Problem 1.

$$M_{xx} = \int_{-h/2}^{h/2} -\frac{E}{1-\nu^2} z^2 \left[ \frac{\partial^2 w}{\partial x^2} + \nu \frac{\partial^2 w}{\partial y^2} \right] dz = -D \left( \frac{\partial^2 w}{\partial x^2} + \nu \frac{\partial^2 w}{\partial y^2} \right) \quad (1)$$

$$M_{yy} = -D \left( \frac{\partial^2 w}{\partial y^2} + \nu \frac{\partial^2 w}{\partial x^2} \right) \quad (2)$$

$$M_{xy} = -D(1-\nu) \frac{\partial^2 w}{\partial x \partial y} \quad (3)$$

$$Q_{xz} = \frac{\partial M_{xx}}{\partial x} + \frac{\partial M_{xy}}{\partial y} = -D \frac{\partial}{\partial x} \left[ \frac{\partial^2 w}{\partial x^2} + \underbrace{\nu \frac{\partial^2 w}{\partial y^2} \right] - D(1-\nu)}_{=\phi} \frac{\partial^3 w}{\partial x \partial y^2}$$

$$= -D \left[ \frac{\partial^3 w}{\partial x^3} + \frac{\partial^3 w}{\partial x \partial y^2} \right] \quad (4)$$

$$Q_{yz} = \frac{\partial M_{xy}}{\partial x} + \frac{\partial M_{yy}}{\partial y} = -D(1-\nu) \frac{\partial^3 w}{\partial x^2 \partial y} - D \frac{\partial}{\partial y} \left[ \frac{\partial^2 w}{\partial y^2} + \nu \frac{\partial^2 w}{\partial x^2} \right]$$

$$= -D \left[ \frac{\partial^3 w}{\partial x^2 \partial y} + \frac{\partial^3 w}{\partial y^3} \right] \quad (5)$$

Problem 2. Using the results obtained in Problem 1, $\delta U$ becomes

$$\delta U = \int_y \int_x \left\{ M_{xx} \frac{\partial^2 \delta w}{\partial x^2} + M_{yy} \frac{\partial^2 \delta w}{\partial y^2} + 2 M_{xy} \frac{\partial^2 \delta w}{\partial x \partial y} \right\} dx\, dy$$

$$= \int_y M_{xx} \frac{\partial \delta w}{\partial x} dy + \int_x M_{yy} \frac{\partial \delta w}{\partial y} dx + \int_y M_{xy} \frac{\partial \delta w}{\partial y} dy + \int_x M_{xy} \frac{\partial \delta w}{\partial x} dx$$

$$- \int_y \int_x \left\{ \frac{\partial M_{xx}}{\partial x} \frac{\partial \delta w}{\partial x} + \frac{\partial M_{yy}}{\partial y} \frac{\partial \delta w}{\partial y} + \frac{\partial M_{xy}}{\partial x} \frac{\partial \delta w}{\partial y} + \frac{\partial M_{xy}}{\partial y} \frac{\partial \delta w}{\partial x} \right\} dx\, dy$$

$$= \int_y M_{xx} \frac{\partial \delta w}{\partial x} dy + \int_x M_{yy} \frac{\partial \delta w}{\partial y} dx + \int_y M_{xy} \frac{\partial \delta w}{\partial y} dy + \int_x M_{xy} \frac{\partial \delta w}{\partial x} dx$$