Homework Exercises for Chapter 17

Solutions

EXERCISE 17.1 Guess $N^e_{11} = c_{11}$(eq. of face124)(eq. of face234)(eq. of face134)(eq. of face123) $= c_{11}\zeta_1\zeta_2\zeta_3\zeta_4$. The condition $N^e_{11}(\frac{1}{4}, \frac{1}{4}, \frac{1}{4}, \frac{1}{4}) = 1$ yields $c_{11} = 256$. Hence

$$N^e_{11} = 256\zeta_1\zeta_2\zeta_3\zeta_4. \tag{E17.1}$$

EXERCISE 17.2 Four corner nodes, plus twelve (2×6) edge (side) nodes, plus four face nodes, plus one center node. Total $4 + 12 + 4 + 1 = 21$. See Figure E17.1.

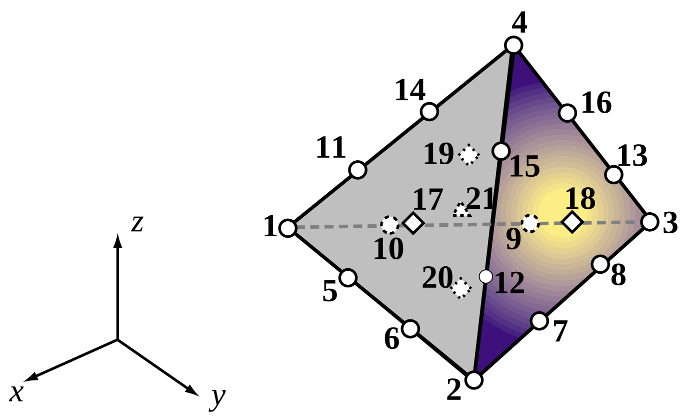

Figure E17.1. The 21-node cubic tetrahedron. For visualization help, face nodes (17,18,19,20) are marked with square symbols, while the centroid node (21) is marked with a triangular symbol.

EXERCISE 17.3 Since

$$\begin{aligned} s_x &= \frac{\partial(\zeta_2 - \zeta_1)}{\partial x} + \frac{\partial(\zeta_3 - \zeta_1)}{\partial x} + \frac{\partial(\zeta_4 - \zeta_1)}{\partial x} \\ &= \frac{\partial(\zeta_1 + \zeta_2 + \zeta_3 + \zeta_4)}{\partial x} - 4\frac{\partial \zeta_1}{\partial x} = -4\frac{\partial \zeta_1}{\partial x} \end{aligned} \tag{E17.2}$$

because $\zeta_1 + \zeta_2 + \zeta_3 + \zeta_4 = 1$. Therefore

$$\frac{\partial \zeta_1}{\partial x} = -\tfrac{1}{4}s_x \tag{E17.3}$$

EXERCISE 17.4

$$p^e = \int_{V^e} \mathbf{N}^T \mathbf{b}\, dV^e \qquad \mathbf{b} = \begin{bmatrix} 0 \\ 0 \\ \rho_z \end{bmatrix} \tag{E17.4}$$

Thus

$$p_{zi} = \int_{V^e} N_i \rho_z \, dV^e = \rho_z \int_{V^e} N_i \, dV^e \tag{E17.5}$$

Components p_{xi} and p_{yi} vanish.

For $i = 1, 2, 3, 4$ the integral is

$$\int_{V^e} (2\zeta_i^2 - \zeta_i)\, dV^e = \left(\frac{2}{10} - \frac{1}{4}\right)\mathcal{V} = -\frac{1}{20}\mathcal{V}, \tag{E17.6}$$

otherwise for $i = 5, 6, \ldots 10$

$$\int_{V^e} 4\zeta_i \zeta_j \, dV^e = \frac{4}{20}\mathcal{V} = \frac{1}{5}\mathcal{V}. \tag{E17.7}$$

It follows that the z-components are given by

$$\mathbf{f}_z^e = \frac{1}{20}\rho_z \mathcal{V} \begin{bmatrix} -1 \\ -1 \\ -1 \\ -1 \\ 4 \\ 4 \\ 4 \\ 4 \\ 4 \\ 4 \end{bmatrix} \tag{E17.8}$$

As a check, the sum of these components is $(4 \times 6 - 4)\rho_z \mathcal{V}/20 = \rho_z \mathcal{V}$, as it should be. An interesting point is that the corners get forces going *against* the body force direction.

The use of the four-point tetrahedron quadrature rule leads to the same result.