ASEN5012/MCEN5023 Mechanics of Solids - Fall 2007: Homework 3

Due at Start of Class on Friday, 21 September 2007

Problem 1. A solid finite element assumes that **u** can be modeled as

$$\begin{aligned}\mathbf{u} &= u_1\mathbf{i} + u_2\mathbf{j} + u_3\mathbf{k}\\ u_1 &= a_0 + a_1x + a_2y + a_3z\\ u_2 &= b_0 + b_1x + b_2y + b_3z\\ u_3 &= c_0 + c_1x + c_2y + c_3z\end{aligned}$$

1.1 Compute the gradient dyadic, $\nabla\mathbf{u}$ where ∇ is given by $\nabla = \frac{\partial}{\partial x}\mathbf{i} + \frac{\partial}{\partial y}\mathbf{j} + \frac{\partial}{\partial z}\mathbf{k}$

1.2 Extract the linear strain components from $\nabla\mathbf{u}$ as discussed in the class.

1.3 Determine the coefficients $(a_i, b_i, c_i, i = 1, 2, 3)$ such that you have only nonzero axial strains, i.e., $(\gamma_{ij} = 0)$.

1.4 Can you obtain a unique set of $(a_i, b_i, c_i, i = 1, 2, 3)$ such that $(\epsilon_{ii} = 0, i = 1, 2, 3)$?

1.5 Find conditions under which all six of the strains are zero.

Problem 2. As discussed in the class, when **x** is the new position vector of a point after deformations from the initial position **X**, we have the following relations:

$$\begin{aligned}\mathbf{x} &= \mathbf{X} + \mathbf{u}, \quad \mathbf{X} = X\,\mathbf{i} + Y\,\mathbf{j} + Z\,\mathbf{k}\\ d\mathbf{x} &= \mathbf{F}\cdot d\mathbf{X}\\ \mathbf{F} &= \frac{d\mathbf{X}}{d\mathbf{X}} = \mathbf{e}^T[F]\mathbf{e}, \quad F_{ij} = \frac{\partial x_i}{\partial X_j}, \quad \mathbf{e}^T = \langle \mathbf{i} \quad \mathbf{j} \quad \mathbf{k} \rangle\end{aligned}$$

2.1 For displacement field

$$\begin{aligned}u_1 &= a_0 + ax - bz\\ u_2 &= b_0 - ax + cz\\ u_3 &= c_0 + bx - cy\end{aligned}$$

Compute [F] and show $det|F| > 0$.

2.2 Extract the strains and spin components and explain what strain conditions and/or spin conditions the assumed displacements physically model.

Problem 3 is described in the next page.

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