Inquiry Brief 1: Validation of Mathematical Proofs

First Findings from a Proof Test

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Aims of the Study

This study addresses our research question about students' cognitive gains from an IBL course and how these gains may differ from non-IBL students' gains.

Study Design

We conducted individual problem- solving interviews with Number Theory students using an assessment test on proofs designed by Weber (2009). The data is based on interviews with 14 IBL and 10 non-IBL students (15 men, 9 women). The test consisted of 10 arguments on algebra, number theory and calculus, each followed by 4 structured questions:

- Did students understand the argument?
- Did they find it to have explanatory power?
- To what extent were students convinced by the argument?
- Did students consider the argument to be a mathematical proof?

Findings

Students Did Well on the Test

- Most students considered the valid arguments to be mathematical proofs; 79-86% of the IBL students and 60-90% of the non-IBL students.
- Students did not have difficulties in understanding the arguments (Table 1-1).
- Students were more convinced by valid arguments than by invalid arguments.
- Students found more explanatory power in valid than in invalid arguments.

Table 1-1: Averages of Students' Assessments of 10 Mathematical Arguments

Assessment	Mean ratings, by course type		*Scale 1-5*
	IBL	Non-IBL	
Understanding	Scale from *1=not understand fundamental details* to *5=understand completely*		
Valid arguments	4.5	4.2	Both groups understood the arguments very well.
Invalid arguments	4.3	4.4	
Conviction	Scale from *1=not convinced at all* to *5=completely convinced*		
Valid arguments	4.3	4.0	Students were less convinced by invalid than valid arguments, but still rather convinced by invalid arguments.
Invalid arguments	3.0	3.4	
Explanatory power	Scale from *1=does not explain* to *5=really illuminates why it is true*		
Valid arguments	4.0	3.8	Students identified explanatory power less confidently than other features. Many saw strong explanatory power in invalid arguments.
Invalid arguments	3.1	3.4	

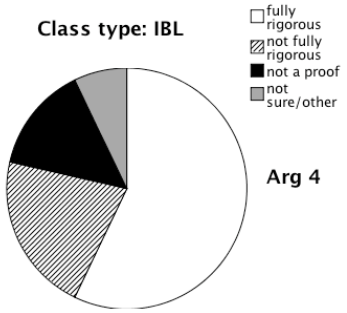
Minor Differences Appeared between IBL and Non-IBL Students

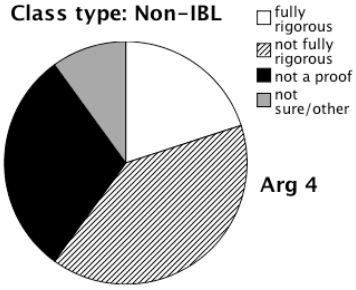
- All IBL students completed all 10 problems; 3 non-IBL students did not.
- IBL students succeeded slightly better in assessing the validity of the arguments.
- IBL students were less often convinced by invalid arguments.

Recognition of a Mathematical Proof

Individual problems showed some interesting differences between IBL and non-IBL students (Figure 1-1), but these differences were not consistent across problems of similar types.

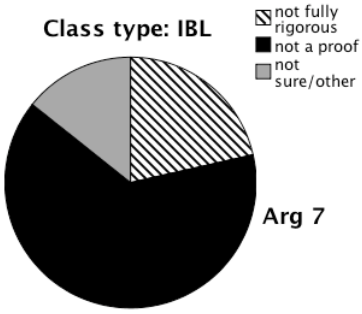
Figure 1-1: Examples of the Distribution of Students' Answers to Specific Problems

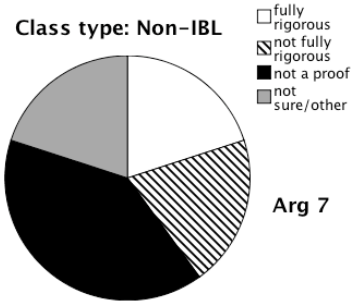




Valid Argument - an equation has no real solutions

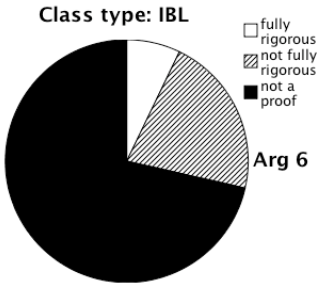
- *IBL students see this as a fully rigorous proof*
- *Non-IBL students are more hesitant and more often see this argument as not a proof*

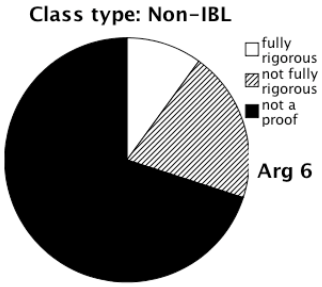




Invalid Argument - deductive number theory, invalid proof structure

- *IBL students more often recognize this as not a proof*
- *Non-IBL students more often identify this argument as a fully rigorous proof*





Invalid Argument – perceptual calculus

- *Both groups recognize this argument as not a proof*

Limitations of the Study and Future Plans

- The data set included only 24 students.
- The students who volunteered were good mathematics students. The test may be more sensitive to differences among less strong or less experienced mathematics students.
- Our aim is to acquire a larger and more deliberately chosen student sample including both IBL and non-IBL students.

Weber, K. (2009). Proving is not convincing. Conference on Research in Undergraduate Mathematics Education, Raleigh, NC, February 26-March 1, 2009.