

ASEN5012/MCEN5023 Mechanics of Solids - Fall 2007: Homework 4

Due at Start of Class on Friday, 28 September 2007

Problem 1. The state of stress at a point in a solid medium in terms of rectangular Cartesian system (x_1, x_2, x_3) is given by the following stress tensor matrix:

$$[\tau] = \begin{bmatrix} 5 & 2 & 0 \\ 2 & -3 & 0 \\ 0 & 0 & 4 \end{bmatrix}$$

1.1 Obtain the stress vectors, $\overset{\nu}{\mathbf{T}}$ for $\nu = \mathbf{i}, \mathbf{j}, \mathbf{k}$

{Hint : $\nu = \mathbf{i}$ meansthat $(\nu_1, \nu_2, \nu_3) = (1, 0, 0)$ }

1.2 When ν is given by $\nu = \frac{1}{\sqrt{2}}(\mathbf{i} + \mathbf{j} + 0\mathbf{k})$, compute $\overset{\nu}{\mathbf{T}}$.

Problem 2. The state of stress at a point in a solid medium in terms of rectangular Cartesian system (x_1, x_2, x_3) is given by the following stress tensor matrix:

$$[\tau] = \begin{bmatrix} 3 & 1 & 4 \\ 1 & 2 & -5 \\ 4 & -5 & 0 \end{bmatrix}$$

2.1 Determine $(\overset{\mathbf{n}}{\mathbf{T}}_m, m = 1, 2, 3.)$ on the plane whose unit normal vector \mathbf{n} is given by

$$\mathbf{n} = \frac{1}{\sqrt{3}}(\mathbf{i} + \mathbf{j} + \mathbf{k})$$

2.2 Determine the three principal normal stresses of the given stress tensor.

Problem 3. Suppose that the principal stresses for a stress tensor in the (x, y, z) -coordinate system

$$[\tau] = \begin{bmatrix} \tau_{11} & \tau_{12} & \tau_{13} \\ \tau_{12} & \tau_{22} & \tau_{23} \\ \tau_{13} & \tau_{23} & \tau_{33} \end{bmatrix}$$

are computed as $(\sigma_1, \sigma_2, \sigma_3)$.

Show that the principal stresses in the (x', y', z') -system are the same where the coordinate transformation relation is given by

$$\begin{Bmatrix} x' \\ y' \\ z' \end{Bmatrix} = [\mathbf{Q}] \begin{Bmatrix} x \\ y \\ z \end{Bmatrix}$$

Hint: eigenvalues under a congruent transformation are preserved!