Solutions for HW#3

1.1 $\nabla \underline{u} = \left(\frac{\partial}{\partial x}\underline{i} + \frac{\partial}{\partial y}\underline{j} + \frac{\partial}{\partial z}\underline{k}\right)\left(u_1\underline{i} + u_2\underline{j} + u_3\underline{k}\right)$

$$= \underline{i}\left(\frac{\partial u_1}{\partial x}\underline{i} + \frac{\partial u_2}{\partial x}\underline{j} + \frac{\partial u_3}{\partial x}\underline{k}\right)$$
$$+ \underline{j}\left(\frac{\partial u_1}{\partial y}\underline{i} + \frac{\partial u_2}{\partial y}\underline{j} + \frac{\partial u_3}{\partial y}\underline{k}\right) + \underline{k}\left(\frac{\partial u_1}{\partial z}\underline{i} + \frac{\partial u_2}{\partial z}\underline{j} + \frac{\partial u_3}{\partial z}\underline{k}\right)$$

$$= [\underline{i}\ \underline{j}\ \underline{k}]\begin{bmatrix} \frac{\partial u_1}{\partial x} & \frac{\partial u_2}{\partial x} & \frac{\partial u_3}{\partial x} \\ \frac{\partial u_1}{\partial y} & \frac{\partial u_2}{\partial y} & \frac{\partial u_3}{\partial y} \\ \frac{\partial u_1}{\partial z} & \frac{\partial u_2}{\partial z} & \frac{\partial u_3}{\partial z} \end{bmatrix}\begin{bmatrix} \underline{i} \\ \underline{j} \\ \underline{k} \end{bmatrix} = \underline{e}^T\left[\frac{\partial u}{\partial x}\right]\underline{e}$$

For $u_1 = a_0 + a_1 x + a_2 y + a_3 z$

$u_2 = b_0 + b_1 x + b_2 y + b_3 z$

$u_3 = c_0 + c_1 x + c_2 y + c_3 z$

$$\left[\frac{\partial u}{\partial x}\right] = \begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix}$$

1.2

$$[D] = [E]_{linear} = [\varepsilon]_{linear} = \frac{1}{2}\left(\left[\frac{\partial u}{\partial x}\right] + \left[\frac{\partial u}{\partial x}\right]^T\right)$$

$$= \begin{bmatrix} \varepsilon_{11} & \varepsilon_{12} & \varepsilon_{13} \\ \varepsilon_{12} & \varepsilon_{22} & \varepsilon_{23} \\ \varepsilon_{13} & \varepsilon_{23} & \varepsilon_{33} \end{bmatrix} = \begin{bmatrix} a_1 & \frac{1}{2}(a_2+b_1) & \frac{1}{2}(a_3+c_1) \\ \frac{1}{2}(a_2+b_1) & b_2 & \frac{1}{2}(b_3+c_2) \\ \frac{1}{2}(a_3+c_1) & \frac{1}{2}(b_3+c_2) & c_3 \end{bmatrix}$$