Homework Exercises for Chapter 2

Solutions

EXERCISE 2.1 Written as vectors:

$$\textbf{grad} = \begin{bmatrix} \partial/\partial x_1 \\ \partial/\partial x_2 \\ \partial/\partial x_3 \end{bmatrix}, \qquad \textbf{div} = [\, \partial/\partial x_1 \quad \partial/\partial x_2 \quad \partial/\partial x_3 \,] \tag{E2.4}$$

hence $\textbf{div}^T = \textbf{grad}$ and $\textbf{grad}^T = \textbf{div}$.

EXERCISE 2.2

(a) Yes. Elimination of e and N gives

$$\frac{d}{dx}\left(EA\frac{du}{dx}\right) + q = 0, \tag{E2.5}$$

Thus the correspondence is $k \to EA$, $q \to -s$ (or, if you prefer, $k \to -EA$ and $q \to s$); u is the same, $x_1 \equiv x$ and ∂ becomes the ordinary differential d. If E is constant, this could be further transformed to $(d/dx)(A\,du/dx) = -q/E$, although this does not buy much.

(b) The diagram is shown in Figure E2.3.

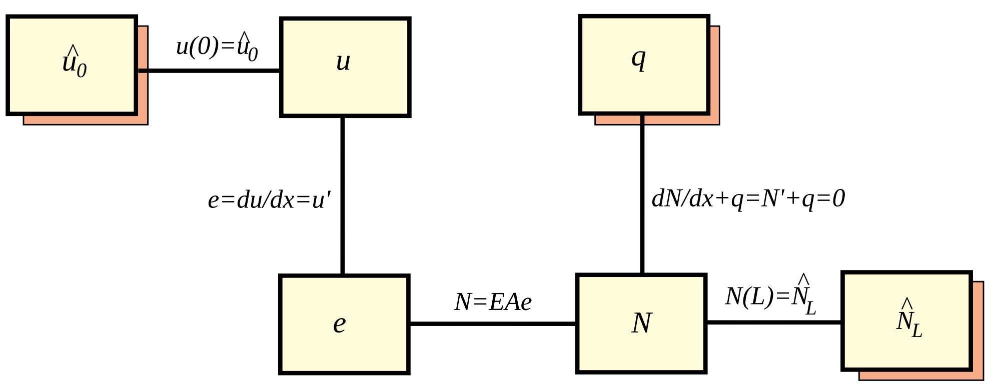

Figure E2.3. Tonti diagram for bar problem. Here prime denotes derivative respect to x.

EXERCISE 2.3

(a) S_T consists of AD and BC, and S_q consists of AB and CD. Mathematically: $S_T : AD \cup BC$, $S_q : AB \cup CD$.

Because of geometry and B.C.s, $x_1 = 0$ is a symmetry plane. That is, $T(x_1, x_2) = T(-x_1, x_2)$. Hence the normal temperature gradient $g_n = \partial T/\partial n = \partial T/\partial x_1$ vanishes there and so does the flux. Consequently one can reduce the problem to one half by placing the boundary condition $\hat{q} = 0$ on $x_1 = 0$.

(b) Elimination of **g** and **q** yields $\nabla \cdot (k\nabla T) = 0$. If k is constant it can be moved out as a factor: $k\nabla \cdot \nabla T = k\nabla^2 T = 0$. Hence $\nabla^2 T = 0$ and the temperature distribution satisfies Laplace's equation.

(c) It is easily checked that $T = 100x_2/3$ satisfies the Laplace's equation (any linear function would) as well as the temperature boundary conditions on S_T. It does not satisfy, however, the zero-flux conditions on S_q. For example on AB $q_n = -(k/\sqrt{2})(100/3) \neq 0$. Therefore that guess temperature distribution is not the exact solution of the boundary value problem.

EXERCISE 2.4 There are several solutions. Two of them are shown in Figure E2.3 and E2.4. The second one is more in the spirit of Tonti's dual diagrams for the potentials.

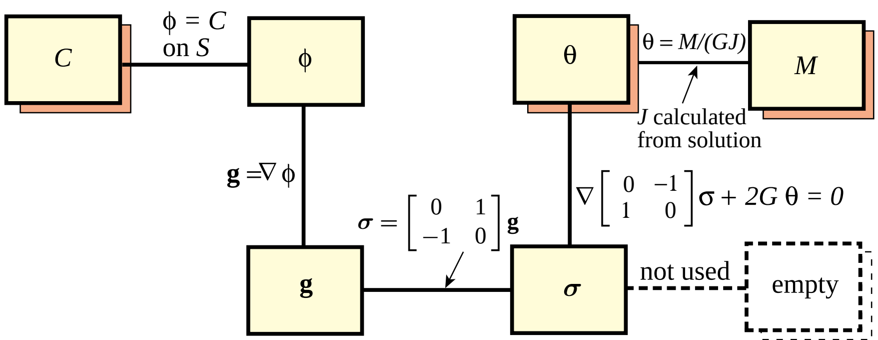

Figure E2.3. Tonti diagram for St.-Venant torsion problem.

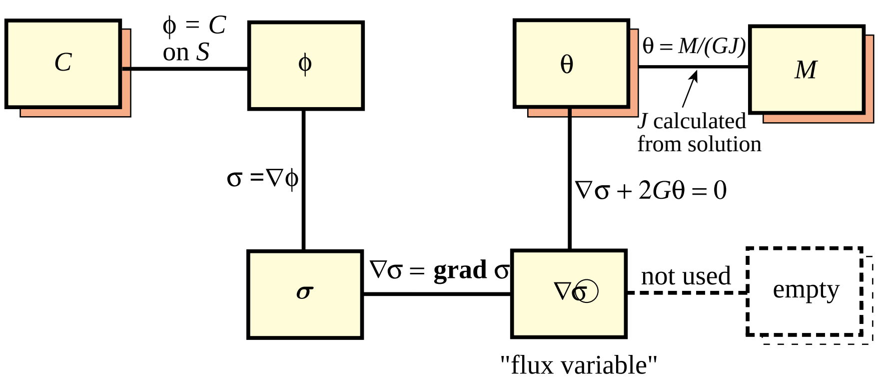

Figure E2.4. Alternative Tonti diagram for St.-Venant torsion problem.

EXERCISE 2.5

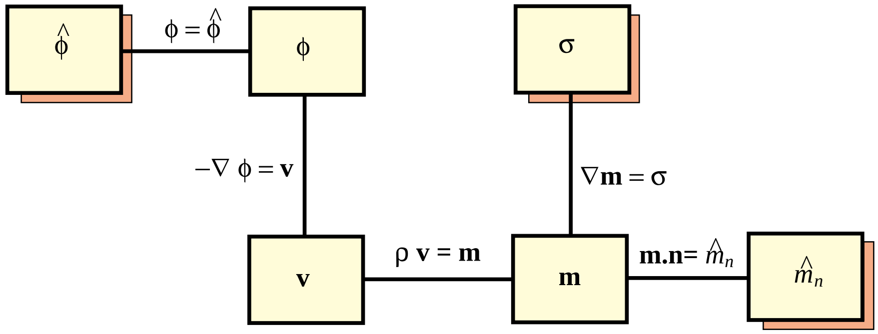

Figure E2.5. Tonti diagram for potential flow problem.

EXERCISE 2.6

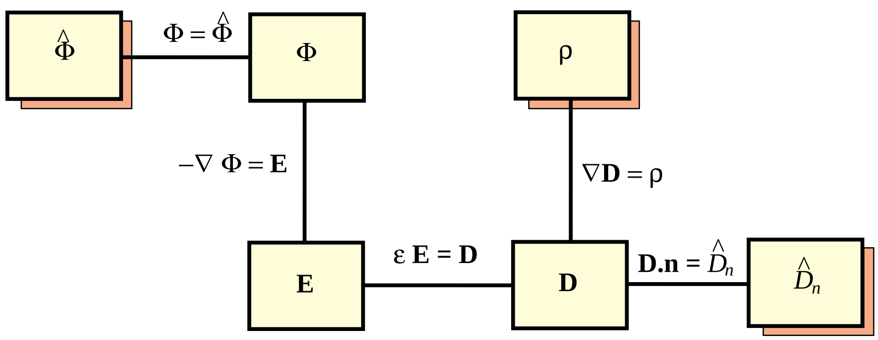

Figure E2.3. Tonti diagram for electrostatics problem.