationary value of a functional

consider a functional

$$J[u] = \int_a^b F(x, u, u')\,dx, \quad u(a) = u_1, \quad u(b) = u_2 \qquad (8.1.1)$$

we perturb

$$u(x) = y(x) + \epsilon\,\eta(x), \qquad \eta(a) = \eta(b) = 0 \qquad (8.1.2)$$

and obtain the perturbed functional

$$\phi(\epsilon) = J[y + \epsilon\eta] = \int_a^b F[x,\ y + \epsilon\eta,\ y' + \epsilon\eta']\,dx \qquad (8.1.3)$$

Terminology

- d-process: For y, dy is "an actual" infinitesimal change in y.
- δ-process: For y, δy is "a virtual" infinitesimal change in y

$$dF = \frac{\partial F}{\partial u}du + \frac{\partial F}{\partial u'}du'$$

$$\delta F \stackrel{\text{def}}{=} \frac{\partial F}{\partial u}\delta u + \frac{\partial F}{\partial u'}\delta u', \qquad \delta u = \epsilon\eta(x),\ \delta u' = \epsilon\eta'(x) \qquad (8.1.4)$$