## ASEN5012/MCEN5023 Mechanics of Solids - Fall 2007: Homework 2

*Due at Start of Class on Friday, 14 September 2007*

**Problem 1.** Show that for two arbitrary vectors, $(\mathbf{u}, \mathbf{v})$, the following holds:

$$|\mathbf{u} - \mathbf{v}|^2 + |\mathbf{u} + \mathbf{v}|^2 = 2(|\mathbf{u}|^2 + |\mathbf{v}|^2)$$

**Problem 2.** Given $(\mathbf{A}, \mathbf{a}, \mathbf{b})$ where $\mathbf{A}$ is a second-order tesnor, and $(\mathbf{a}, \mathbf{b})$ are vectors, perform the following operations in terms of the resulting components.

2.1 $\mathbf{a} \times \mathbf{A}$

2.2 $\mathbf{A} \times \mathbf{a}$

2.3 $(\mathbf{a} \times \mathbf{A})\mathbf{b}$

2.4 $\mathbf{a} \cdot \mathbf{A}\mathbf{b}$

**Problem 3.** Given two coordinate transformations as

$$\begin{Bmatrix} x' \\ y' \\ z' \end{Bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ -1 & 0 & 0 \end{bmatrix} \begin{Bmatrix} x \\ y \\ z \end{Bmatrix}, \qquad \begin{Bmatrix} x'' \\ y'' \\ z'' \end{Bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} x' \\ y' \\ z' \end{Bmatrix}$$

obtain $\mathbf{x}'' = \mathbf{A}\mathbf{x}$ and also $\mathbf{x} = \mathbf{B}\mathbf{x}''$, and offer your geometrical interpretations of the two transformations.

**Problem 4.** Using the rotational dyadic operator given by

$$\mathbf{Q}(\mathbf{n}, \theta) = \mathbf{n} \otimes \mathbf{n} + (\mathbf{I} - \mathbf{n} \otimes \mathbf{n}) \cos\theta + (\mathbf{n} \times \mathbf{I}) \sin\theta$$

Prove the following Euler angle transformation

$$\mathbf{Q} = \mathbf{Q}_3(\mathbf{k}'', \psi) \cdot \mathbf{Q}_2(\mathbf{i}', \theta) \cdot \mathbf{Q}_1(\mathbf{k}, \phi) = \mathbf{Q}_1(\mathbf{k}, \phi) \cdot \mathbf{Q}_2(\mathbf{i}, \theta) \cdot \mathbf{Q}_3(\mathbf{k}, \psi)$$

and obtain the resulting $(3 \times 3)$ Euler angle transforamtion matrix.

*Hint:* Use the identities

$$\mathbf{Q}(\mathbf{n}', \beta) = \mathbf{Q}(\mathbf{m}, \alpha) \cdot \mathbf{Q}(\mathbf{n}, \beta) \cdot \mathbf{Q}^T(\mathbf{m}, \alpha), \quad \mathbf{Q}^T(\mathbf{m}, \alpha) = \mathbf{Q}(\mathbf{m}, -\alpha)$$

$$\mathbf{Q}(\mathbf{i}', \theta) \cdot \mathbf{Q}(\mathbf{k}, \phi) = \mathbf{Q}(\mathbf{k}, \phi) \cdot \mathbf{Q}(\mathbf{i}, \theta)$$