

ASEN 5010 - Mid-Term Quiz III - 13 April 2006

Your name: _____

Problem 1. Rigid Body in a circular orbit (40 points):

We have learned in Section 6.10 that the equations of motion for rigid body in a circular orbit in terms of its body-fixed coordinates, $(\mathbf{b}_1, \mathbf{b}_2, \mathbf{b}_3)$, is given by

$$\begin{aligned} J_1 \dot{\omega}_1 - (J_2 - J_3) \omega_2 \omega_3 &= -3n^2 (J_2 - J_3) \theta_1 \\ J_2 \dot{\omega}_2 - (J_3 - J_1) \omega_3 \omega_1 &= 3n^2 (J_3 - J_1) \theta_2 \\ J_3 \dot{\omega}_3 - (J_1 - J_2) \omega_1 \omega_2 &= 0 \end{aligned} \quad (1)$$

where $(\theta_1, \theta_2, \theta_3)$ are small angle perturbations from the roll (\mathbf{a}_1), pitch (\mathbf{a}_2), and yaw (\mathbf{a}_3) angles, respectively; and, n is the orbital rate.

(1.1) (20 points) Using small angle approximations

$$\boldsymbol{\omega} = \omega_1 \mathbf{b}_1 + \omega_2 \mathbf{b}_2 + \omega_3 \mathbf{b}_3 \approx (\dot{\theta}_1 - n\theta_3) \mathbf{b}_1 + (\dot{\theta}_2 - n) \mathbf{b}_2 + (\dot{\theta}_3 + n\theta_1) \mathbf{b}_3 \quad (2)$$

show that the linearized equations, that is, retaining only linear terms, are given by

$$\begin{aligned} J_1 \ddot{\theta}_1 - n(J_1 - J_2 + J_3) \dot{\theta}_3 + 4n^2 (J_2 - J_3) \theta_1 &= 0 \\ J_3 \ddot{\theta}_3 + n(J_1 - J_2 + J_3) \dot{\theta}_1 + n^2 (J_2 - J_1) \theta_3 &= 0 \\ J_2 \ddot{\theta}_2 + 3n^2 (J_1 - J_3) \theta_2 &= 0 \end{aligned} \quad (3)$$

(1.2) (10 points) Obtain the characteristic equation of the linearized equations of motion.

(Hint: Note that θ_2 -equation is not coupled and hence you may treat it as if it is a single degrees of freedom equation. For equation of motion $A\ddot{x} + B\dot{x} + Cx = 0$, its characteristic equation is obtained by $|s^2 A + sB + C| = 0$)

(1.3) (10 points) Using the stability criterion

$$a_0 s^4 + a_2 s^2 + a_4 = 0, \quad \{a_0 > 0, a_2 > 0, a_4 > 0, \text{ and } (a_2^2 - 4a_0 a_4) > 0\} \quad \text{for stability,} \quad (4)$$

obtain the stability conditions in terms of the principal moments of inertia and the orbital rate, n .

Problem 2. Kinematics in Rotational Dynamics (30 points):

(2.1) (10 points) The orientation of a rigid satellite is to be described by a sequence of $C_3(\theta_3) \leftarrow C_2(\theta_2) \leftarrow C_1(\theta_1)$. Express the three rotational matrices and obtain an approximate linearized compounded transformation matrix when $\{\theta_j \ll 1, j = 1, 2, 3\}$, that is, $\mathbf{b} = \mathbf{C}\mathbf{a}$.

Use $\mathbf{b}^T = [\mathbf{b}_1 \ \mathbf{b}_2 \ \mathbf{b}_3]$ as the unit vectors attached on the satellite after the three rotations, and $\mathbf{a}^T = [\mathbf{a}_1 \ \mathbf{a}_2 \ \mathbf{a}_3]$ as the reference orientation.

(2.2) (10 points) If the orbital rate of the satellite is given by $-\mathbf{n}\mathbf{a}_2$, express the the absolute angular velocity of the satellite after undergoing the small rotations described in Problem (2.1)?

(2.3) (10 points) The \mathbf{b} -frame rotational inertia matrix is given by

$$\mathbf{J} = \begin{bmatrix} J_1 & 0 & 0 \\ 0 & J_2 & 0 \\ 0 & 0 & J_3 \end{bmatrix} \quad (5)$$

Transform the above rotational inertia matrix into the corresponding one in the \mathbf{a} -frame, using the small angle approximation.

Problem 3. Nutation Dynamics of a Satellite (30 points):

The governing equations for studying the active nutation control may be given by

$$\begin{aligned}\dot{\omega}_1 &= \lambda\omega_2 + \mu, & \mu &= M_1/J \\ \dot{\omega}_2 &= -\lambda\omega_1 \\ J_3\dot{\omega}_3 &= 0 & \Rightarrow & \omega_3 = n \\ \lambda &= n(J - J_3)/J > 0, & J_1 &= J_2 = J > J_3\end{aligned}\tag{6}$$

where a constant control torque is applied only to \mathbf{b}_1 .

(3.1) (15 points) Show that, if $\omega_1(0) = \omega_2(0) = 0$, then the solution is given by

$$\begin{aligned}\omega_1(t) &= \frac{\mu}{\lambda} \sin \lambda t \\ \omega_2(t) &= -\frac{\mu}{\lambda} (1 - \cos \lambda t)\end{aligned}\tag{7}$$

(Hint: Differentiate the second of equation(6) once and eliminate $\dot{\omega}_1$ by using the first of equation(6). The resulting differentialequation should read in the form of: $\ddot{x} + k^2x = f$.)

(3.2) (15 points) Compute the maximum nutation angle. The nutation angle (θ) is defined by

$$\tan \theta = \frac{\sqrt{H_1^2 + H_2^2}}{H_3}, \quad \mathbf{H} = H_1\mathbf{b}_1 + H_2\mathbf{b}_2 + H_3\mathbf{b}_3, \quad H_i = J_i\omega_i\tag{8}$$