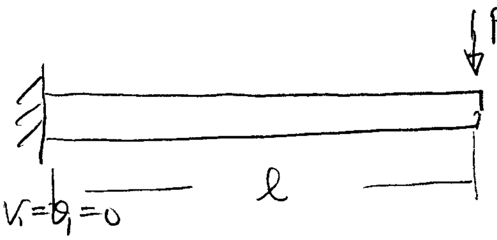
Solution of HW10: Summary

Given

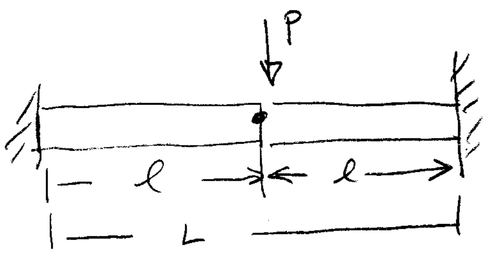
$$K_E = \frac{EI}{\ell^3}\begin{bmatrix} 12 & 6\ell & -12 & 6\ell \\ 6\ell & 4\ell^2 & -6\ell & 2\ell^2 \\ -12 & -6\ell & 12 & -6\ell \\ 6\ell & 2\ell^2 & -6\ell & 4\ell^2 \end{bmatrix}$$



$$\frac{EI}{\ell^3}\begin{bmatrix} 12 & -6\ell \\ -6\ell & 4\ell^2 \end{bmatrix}\begin{pmatrix} V_2 \\ \theta_2 \end{pmatrix} = \begin{pmatrix} -P \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} V_2 \\ \theta_2 \end{pmatrix} = \frac{\ell^3}{EI}\cdot\frac{1}{12\ell^2}\begin{bmatrix} 4\ell^2 & 6\ell \\ 6\ell & 12 \end{bmatrix}\begin{bmatrix} -P \\ 0 \end{bmatrix} = \begin{bmatrix} -\frac{P\ell^3}{3EI} \\ -\frac{P\ell^2}{2EI} \end{bmatrix}$$

$$\begin{pmatrix} Q_1 \\ M_1 \\ Q_2 \\ M_2 \end{pmatrix} = \frac{EI}{\ell^3}\begin{bmatrix} 12 & 6\ell & -12 & 6\ell \\ 6\ell & 4\ell^2 & -6\ell & 2\ell^2 \\ -12 & -6\ell & 12 & -6\ell \\ 6\ell & 2\ell^2 & -6\ell & 4\ell^2 \end{bmatrix}\begin{bmatrix} 0 \\ 0 \\ -\frac{1}{3} \\ -\frac{1}{2\ell} \end{bmatrix}\frac{P\ell^3}{EI} = \begin{bmatrix} 1 \\ \ell \\ -1 \\ 0 \end{bmatrix} P$$



After assembling & eliminating the boundary conditions, we obtain

$$\frac{EI}{\ell^3}\begin{bmatrix} 24 & 0 \\ 0 & 8\ell^2 \end{bmatrix}\begin{bmatrix} V_2 \\ \theta_2 \end{bmatrix} = \begin{bmatrix} -P \\ 0 \end{bmatrix}$$

$$V_2 = -\frac{P\ell^3}{24EI} = -\frac{PL^3}{192EI} \qquad \theta_2 = 0$$

$$\begin{pmatrix} Q_1 \\ M_1 \\ Q_2 \\ M_2 \end{pmatrix} = \frac{EI}{\ell^3}\begin{bmatrix} 12 & 6\ell & -12 & 6\ell \\ 6\ell & 4\ell^2 & -6\ell & 2\ell^2 \\ -12 & -6\ell & 12 & -6\ell \\ 6\ell & 2\ell^2 & -6\ell & 4\ell^2 \end{bmatrix}\cdot\frac{P\ell^3}{24EI}\begin{bmatrix} 0 \\ 0 \\ -1 \\ 0 \end{bmatrix} = \begin{bmatrix} \frac{P}{2} \\ \frac{PL}{8} \\ -\frac{P}{2} \\ \frac{PL}{8} \end{bmatrix}$$

Beam FEM provides the exact solutions as compared to E-B theory.