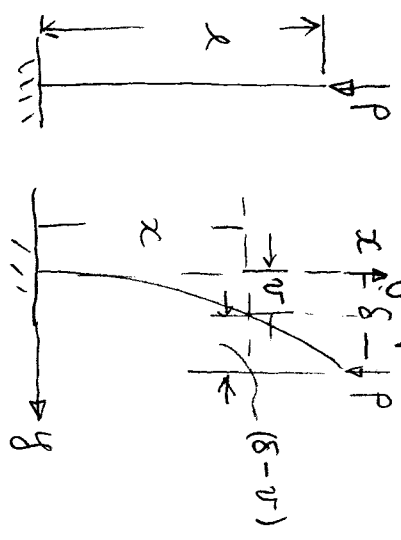


8.6 Application 1 - Stability of structures under quasi-static loads

8.6.1 Buckling of beams under axial compressive loads.



$$EI \frac{\partial^2 v}{\partial x^2} = P(\delta - v), \quad (8.6.1)$$

$$\frac{\partial^2 v}{\partial x^2} + k^2 v = k^2 \delta, \quad k = \frac{P}{EI} \quad (8.6.2)$$

$$v(x) = A \cos kx + B \sin kx + \delta \quad (8.6.3)$$

B.C.s $v=0, \frac{\partial v}{\partial x}=0$ at $x=0 \Rightarrow A=-\delta, B=0 \quad (8.6.4)$

$v(x) = \delta(1 - \cos kx)$ which must satisfy $v(l) = \delta$

$\delta = \delta(1 - \cos kl)$ or $\cos kl = 0 \quad (8.6.4)$

$kl = (2n-1)\frac{\pi}{2} = \sqrt{\frac{P}{EI}} l \quad (8.6.5)$

$\Rightarrow P_{cr} = \frac{\pi^2 (2n-1)^2}{4} \cdot \frac{EI}{l^2} \quad (8.6.6)$

which is known as the critical buckling load for a cantilever column.