

1

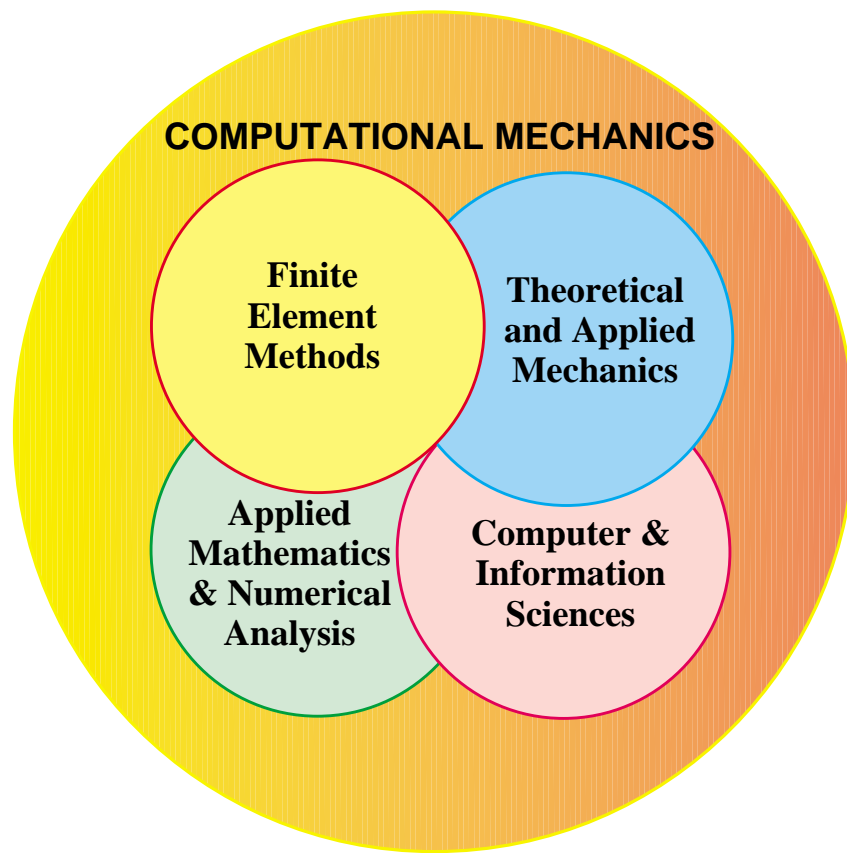
Overview

Course Contents

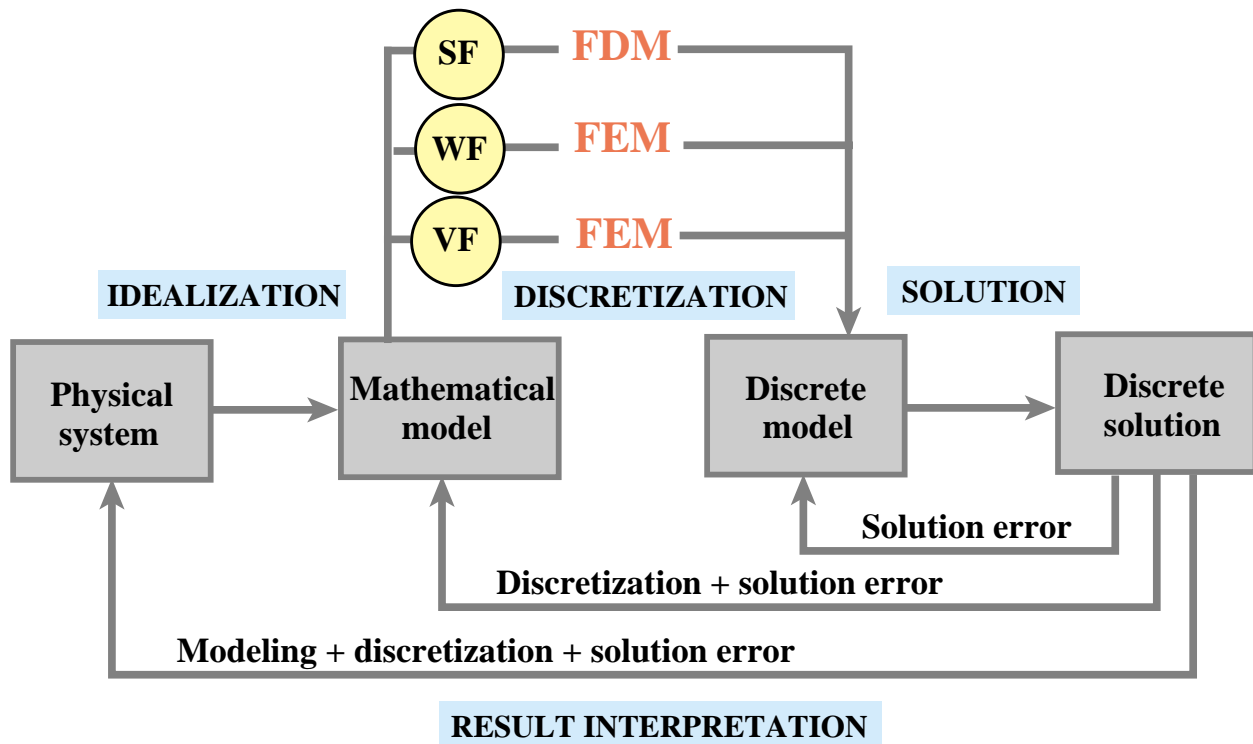
The course embodies five parts:

- I. Review of Variational Methods**
- II. Three-Dimensional Finite Elements**
- III. High Performance Element Formulations**
- IV. Beams, Plates and Shells**
- V. Miscellaneous and Special Project Topics**

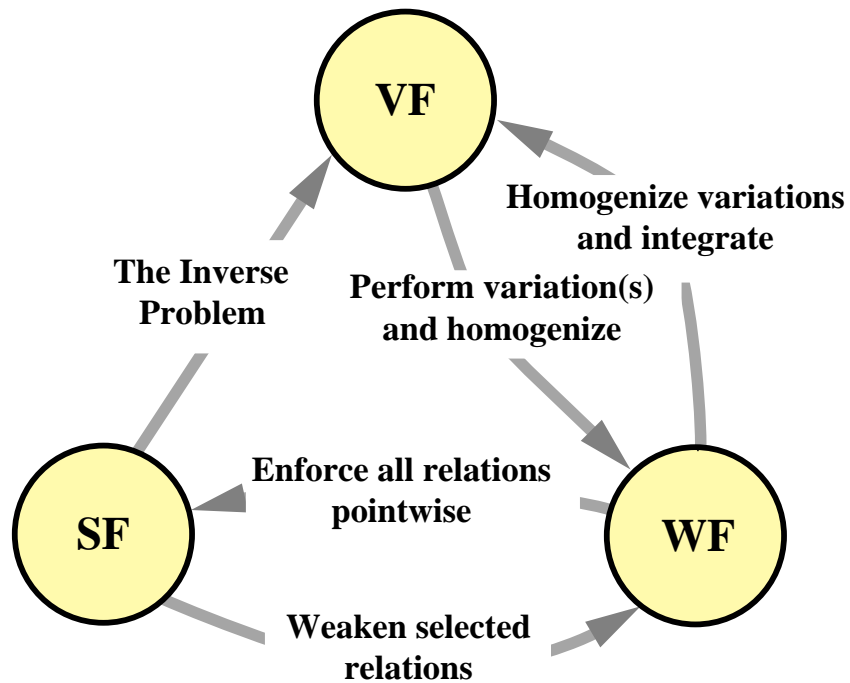
Computational Mechanics is a Fancy Pizza



The Computer Simulation Process



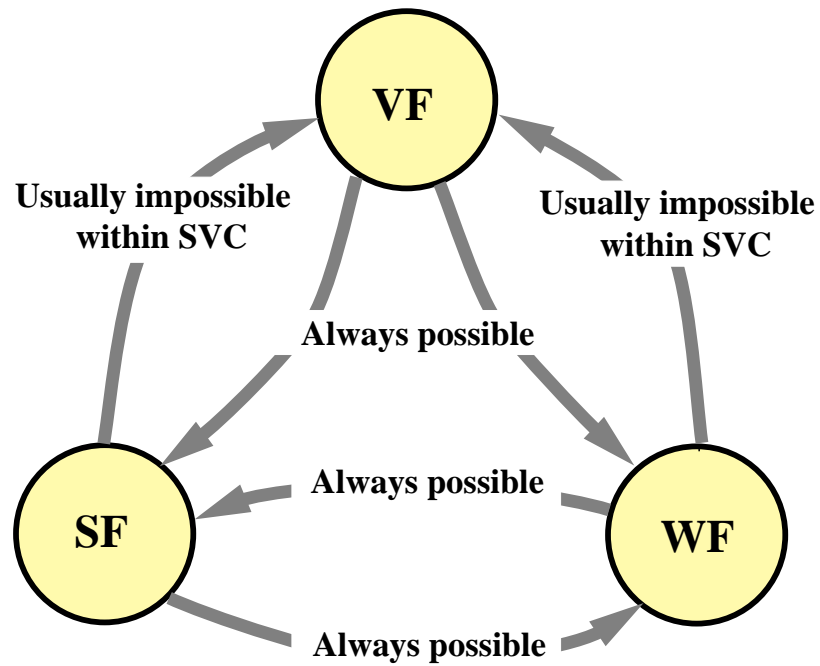
Strong, Weak and Variational Forms of the Mathematical Model



Strong, Weak and Variational Forms (Cont'd)

- SF Strong Form.** Presented as a system of *ordinary or partial differential equations* in space and/or time, complemented by appropriate boundary conditions. Occasionally this form may reduce to algebraic equations.
- WF Weak Form.** Presented as a *weighted integral statement* that "relaxes" the point-by-point enforcement of the SF into a domain-averaging statement.
- VF Variational Form.** Presented as a *functional* whose stationary conditions generate the WF and SF.
- Variational Calculus** is a set of rules and techniques to pass from one form to another.

Feasibility of Transformations Between Forms

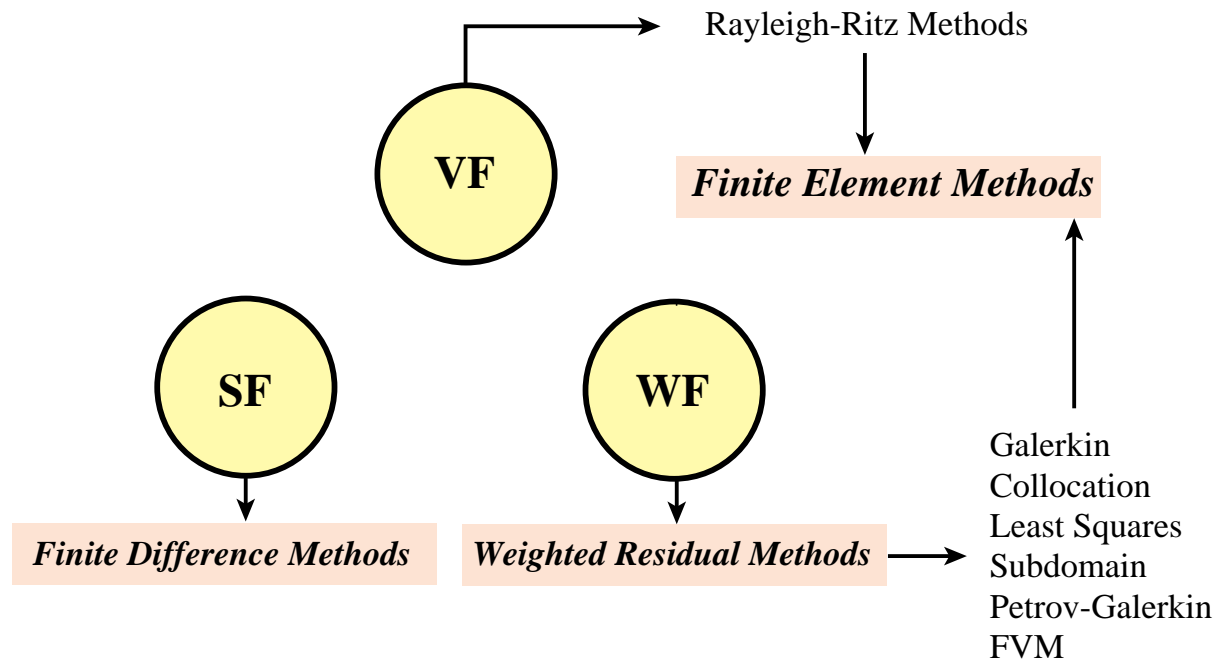


Why Weak and Variational Forms?

The following reasons may be offered:

1. Unification and coordinate-invariance properties
2. Provide basis for discrete methods of approximation, notably FEM
3. Characterize overall quantities of interest to engineers
4. Clarify treatment of boundary & interface conditions
5. Permit unified mathematical treatment of questions of existence, stability, error, etc. Also provide guidelines as to how to achieve desirable behavior in discrete models

Mathematical Model Forms as Sources of Discretization Methods



Discretization Methods

Universal Methods

Finite Difference Methods

Weighted Residual Methods

Galerkin

Least Squares

Collocation

Subdomain

Finite Element Methods

Special Methods

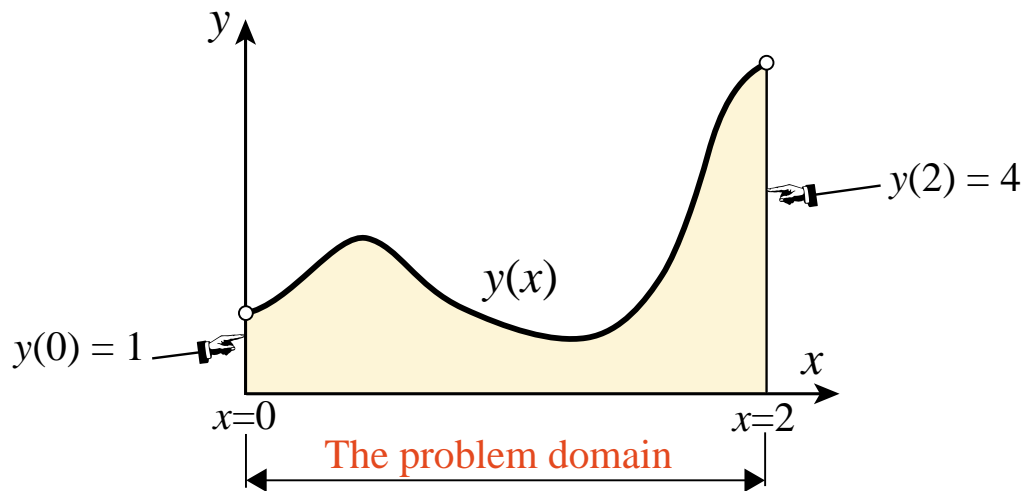
Rayleigh-Ritz (classical)

Boundary Element Methods

Fluid Volume Methods

Semi-Analytical Methods

A Simple Example



A Simple Example (Cont'd)

VF

Functional

$$J[y] = \int_0^2 \left[\frac{1}{2}(y')^2 - \frac{1}{2}y^2 + 2y \right] dx$$

$$y(0) = 1 \quad y(2) = 4$$

SF

$$y'' = y + 2 \quad \text{in } 0 \leq x \leq 2$$

$$y(0) = 1 \quad y(2) = 4$$

Boundary value problem

WF

$$\int_0^2 r(x) \delta v(x) dx + r_0 \delta v_0 + r_2 \delta v_2 = 0$$

Variational statement

$$\int_0^2 r(x) w(x) dx + r_0 w_0 + r_2 w_2 = 0$$

Weighted residual form