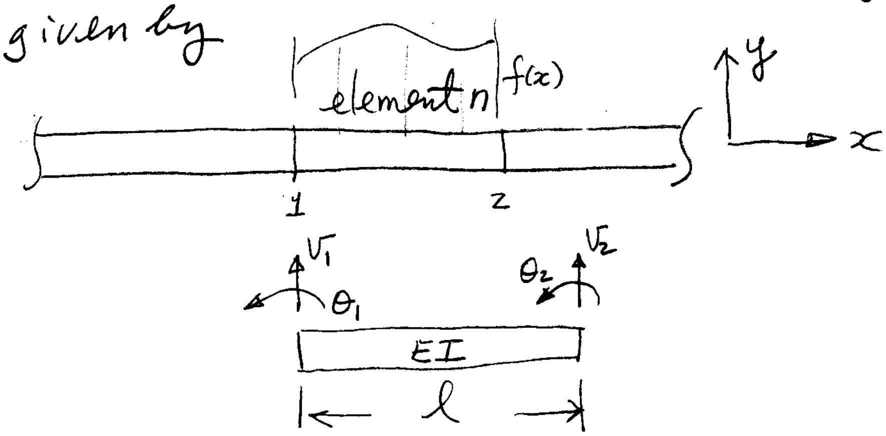
Problem: Application of Variational Formulation

The cubic approximation of beam bending for an element is given by



$$\delta U = \delta(v_1\ \theta_1\ v_2\ \theta_2)[K_e]\begin{Bmatrix} v_1 \\ \theta_1 \\ v_2 \\ \theta_2 \end{Bmatrix} = \delta(v_1\ \theta_1\ v_2\ \theta_2)\begin{bmatrix} Q_1 \\ M_1 \\ Q_2 \\ M_2 \end{bmatrix} \qquad (H10.1)$$

$$\begin{bmatrix} Q_1 \\ M_1 \\ Q_2 \\ M_2 \end{bmatrix} \text{(elemental force \& moment)} = \frac{EI}{l^3}\begin{bmatrix} 12 & 6l & -12 & 6l \\ 6l & 4l^2 & -6l & 2l^2 \\ -12 & -6l & 12 & -6l \\ 6l & 2l^2 & -6l & 4l^2 \end{bmatrix}\begin{Bmatrix} v_1 \\ \theta_1 \\ v_2 \\ \theta_2 \end{Bmatrix} \qquad (H10.2)$$

where Q_i & M_i are elemental shear force and bending moment, respectively.

For the lateral load $f(x)$, we have for a symbolic representation:

$$\delta W = \int_0^l f(x)\cdot\delta v = \delta(v_1\ \theta_1\ v_2\ \theta_2)\begin{bmatrix} f_{v_1} \\ f_{\theta_1} \\ f_{v_2} \\ f_{\theta_2} \end{bmatrix} \qquad (H10.3)$$

$$\begin{bmatrix} f_{v_1} \\ f_{\theta_1} \\ f_{v_2} \\ f_{\theta_2} \end{bmatrix} = \int_{-1}^{1}\begin{bmatrix} H_1(\xi) \\ H_2(\xi) \\ H_3(\xi) \\ H_4(\xi) \end{bmatrix} f(\xi)\left(\frac{l}{2}d\xi\right),$$ where $H_i(\xi)$ are given in Eq. (8.6.2) of Lecture 08.