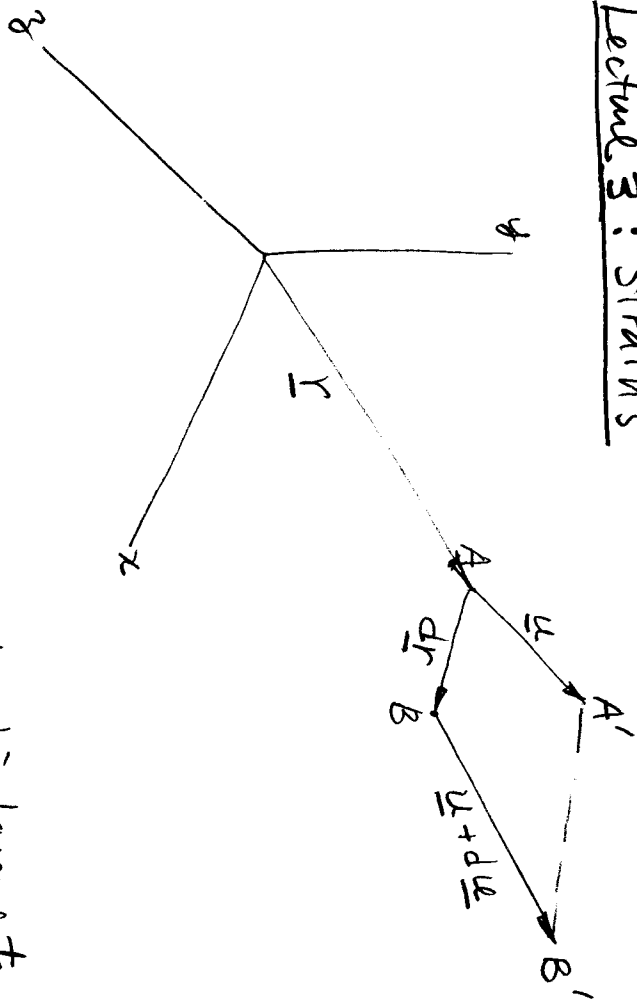


Lecture 3: Strains



The difference of the displacements at A and B, or the "shift", is given by

$$d\underline{u} = \frac{\partial u}{\partial x} dx + \frac{\partial u}{\partial y} dy + \frac{\partial u}{\partial z} dz = d\underline{r} \cdot \nabla \underline{u} \quad (|d\underline{u}| \ll |d\underline{r}|)$$

$$d\underline{r} = dx_1 \underline{i} + dx_2 \underline{j} + dx_3 \underline{k}, \quad \nabla \underline{u} = \text{grad } \underline{u}, \quad \nabla = \frac{\partial}{\partial x_1} \underline{i} + \frac{\partial}{\partial x_2} \underline{j} + \frac{\partial}{\partial x_3} \underline{k}$$

Displacement Gradient: $\nabla \underline{u} = \frac{\partial u_1}{\partial x} \underline{i} \underline{i} + \frac{\partial u_2}{\partial x} \underline{i} \underline{j} + \frac{\partial u_3}{\partial x} \underline{i} \underline{k}$

$$+ \frac{\partial u_1}{\partial y} \underline{j} \underline{i} + \frac{\partial u_2}{\partial y} \underline{j} \underline{j} + \frac{\partial u_3}{\partial y} \underline{j} \underline{k}$$

$$+ \frac{\partial u_1}{\partial z} \underline{k} \underline{i} + \frac{\partial u_2}{\partial z} \underline{k} \underline{j} + \frac{\partial u_3}{\partial z} \underline{k} \underline{k}$$

1. Assume a point A is displaced to A'

$$\underline{r} \rightarrow \underline{r} + \underline{u}$$

2. A neighboring point B is displaced to B'

$$\underline{r} + d\underline{r} \rightarrow \underline{r} + d\underline{r} + \underline{u} + d\underline{u}$$

$$\underline{u} = u_1 \underline{i} + u_2 \underline{j} + u_3 \underline{k}, \quad \underline{r} = x_1 \underline{i} + x_2 \underline{j} + x_3 \underline{k}$$