Solutions for Homework #2 (Fall 2007, MCEN5023/ASEN5012)

1. $\underline{u} = u_1 \underline{i} + u_2 \underline{j} + u_3 \underline{k}$, $\underline{v} = v_1 \underline{i} + v_2 \underline{j} + v_3 \underline{k}$

$$\underline{u} - \underline{v} = (u_1 - v_1)\underline{i} + (u_2 - v_2)\underline{j} + (u_3 - v_3)\underline{k}$$
$$\underline{u} + \underline{v} = (u_1 + v_1)\underline{i} + (u_2 + v_2)\underline{j} + (u_3 + v_3)\underline{k}$$
$$|\underline{u} - \underline{v}|^2 = (\underline{u} - \underline{v}) \cdot (\underline{u} - \underline{v}) = (u_1 - v_1)^2 + (u_2 - v_2)^2 + (u_3 - v_3)^2$$
$$|\underline{u} + \underline{v}|^2 = (\underline{u} + \underline{v}) \cdot (\underline{u} + \underline{v})^2 = (u_1 + v_1)^2 + (u_2 + v_2)^2 + (u_3 + v_3)^2$$
$$|\underline{u}|^2 = \underline{u} \cdot \underline{u} = u_1^2 + u_2^2 + u_3^2, \quad |\underline{v}|^2 = \underline{v} \cdot \underline{v} = v_1^2 + v_2^2 + v_3^2$$

$$|\underline{u} - \underline{v}|^2 + |\underline{u} + \underline{v}|^2 = [(u_1 - v_1)^2 + (u_1 + v_1)^2] + [(u_2 - v_2)^2 + (u_2 + v_2)^2] + [(u_3 - v_3)^2 + (u_3 + v_3)^2]$$
$$= 2\left([u_1^2 + v_1^2] + [u_2^2 + v_2^2] + [u_3^2 + v_3^2] \right)$$
$$= 2\left([u_1^2 + u_2^2 + u_3^2] + [v_1^2 + v_2^2 + v_3^2] \right)$$
$$= 2\left(|\underline{u}|^2 + |\underline{v}|^2 \right)$$

2. $\underline{a} = a_m \underline{e}_m = \underline{e}^T [a]$, $\underline{e}^T = [\underline{e}_1 \ \underline{e}_2 \ \underline{e}_3]$, $[a]^T = [a_1 \ a_2 \ a_3]$

$$\underline{A} = A_{ij} \underline{e}_i \otimes \underline{e}_j = \underline{e}_n A_{np} \underline{e}_p = \underline{e}^T [A] \underline{e}, \quad [A] = \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix}$$

$$\underline{a} \times \underline{A} = (a_1 \underline{e}_1 + a_2 \underline{e}_2 + a_3 \underline{e}_3) \times (\underline{e}_1 \ \underline{e}_2 \ \underline{e}_3)[A]\underline{e}$$
$$= \left([-a_2 e_3 + a_3 e_2] \ [a_1 \underline{e}_3 - a_3 \underline{e}_1] \ [-a_1 \underline{e}_2 + a_2 \underline{e}_1] \right)[A]\underline{e}$$
$$= [\underline{e}_1 \ \underline{e}_2 \ \underline{e}_3] \underbrace{\begin{bmatrix} 0 & -a_3 & a_2 \\ a_3 & 0 & -a_1 \\ -a_2 & a_1 & 0 \end{bmatrix}}_{[\tilde{a}]} [A] \underline{e} = \underline{e}^T [\tilde{a}][A]\underline{e}$$

$$= a_m \underline{e}_m \times \underline{e}_n A_{np} \underline{e}_p$$
$$= \underline{e}_\ell \varepsilon_{mn\ell} a_m A_{np} \underline{e}_p$$

given in the handout ↓

$$\underline{A} \times \underline{a} = \underline{e}^T [A][\tilde{a}]\underline{e}$$
$$= \underline{e}_n A_{np} u_m \varepsilon_{pm\ell} \underline{e}_\ell$$
$$= \underline{e}^T [A][\tilde{a}]\underline{e}$$