

## ASEN5012/MCEN5023 Mechanics of Solids - Fall 2007: Homework 9

*Due at Start of Class on Friday, 09 November 2007*

### Problem . Variational Derivation of Thin Plate Theory and the Associated Boundary Conditions

*Background Materials: Derivation of Thin Plate Theory from Equilibrium Equations*

1. Equilibrium equations:

$$\begin{aligned} \frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \sigma_{xy}}{\partial y} + \frac{\partial \sigma_{xz}}{\partial z} &= 0, & F_x &= 0 \\ \frac{\partial \sigma_{xy}}{\partial x} + \frac{\partial \sigma_{yy}}{\partial y} + \frac{\partial \sigma_{yz}}{\partial z} &= 0, & F_y &= 0 \\ \frac{\partial \sigma_{xz}}{\partial x} + \frac{\partial \sigma_{yz}}{\partial y} + F_z &= 0, & \frac{\partial \sigma_{zz}}{\partial z} &= 0 \end{aligned} \quad (1)$$

2. Kinematical or Strain-displacement relations:

$$\begin{Bmatrix} \epsilon_{xx} \\ \epsilon_{yy} \\ \epsilon_{xy} \end{Bmatrix} = -z \begin{Bmatrix} \kappa_{xx} \\ \kappa_{yy} \\ \kappa_{xy} \end{Bmatrix} = -z \begin{Bmatrix} \frac{\partial^2 w}{\partial x^2} \\ \frac{\partial^2 w}{\partial y^2} \\ \frac{\partial^2 w}{\partial x \partial y} \end{Bmatrix}, \quad -h/2 \leq z \leq h/2 \quad (2)$$

where  $w$  is the vertical deflection and  $h$  is the plate thickness. Note with the above strain displacement relations, we have  $\epsilon_{xz} = \epsilon_{yz} = 0$ . However, the shear stresses  $\sigma_{xz}$  and  $\sigma_{yz}$  are not zero. This is akin to the Euler-Bernoulli beam theory.

3. Constitutive equations

$$\begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{xy} \end{Bmatrix} = \frac{E}{1-\nu^2} \begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & (1-\nu) \end{bmatrix} \begin{Bmatrix} \epsilon_{xx} \\ \epsilon_{yy} \\ \epsilon_{xy} \end{Bmatrix} \quad (3)$$

4. Moment ( $M_{xx}$ ,  $M_{yy}$ ,  $M_{xy}$ ) and Shear forces ( $Q_{xz}$ ,  $Q_{yz}$ ) unit plate side length:

$$\begin{Bmatrix} M_{xx} \\ M_{yy} \\ M_{xy} \end{Bmatrix} = \int_{-h/2}^{h/2} z \begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{xy} \end{Bmatrix} dz, \quad \begin{Bmatrix} Q_{xz} \\ Q_{yz} \end{Bmatrix} = \int_{-h/2}^{h/2} \begin{Bmatrix} \sigma_{xz} \\ \sigma_{yz} \end{Bmatrix} dz \quad (4)$$