Recent Developments in External Acoustic Models for Flexible Underwater Vehicle Dynamics **

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Essentials in Developing New Technologies or Advancing Existing Technologies

1. Articulate Technology Needs (e.g., Floating City);

2. Identify Present and Future Expected Customers;

3. Survey Pillar Technology Base and Match with Existing Ones;

4. Identify New Technology Needs, and Existing Technologies that need to be significantly improved;

University Roles for New Technology Development

They say students do not learn much in university; yet it is where a student experiences profound transformation in terms of maturity and career readiness.

1. **Base Research (or Basic Research) and Professional Manpower**;

2. **However, university is a self-surviving entity; it wishes that industry would depend on university for technology innovation, expert professional production, even patents to utilize**…

3. **Within university community, each campus must compete for talents (faculty and students recruits) and resources (funds)**.
My Personal Preference/Goals as a Professor

1. Educate both undergraduate and graduate students;

1. Conduct Basic Research, in particular:
   1. Analysis methods developments – FEM, transient algorithms, etc;
   2. New modeling theory – acoustic-structure interaction models (Today’s talk);
   3. Multiphysics simulation, system identification, etc.

2. Conduct Engineering Analysis such as:
   1. Space rendezvous dynamics and control;
   2. Underwater vehicle dynamics;
   3. Shell structures and membranous reflectors
   4. Containment vessels health monitoring
   5. MEMS resonators and Gyroscope performance
   6. Health monitoring of structures
   7. Contact-impact problems
   ..
   ..
My Past Activities

Past Activities:

- Transient analysis methods (direct time integration)
- Helicopter and general aviation crashworthiness
- Shell finite elements
- Computational methods for fluid-structure interactions
- Large space structures and control
- Space construction and space rendezvous dynamics
- Partitioned analysis methods for coupled-field problems
- Multi-body dynamics and space assembly
- Structural system identification and damage detection
- Contact-impact mechanics
Current Activities:

- Multi-physics dynamics and computational methods
- Health monitoring of structural systems
- MEMS Gyroscopes for space applications
- Membranous structures for solar sails and reflectors
- Hilbert-Huang transform for nonlinear dynamics
- Loose coupling of tightly coupled multi-physics problems via partitioned analysis procedures.
Background in Today’s Talk

1. Today’s talk is considered as a new theory as opposed to improving existing theories;

2. It was a test case in that I have consistently advocated that more KAIST engineering theses should take on developing new methodologies, new theories and new modeling instead of improving existing engineering tools;

3. It has been carried out at KAIST during the past four years by Dr. Moonseok Lee as his PhD thesis;

4. I am hoping that you would get a glimpse of what it takes to take on research that requires risk and open-ended time commitment. . .
My Objectives as WCU-OSE Faculty Member

• First, work with students!

• Assist OSE faculty to promote ‘basic research’ emphasis;

• By engaging in the above two activities, help them write fundamental papers that will be cited by others in years to come (out of many papers, I am sure).

• In other words, I am intensely academically oriented. But then beware of ‘많은 물엔 고기가 크게 자라지 않습니다.’

• I am an American and we Americans believe in pursuit of happiness! Hence, I will enjoy doing the above activities.
Classification of Fluid-Structure Interactions

- Waves (e.g., hurricanes) bombarding the structures
- Liquid Sloshing in Container Compartment
- Intense Pressure Originating from Tsunami and Explosions – Today’s Topics
Waves (e.g., hurricanes & North Sea) bombarding the structures (Height: 10-25 m and Period: 10 - 15 seconds, speed of wave ~ 1-5 /s)
Liquid Sloshing in Container Compartment
Today’s Topics: Intense Pressure Originating from Tsunami and/or Explosions (wave speed ~ 1450 m/s)

Also applicable to Surface Ships and Floating Large Structures
Why simulation and why not real experiments?

• A bicycle can be tested not only for parts but the whole bicycle;

• A car can be crash-tested, although expensive. Even for cars, simulation using component test data saves cost and expedite from design to market turnaround.

• To test an airplane for its crash landing safety? No way! Too expensive and time consuming.

• How about satellites? How about a submarine? How about a large floating structures such as oil/gas drilling platforms?
Acoustic-Structure Interaction Problems in Ocean Engineering

Sonar Detection Problem:
Send pressure waves and measure the scattered (return) pressure. By analyzing the scattered pressure, one can identify the location, the shape and the speed by which the ship or submarine is moving.

Dynamic Response of Surface and Underwater Ships:
Vibration and vulnerability of vehicles subjected to intense pressure waves (Professor Young Shin is an expert in this field). All future ocean infrasystems must withstand hostile attacks as well as tsunami shock waves.
Aren’t There Theories Available to Model Dynamics of Acoustic-Structure Interactions?

At present, we use two separate modeling techniques:

a. Various acoustic-rigid scattering model for underwater detection

b. Acoustic-flexible interaction model for structural response
Objective of Present Research:

Develop one model that is applicable both to acoustic scattering and flexible structural dynamics response.
Related Literature:

Baker and Copson (1949): My favorite theoretical text

G. Carrier (1951): Cylindrical shells excited by incident waves

Junger and Feit (1950s): Sound scattering by thin elastic shells

Mindlin and Bleich (1953): Perhaps the first plane wave approximation

Huang (1969): A successful solution of retarded potential for a sphere

Geers (1978): The seminal paper on doubly asymptotic approximation (DAA)

Felippa (1980): A systematic derivation for early-time response

Geers and Felippa (1983): Application of DAA to vibration problems

Astley et al (1998): Wave envelope elements that treats the scattering in addition to radiation in applying retarded potential.
Two Views on External Acoustic-Structure Interactions

• Solve the wave equation with approximate infinite radiation boundary conditions;

• Approximate Kirchhoff’s retarded potential that properly incorporates the infinite radiation boundary conditions.

• There are, of course, a host of approaches that try to exploit the desirable features of the two views.
Motivations for Present Study

When the primary interest is to obtain structural response arising from external acoustic-structural interactions, existing DAA (Doubly Asymptotic Approximation; Geers, 1978) models seem adequate;

However, when one is interested in the acoustic field, e.g., for identification of sound sources, existing DAAs are considered inadequate, primarily due to its inconsistent impulse response characteristics.

In addition, DAAs are obtained via an impedance matching procedure between early-time and late-time approximations. New approximations without resorting to impedance matching should be at least intellectually pleasing!
Motivations for Present Study - continued

For model determination through system identification, one assumes the system model in terms of discrete event equation as

- Internal dynamics: \( x^{(n+1)} = A \, x^n + B \, u^n \)
- Measure output: \( y^n = C \, x^n + D \, u^n \)

System identification is to obtain \((A, B, C, D)\) with given input \((u^n)\) and measured output \((y^n)\)

Kirchhoff’s retarded potential (as will be shown shortly) does not lend to this standard system identification model!

Hence, a high-fidelity ordinary differential acoustic model is needed.
(Why spherical waves?) - That is how waves propagates in homogeneous medium.

Isotropic Spherical Waves of Sound

For spherical coordinates, Poisson’s operator takes the form of

\[ \nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{2}{r} \frac{\partial}{\partial r} + \frac{1}{r^2} \frac{\partial^2}{\partial \theta^2} + \frac{\cot \theta}{r^2} \frac{\partial}{\partial \theta} + \frac{1}{r^2 \sin^2 \theta} \frac{\partial^2}{\partial \psi^2} \] (14)

where \( r \) is the radial coordinate, \( \theta \) and \( \psi \) are the two spherical angles.

For the case of isotropic wave propagation, the velocity potential depends only on \( r \), which reduces \( \nabla^2 \) to

\[ \nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{2}{r} \frac{\partial}{\partial r} \]

so the wave equation (8) becomes

\[ \frac{\partial^2 \phi}{\partial r^2} + \frac{2}{r} \frac{\partial \phi}{\partial r} = \frac{1}{c^2} \frac{\partial^2 \phi}{\partial t^2} \]

\[ \downarrow \]

\[ \frac{\partial^2}{\partial r^2} (r \phi) = \frac{1}{c^2} \frac{\partial^2}{\partial t^2} (r \phi) \] (This has the form of the equation for plane waves!)

whose solution is given by

\[ \phi = \frac{1}{r} \{ f(ct - r) + F(ct + r) \} \] (17)
Kirchhoff’s retarded potential formula

\[ 4\pi \delta(P, t) = \int \left\{ \frac{\rho}{R_{PQ}} \hat{u}(Q, t_R) - \frac{1}{R_{PQ}} \frac{\partial R_{PQ}}{\partial n} \left[ p(Q, t_R) + c^{-1} R_{PQ} \hat{p}(Q, t_R) \right] \right\} dS_Q \]

\[ t_R = t - \frac{R_{PQ}}{c} \]

\[ \varepsilon = \begin{cases} 
0 & \text{Outside acoustic medium} \\
\frac{1}{2} & \text{On the surface} \\
1 & \text{Inside acoustic medium} 
\end{cases} \]

External acoustic problems

\[ 4\pi \delta(P, t) = \int \left\{ \frac{\rho}{R_{PQ}} \hat{u}(Q, t_R) - \frac{1}{R_{PQ}} \frac{\partial R_{PQ}}{\partial n} \left[ p(Q, t_R) + c^{-1} R_{PQ} \hat{p}(Q, t_R) \right] \right\} dS_Q \]

External value

Boundary values

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Laplace Transform of the Retarded Potential

The retarded potential equation derived in (42) is extremely difficult to evaluate in time domain. Although a limited solution exists for external wave problems such as spheres and infinite cylinder, it is not tractable for general structural shapes. Hence, various approximations have been proposed (see, e.g., Geers[1-3] and Felippa[4-5]). The underlying observations for approximating the pressure field approximations have been to obtain to two limiting temporal cases, transient early time approximations when the waves start to initiate propagation and late-time approximations after waves reach their steady motions. Mathematically, these approximations to a varying degree utilize the following properties:

\[
\begin{align*}
\text{Initial value theorem:} \quad & \lim_{s \to \infty} s F(s) = \lim_{t \to 0} f(t) \\
\text{Final value theorem:} \quad & \lim_{s \to 0} s F(s) = \lim_{t \to \infty} f(t)
\end{align*}
\] (43)

Before we Laplace-transform the retarded velocity potential equation derived in equation (42), it is convenient to express the third term in terms of the normal velocity on the surface, viz.,

\[
4\pi \epsilon \phi(P, t) = \int_S \left\{ -[\phi] \frac{\partial r}{\partial n} \frac{1}{r^2} - \frac{1}{c r} \frac{\partial r}{\partial n} \left[ \frac{\partial \phi}{\partial t} \right] + \frac{1}{r} [u_n] \right\} dS \quad \text{since} \quad [u_n] = -\left[ \frac{\partial \phi}{\partial n} \right] \] (44)

With \( \Phi = \int_0^\infty e^{-st} \phi(x, y, z, t) \, dt \) and \( U_n(s) = \int_0^\infty e^{-st} u_n(x, y, z, t) \, dt \), the preceding equation is
transformed into

\[
4\pi \varepsilon \Phi(P, s) = \int_S \left\{ -\Phi(s) \frac{\partial r}{\partial n} \frac{1}{r^2} - \frac{1}{cr} \frac{\partial r}{\partial n} \{s \Phi(s) - \phi(0)\} + \frac{1}{r} U_n(s) \right\} e^{-rs/c} dS
\]

(45)

In the above equation, the delayed exponential \(e^{-sr/c}\) comes from the following Laplace transform property:

\[
\int_0^\infty e^{-st}[\phi]dt = \int_0^\infty e^{-st} \phi(x, y, z, t - r/c)dt = \int_0^\infty e^{-s(t'+r/c)} \phi(x, y, z, t')dt'
\]

\[
= e^{-sr/c} \int_0^\infty e^{-st'} \phi(x, y, z, t')dt' = e^{-sr/c} \Phi(x, y, z, s) = e^{-sr/c} \Phi(s)
\]

(46)

A similar transformation can be carried out for \([u_n(t)]\).

The solution of the pressure distributions and the normal velocity on the structural surface employing either the time-domain retarded potential(44) or the Laplace-transformed equation(45) presents considerable challenge for general structural geometries. This has led many investigators to seek approximations of them. This is discussed below.
Early and late time ranges

- Acoustic impedance can be characterized by $\lambda_{st} / \lambda_{ac}$.
  - Characteristic structural wavelength for the surface motion $\lambda_{st}$
  - Acoustic wavelength $\lambda_{ac} = 2\pi c / \omega$

Modal impedance for a sphere at $n=1$

For a sphere:

- $\lambda_{st} = 2\pi a / (n + 1)$
- $\lambda_{st} / \lambda_{ac} = \omega a / (n + 1)c$
Approximate models

(adopted in existing models)

- Plane wave approximation (PWA)
  - Early time approximation \((s \rightarrow \infty)\)
  - High frequency approximation of an infinite cylindrical shell proposed by Mindlin and Bleich.
    \[
    \frac{-s}{p_n} = \rho c \frac{H_n(s)}{H'_n(s)} \frac{s}{u_n(s)} \quad (s \rightarrow \infty) \quad \frac{-s}{p_n} = \rho c u_n(s)
    \]

- Virtual mass approximation (VMA)
  - Late time approximation \((s \rightarrow 0)\)

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Acoustic – Structural Interaction

SDAC, NoVic, KAIST
Doubly asymptotic approximation (1978)

- **Doubly asymptotic approximation (DAA)**
  - Geers proposed the boundary based approximations for shock analysis of a submerged structure.
  - It combines the **plane wave** and **virtual mass approximations**.

- **Inaccuracy at intermediate frequency**
  - Impedance of spherical wave \( n=2 \)
Approximation via filtering concept

- Laplace transformed Kirchhoff’s retarded potential formula

\[ 4 \pi \varepsilon \bar{\rho}(P) = \iint_S e^{-R_{PQ}/c} \left\{ -\bar{p} \frac{\partial R_{PQ}}{\partial n} \frac{1}{R_{PQ}^2} - s\bar{p} \frac{\partial R_{PQ}}{\partial n} \frac{1}{R_{PQ}^2} + su \frac{1}{R_{PQ}} \right\} ds \]

- Approximation of delayed operator

Late time Unstable Filter \( e^{-sr/c} \approx 1 - sr/c, \ |s| \ll 1 \)

Early time stable Filter \( e^{-sr/c} \approx \frac{1}{1 + sr/c}, \ |s| \gg 1 \)

All frequency Filter \( e^{-sr/c} \approx 1, \ 0 \leq |s| \leq \infty \)

Asymptotic approximation is required for all frequency range.
Theoretical Foundation for Present Approximations

Series expansion of the retarded operator, $e^{-Rs/c}$, in

$$\int_0^\infty e^{-st}g(Q, t_r)dt = e^{-sR/c}\overline{g}(Q, s), \quad \overline{g}(Q, s) = \int_0^\infty e^{-st}g(Q, t)dt$$

where $t_r$ denotes the retarded time $t - \frac{R}{c}$

has been responsible for unstable form unless a modification is incorporated.

Possible stabilization: Employ advanced potential!
The Role of Advanced Potential in Computational Context

Physically, while the retarded operator (RO) is to foresee the wave front, the advanced operator (AO) looks back the wave front.

Hence, computationally RO is a predictor while AO is a backward interpolator, hence a stable operator.

\[ \phi_a = \phi(Q, t_a) = \phi(Q, t + \frac{R}{c}), \quad \int_0^\infty e^{-st} \phi(Q, t_a) dt = e^{sR/c_\odot} \phi(Q, s) \]
Geometrical Interpretation of Retarded and Advanced Waves

Wave Front, Time = \( t \)

Advanced Wave Front, time = \( t + \frac{r}{c} \)

( It is a backward interpolation. Hence, generally stable to compute)

Retarded Wave Front, time = \( t - \frac{r}{c} \)

( It is a forward extrapolation. Hence, generally unstable to compute!)
Key Idea for Present Approximations

- Proposed modified potential: \[ \phi = \frac{1}{2} (1 - \chi) [\phi]_r + \frac{1}{2} (1 + \chi) [\phi]_a \]

- Retarded potential: \[ [\phi]_r = \phi(Q, t - R_{PQ} / c) \]

- Advanced potential: \[ [\phi]_a = \phi(Q, t + R_{PQ} / c) \]

- Laplace transformed present modified potential
  \[ \overline{\phi}_{\text{mod}} = \left\{ \frac{1}{2} (1 - \chi) e^{-\frac{sR_{PQ}}{c}} + \frac{1}{2} (1 + \chi) e^{\frac{sR_{PQ}}{c}} \right\} \overline{\phi}(Q, s) \]

- Resulting modified Kirchhoff’s formula
  \[ 4\pi \varepsilon \overline{p}(P) = \int_{S} \left\{ \frac{1 - \chi}{2} e^{-\frac{R_{PQ} s}{c}} + \frac{1 + \chi}{2} e^{\frac{R_{PQ} s}{c}} \right\} \left\{ -\overline{p} \frac{\partial R_{PQ}}{\partial n} \frac{1}{R_{PQ}^2} - s\overline{p} \frac{\partial R_{PQ}}{\partial n} \frac{1}{R_{PQ}} + su \frac{1}{R_{PQ}} \right\} ds \]
Proposed approximate model

- **Key Idea:**
  Expand the retarded and advanced operators such that no spatially divergent Rp-term with p>0 should result, viz..

\[
\frac{1 - \chi e^{-\mu}}{2} + \frac{1 + \chi}{2} e^\mu
\]

\[
= \frac{1 - \chi}{2} (1 - \mu + O(\mu^2)) + \frac{1 + \chi}{2} (1 + \mu + O(\mu^2))
\]

\[
\approx 1 + \chi \mu, \quad \mu = R_{\text{PQ}} s / c \quad R_{\text{PQ}} s / c < 1
\]

- **Observation:** $\chi = -1$ corresponds to the classical retarded potential formula which is an inherently unstable approximation.
Proposed approximate model

- Present second-order approximate model

\[ \chi B s^2 \bar{p} + c(1 + \chi) B_1 s \bar{p} + c^2 B_2 \bar{p} = \rho c \chi A s^2 \bar{u} + \rho c^2 A_1 s \bar{u} \]

\[ Bq(P,t) = \int_s \frac{\partial R_{pq}}{\partial n} q(Q,t) dS_Q, \quad B_1 q(P,t) = \int_s \frac{1}{R_{pq}} \frac{\partial R_{pq}}{\partial n} q(Q,t) dS_Q \]

\[ B_2 q(P,t) = \int_s \frac{1}{R_{pq}^2} \frac{\partial R_{pq}}{\partial n} q(Q,t) dS_Q + 4\pi \delta(P-Q) \]

\[ Aq(P,t) = \int_s q(Q,t) dS_Q, \quad A_1 q(P,t) = \int_s \frac{1}{R_{pq}} q(Q,t) dS_Q \]

- \( \chi = -1 \) : corresponds to the classical retarded potential-based approximation and leads to an unstable model; and,
- \( \chi = 0 \) : leads to a first-order model akin to DAA\(_1\).
Observations on the Present Basic Approximation

The fact that $sR/c < 1$ implies that the approximation is accurate for late-time response due to the inherent property of the Final Value Theorem of the Laplace Transform.

Observe that $s^2 B\bar{P}(P, s)$ and $s c B_1 \bar{p}(P, s)$ are transient terms, hence dominating early-time responses.

Hence, these terms would not be accurate with the expansion restriction of $sR/c < 1$. 
Modifications for Transient or Early-Time Response

(a) Consistency for Impulse Response compared with analytical solutions;

(b) Incorporation of classical results wherein early-time response is dominated by plane waves;

The preceding requirements are met fortuitously if one approximates \( \frac{\partial R}{\partial n} \) in the following terms

\[
B\overline{p}(P, s) = \int_S \frac{\partial R}{\partial n} \overline{p}(Q, s) dS_Q, \quad B_1\overline{p}(P, s) = \int_S \frac{1}{R} \frac{\partial R}{\partial n} \overline{p}(Q, s) dS_Q
\]

by replacing with \( \left( \frac{\partial R}{\partial n} \to 1 \right) \)
Approximate models

- Proposed approximate model

\[
[\chi \bar{A} s^2 + c(I + \chi) A_1 s + c^2 B_2] \ddot{p} = \rho c s [\chi \bar{A} s + c A_1] u, \quad \bar{A} = A_1 \frac{1}{2} B_1 (P)^{-1} A
\]

- 2\textsuperscript{nd} order Doubly Asymptotic Approximation (DAA2)

\[
M_f \ddot{p} + \rho c A \ddot{p} + \rho c \Omega A p = \rho c M_f \ddot{u} + \rho c \Omega M_f \ddot{u}
\]

\[M_f : \text{fluid - mass matrix}, \quad M_f = \frac{1}{2} \rho \left( B_2^{-1} A_1 A + A_1^T A B_2^{-T} \right)\]

\[\Omega = g \rho c A M_f^{-1}\]

\[\rightarrow\]

\[A \ddot{p} + \rho c [A M_f^{-1} A] \ddot{p} + \rho c [A M_f^{-1} \Omega A] p = \rho c A u + \rho c^2 [A M_f^{-1} \Omega M_f] u\]
Modal solutions for a sphere

- Mode by mode form for a sphere
  - Proposed approximation
    \[ \chi_n s^2 \bar{p}_n + (1 + \chi_n) s \bar{p}_n + (1 + n) \bar{p}_n = \chi_n s^2 \bar{u}_n + s \bar{u}_n \]
  - 1\textsuperscript{st} DAA and 2\textsuperscript{nd} DAA for a sphere
    \[ DAA_1 : s \bar{p}_n + (1 + n) \bar{p}_n = s \bar{u}_n \]
    \[ DAA_2 : s^2 \bar{p}_n + (1 + n) s \bar{p}_n + (1 + n)^2 \bar{p}_n = s^2 \bar{u}_n + (1 + n) s \bar{u}_n \]
  - Exact solution
    \[ \kappa'_n(s) \bar{p}_n = -\kappa_n(s) \bar{u}_n \]
    \[ \kappa_n(s) : \text{Modified spherical bessel function of the third kind} \]
    \[ n = 0 : (s + 1) \bar{p}_n(s) = s \bar{u}_n(s) \]
    \[ n = 1 : (s^2 + 2s + 2) \bar{p}_n(s) = (s^2 + s) \bar{u}_n(s) \]
    \[ n = 2 : (s^3 + 4s^2 + 9s + 9) \bar{p}_n(s) = (s^3 + 3s^2 + 3s) \bar{u}_n(s) \]
    \[ \vdots \]
    \[ \Rightarrow \text{If } \chi \text{ is zero, the present and } 1\textsuperscript{st} \text{ DAA are same.} \]
Impulse Response Consistency

- The early time consistency is important for inverse acoustic problems.
- Consistent initial impedance

\[ u_n(t) = \delta(t) \]

\[ \lim_{t \to 0} \left[ \frac{p_n(t)}{u_n(t)} \right] = \delta(0) - 1 \quad \text{for the exact solution} \]

\[ \lim_{t \to 0} \left[ \frac{p_n(t)}{u_n(t)} \right]_{DAA} = \begin{cases} 
\delta(0) - (n + 1) & \text{for } DAA_1 \\
\delta(0) & \text{for } DAA_2 \text{ (1978)}
\end{cases} \]

Present model satisfies the early-time consistency independent of the choice of the weighting parameter, \( \chi \)
Determination of Weighting Parameter, $\chi$

1. Plot the characteristic roots of the exact analytical solution for a sphere

2. Assume $\chi$ in the following form:

$$\chi_n^{-1} = \begin{cases} 
\bar{n}, & \text{when } \{\bar{n} = 1, n = 0\} \text{ and } \{\bar{n} = 1, n = 1\} \\
b_0 + b_1 n + b_2 n^2, & \left(b_0, b_1, b_2\right) \geq 0 \text{ when } n > 1
\end{cases}$$

And determine the best fitting constants, $\left(b_0, b_1, b_2\right)$
Analytical Pressure Characteristic Root Loci for a Sphere
Model parameter selection

- Mode by mode parameterization
  - General expression of $\alpha$

\[
\frac{\bar{p}_n(s)}{\bar{u}_n(s)} = \frac{Q_n(s)}{R_n(s)}
\]

\[
\begin{align*}
  s^2 \bar{p}_n + (1 + n^*) s \bar{p}_n + n^* (n + 1) \bar{p}_n &= s^2 \bar{u}_n + n^* s \bar{u}_n 
\end{align*}
\]
Determination of Weighting Parametric Matrix, \( \chi \)

- It specializes to the modal form of \( \chi_n \) when applied to spherical geometries.

- It should be robust with respect to computational errors for general geometries.

A general matrix form of \( \chi \) that satisfies the above requirements is found to be

\[
X^{-1} = N^{-1}B_2 - I + S \\
S = 2B_1A_1^{-1} \\
N = A_1\bar{A}^{-1}A_1 \\
\bar{A} = \left[ \int_S \frac{1}{r} dS \right] \left[ 2 \int_S \frac{1}{r} \frac{\partial r}{\partial n} dS_p \right]^{-1} \left[ \int_S dS_p \right]
\]

The case of \( S=0 \) is symbolically equivalent to DAA2 with curvature Correction which is found to be prone to instability for \( n = 0 \) mode.
Proposed modal equations and DAA$_2$ with curvature correction

<table>
<thead>
<tr>
<th>$n = 0$</th>
<th>Proposed model</th>
<th>curvature DAA$_2$</th>
</tr>
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<tbody>
<tr>
<td></td>
<td>$[s + 1]^2 \bar{p}_0 = s[s + 1]u_0$</td>
<td>$s[s + 1]p_0 = s^2u_0$</td>
</tr>
<tr>
<td>$n &gt; 0$</td>
<td>$[s^2 + (1 + n)s + n(n + 1)] \bar{p}_n = s[n + 1]u_n$</td>
<td></td>
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</tbody>
</table>
Comparison of Analytical and Present Principal Roots with $\chi^{-1}_n = n$, $\chi^{-1}_0 = 1$
Computer Implementation of Interaction Problems

- Coupled problems for **acoustic-structural interaction**
  - Each field has different governing equations and physical variables.
  - Both of structural and acoustic fields should satisfy geometric compatibility conditions on their interface surface.

\[ M\ddot{x} + Kx = G^T Ap \]
**Structure**

\[ G^T \dot{x} = u_0 + u_s \]

**Exterior acoustic fluid**
\[ c^2 \nabla^2 \phi(r, t) = \ddot{\phi}(r, t) \]
\[ p(Q, t) = \rho \dot{\phi}(Q, t) \]
\[ u(Q, t) = -\nabla \phi(Q, t) \]
Problems that can be treated by the present models

- Estimating scattered pressures caused by a vibrating structure
  - Reducing structure borne noise
  - Detecting enemy’s submarines

- It requires accurate impedance on boundary surface.
Governing Interaction Equations

- Pressure in exterior domain using approximate models
  - Known boundary values
    \[
    \begin{bmatrix}
    Ms^2 + K \\
    \rho c \chi AG s^3 + \rho c^2 A s^2 \\
    \end{bmatrix}
    \begin{bmatrix}
    G^T A \\
    \chi AS^2 + c(I + \chi)A_1s + c^2 B_2
    \end{bmatrix}
    \begin{bmatrix}
    x(Q,t) \\
    p_z(Q,t)
    \end{bmatrix} = \begin{bmatrix}
    -G^T Ap_1 \\
    -\rho c[\chi AS^2 + cA_1s]u_t
    \end{bmatrix}
    \]
  - Unknown values
    \[
    4\pi p_z(P,t) = \int \left\{ \frac{\rho}{R_{PQ}} \hat{u}_z(Q,t_R) - \frac{1}{R_{PQ}^2} \frac{\partial R_{PQ}}{\partial n} \left[ p_z(Q,t_R) + c^{-1} R_{PQ} \hat{p}_z(Q,t_R) \right] \right\} dS_Q
    \]
    External value
    Boundary values
    \[ p = p_I + p_z \]
Benchmark Test: elastic sphere subjected to Incident acoustic excitations

- Spherical shell
  - It is a simple geometry where the exact solution exists.

- Dimensionless values and formulations are used
  - Pressure \( p = \frac{p}{\rho c^2} \)
  - Time \( T = \frac{tc}{a} \)
  - Length \( r = \frac{R}{a} \)  \( w = \frac{W}{a} \)  \( v = \frac{V}{a} \)

\( \rho, c \) in air
\( \rho : \text{density of air} \)
\( c : \text{acoustic velocity} \)
\( a : \text{radius of a sphere} \)
Transient responses of a submerged spherical shell excited by cosine-type impulse pressure

- A spherical shell surrounded with fluid medium.
- In water medium
- $h/a=0.01$, $\rho_s/\rho=7.7$, $c_s/c=3.7$

Numerical Simulation

$$P_I = -\cos \theta \delta(t)$$
Numerical Simulation

- Velocity responses on the surface
  - On front side

![Graphs showing velocity responses and errors](image-url)
Numerical Simulation

- Velocity responses on the surface
  - On back side

![Graphs showing normalized velocity and error as functions of Tc/a](image-url)
Numerical Simulation

- Pressure responses at measured points

\[ 4\pi p_s(P, t) = \int \left\{ \frac{\rho}{R_{PQ}} \ddot{u}_s(Q, t_R) - \frac{1}{R_{PQ}^2} \frac{\partial R_{PQ}}{\partial n} \left[ p_s(Q, t_R) + c^{-1} R_{PQ} \dot{p}_s(Q, t_R) \right] \right\} dS_Q \]

R=1.5a

R=3a

Acoustic – Structural Interaction  SDAC, NoVic, KAIST
Numerical example 3 (Impulse incident velocities)

- Impulse incident velocities are acting on the spherical shell
  - Impulse incident velocities excite the pressure directly.
  - Pressure modes become dominant.

- Initial values order by order
  - \( \text{Exact} : \delta(t) + 1 + \mu \)
  - \( \text{Present} : \delta(t) + 1 + \mu \)
  - \( \text{DAA}_2 : \delta(t) + \mu \)

- If \( \mu \) is small, 2\textsuperscript{nd} DAA introduces large errors.
Numerical example 3 (Impulse incident velocities)

- The pressure responses on the front side of the sphere $\theta = \pi$
Cylindrical shell with stiffeners

**Shock analysis of a complex structure**

- A steel ring-stiffened, right circular cylinder with flat aluminum end caps
  - A test vehicle of Defense Nuclear Agency (DNA) and the Office of Naval Research (ONR) of USA for demonstrating the ability of the DAA-based computer code
  - Shock analysis of the submerged structure excited by the underwater explosion

Finite element model

![Diagram of a shock analysis model](image)

**Measured incident pressure**

![Graph showing pressure over time](image)
- Incident pressure
  - Incident pressure is approximated as the summation of linear and exponential functions.

![Graph of measured incident pressure.](image)

![Graph comparing experiment and simulation results.](image)

Figure 2. Measured incident pressure.
Shock analysis of a complex structure

- Shock responses
  - Radial velocity

**A points**

**B points**

Center for Noise and Vibration Control / KAIST  Structural Dynamics & Applied Control
Shock analysis of a complex structure

- Shock responses
  - Radial velocity

C points

D points

Center for Noise and Vibration Control / KAIST Structural Dynamics & Applied Control
Conclusions

- A stable approximate model for external acoustic and structural interaction problems has been derived by employing a combination of retarded and advanced potentials.

- The maximum order of “regular” approximation is found to be two.

- The present approximate model is consistent with respect to impulse response, important for inverse identification as the Frequency Response Functions are in fact the time-domain counterpart of impulse response functions.

- It remains to assess the present approximations as applied to more realistic problems.