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The Patch Test

FEM as a Form of Rayleigh-Ritz

For FEM to fit within the classical Ritz method:

(1) Trial functions must be **admissible in the sense of providing**

**continuity (intra- and inter-element)
completeness**

as required by variational index of master variables

(2) Functional integration must be **exact and provide **rank sufficiency****

(3) Essential BCs satisfied **strongly (pointwise)**

If all of these conditions are met, "convergence in energy" as the mesh is refined follows from Ritz theory

Variational Crimes

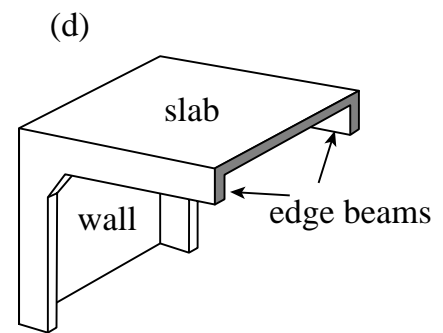
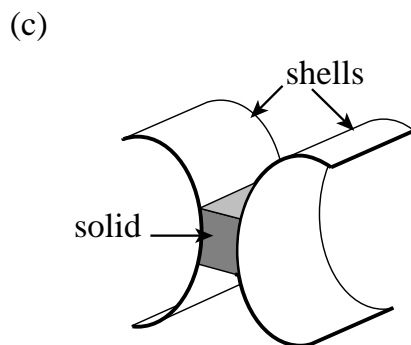
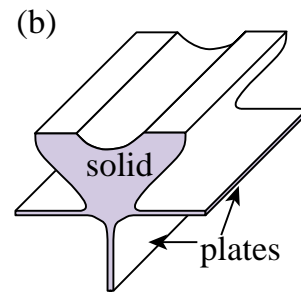
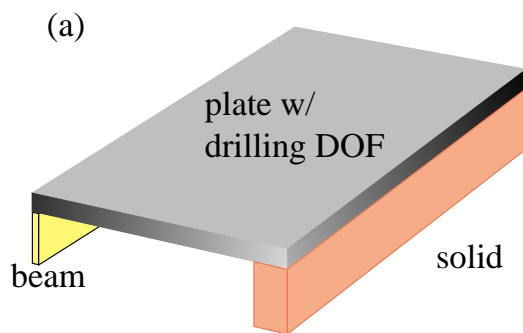
Advanced FEM

In FEM, **as actually used**, those requirements may be at least partly violated. Such violations were labeled by Strang **variational crimes**. Here is a list of possible offenses.

- (1) **Lack of completeness**
- (2) **Lack of invariance:
element response depends on observer frame**
- (3) **Rank deficiency**
- (4) **Nonconformity: violation of interelement
continuity as required by variational index**
- (5) **Inexact, but rank sufficient, numerical integration**
- (6) **Inexact treatment of curved boundaries and essential BC**

(1) and (2) are **capital crimes** (3) a **misdemeanor**
(4)-(5) could be in fact beneficial, (6) generally unimportant

Heterogeneous Patches in 3D (mesh not pictured)



There are Many Versions of the Physical Patch Test

Multielement Tests

Homogeneous (Irons et al)

Displacement Patch Test

Force (aka Stress) Patch Test

Mixed Patch Tests

**in this
Chapter**

Heterogeneous

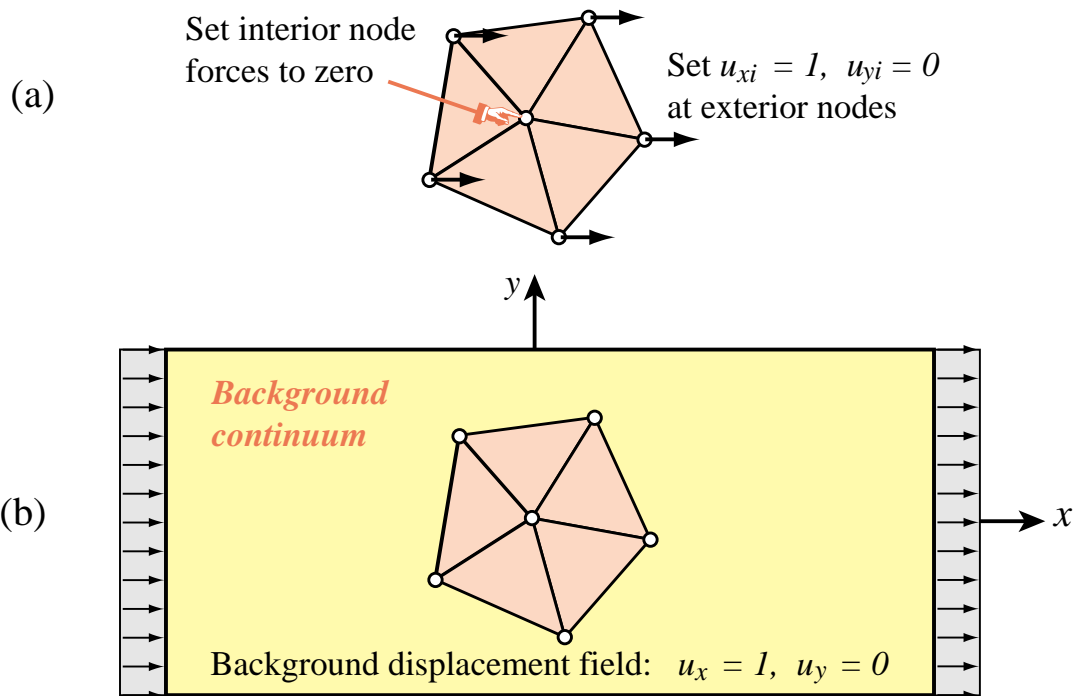
One Element Tests

Individual Element Test (IET: Bergan-Hanssen)

Single Element Test (SET: Taylor et al)

Can be done numerically or symbolically

Multielement Displacement Patch Test (DPT) for a Rigid Body Mode (RBM): Translation in x Direction



Multielement Displacement Patch Test (DPT) for a Constant Strain State $e_{xx} = 1$ in x Direction

