Solutions

ASEN5012/MCEN5023 Mechanics of Solids - Fall 2007: Exam 1

Problem 1. Given (**A**, **a**, **b**) where **A** is a second-order tesnor, and (**a**, **b**) are vectors, perform the following operations. State also whether your answer is a scalar, vector or tensor.

Help: In carying out your solution, assume the following.

$$\mathbf{a} = a_1\mathbf{i} + a_2\mathbf{j} + a_3\mathbf{k} = \langle \mathbf{i} \quad \mathbf{j} \quad \mathbf{k} \rangle \begin{Bmatrix} a_1 \\ a_2 \\ a_3 \end{Bmatrix} = \mathbf{e}^T \begin{Bmatrix} a_1 \\ a_2 \\ a_3 \end{Bmatrix} = \langle a_1 \quad a_2 \quad a_3 \rangle\, \mathbf{e}$$

$$\mathbf{e} = \begin{Bmatrix} \mathbf{i} \\ \mathbf{j} \\ \mathbf{k} \end{Bmatrix}, \quad \mathbf{e}^T = \langle \mathbf{i} \quad \mathbf{j} \quad \mathbf{k} \rangle \tag{1}$$

$$\mathbf{A} = \mathbf{e}^T\,[\,A\,]\,\mathbf{e}, \quad [\,A\,] = \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix}, \quad \mathbf{Ab} = \mathbf{e}^T[\,A\,]\begin{Bmatrix} b_1 \\ b_2 \\ b_3 \end{Bmatrix}$$

and the vector **b** is similarly expressed as the vector **a**

1.1 $\mathbf{a}\cdot\mathbf{Ab}$ $= (a_1 \; a_2 \; a_3)\begin{bmatrix} \underline{i} \\ \underline{j} \\ \underline{k} \end{bmatrix} \cdot \left\{ (\underline{i} \; \underline{j} \; \underline{k})\,[A]\begin{pmatrix} \underline{i} \\ \underline{j} \\ \underline{k} \end{pmatrix} \right\} (\underline{i} \; \underline{j} \; \underline{k})\begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$

$= (a_1 \; a_2 \; a_3)\,[A]\begin{Bmatrix} b_1 \\ b_2 \\ b_3 \end{Bmatrix}$ scalar

1.2 $\mathbf{a}\times\mathbf{A}$ $= (a_1 \; a_2 \; a_3)\begin{pmatrix} \underline{i} \\ \underline{j} \\ \underline{k} \end{pmatrix} \times (\underline{i} \; \underline{j} \; \underline{k})\,[A]\begin{pmatrix} \underline{i} \\ \underline{j} \\ \underline{k} \end{pmatrix}$

$= (a_1 \; a_2 \; a_3)\begin{bmatrix} 0 & \underline{k} & -\underline{j} \\ -\underline{k} & 0 & \underline{i} \\ \underline{j} & -\underline{i} & 0 \end{bmatrix}[A]\begin{pmatrix} \underline{i} \\ \underline{j} \\ \underline{k} \end{pmatrix}$

$= (\underline{i} \; \underline{j} \; \underline{k})\begin{bmatrix} 0 & -a_3 & a_2 \\ a_3 & 0 & -a_1 \\ -a_2 & a_1 & 0 \end{bmatrix}[A]\begin{pmatrix} \underline{i} \\ \underline{j} \\ \underline{k} \end{pmatrix} = \underline{e}^T[\tilde{a}][A]\,\underline{e}$

tensor.