Problem 1: with $\sigma_{zz} = \sigma_{zx} = \sigma_{zy} = \delta W = F_z = \overset{\nu}{T}_z = \overset{\nu *}{T}_z = 0$, we have

$$-\int_V \left\{ \left(\frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \sigma_{xy}}{\partial y} + F_x \right) \delta u + \left(\frac{\partial \sigma_{xy}}{\partial x} + \frac{\partial \sigma_{yy}}{\partial y} + F_y \right) \delta v \right\} dV$$

$$+ \int_{S_2} \left\{ \left(\overset{\nu}{T}_x - \overset{\nu *}{T}_x \right) \delta u + \left(\overset{\nu}{T}_y - \overset{\nu *}{T}_y \right) \delta v \right\} dS = 0 \qquad (1)$$

Where

$$\overset{\nu}{T}_x = \sigma_{xx} \nu_1 + \sigma_{xy} \nu_2$$
$$\overset{\nu}{T}_y = \sigma_{xy} \nu_1 + \sigma_{yy} \nu_2 \qquad (2)$$

(see Lecture04, page 01-1 & page 11)

First, we note

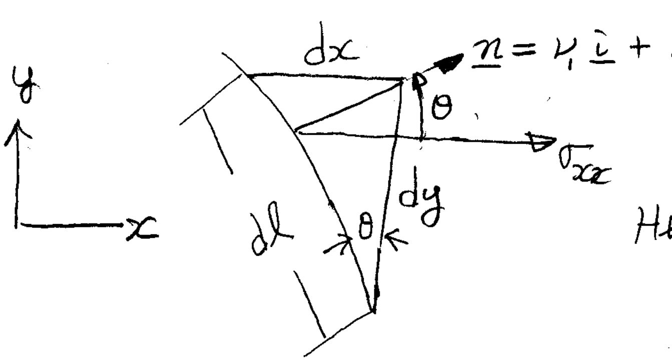
$$\frac{\partial \sigma_{xx}}{\partial x} \delta u = \frac{\partial (\sigma_{xx} \delta u)}{\partial x} - \sigma_{xx} \frac{\partial \delta u}{\partial x} \qquad (3)$$

Integration of (3) yields:

$$\int_V \frac{\partial \sigma_{xx}}{\partial x} \delta u \, dV = \int_V \frac{\partial (\sigma_{xx} \delta u)}{\partial x} dV - \int \sigma_{xx} \frac{\partial \delta u}{\partial x} dV \qquad (4)$$

Let $dV = dx\,dy\,dz$ so that (3) becomes

$$\int_V \frac{\partial \sigma_{xx}}{\partial x} \delta u \, dV = \int_z \int_y \sigma_{xx} \delta u \, dy\,dz - \int \sigma_{xx} \frac{\partial \delta u}{\partial x} \qquad (5)$$



$\cos\theta = \cos(\underline{n}, \underline{i}) = \nu_1$

Hence, $dy\,dz = \nu_1 \, dl\,dz = \nu_1 \, dS \qquad (6)$

$dx\,dz = \nu_2 \, dl\,dz = \nu_2 \, dS$