Executive Summary

Evaluation of the IBL Mathematics Project: Student and Instructor Outcomes of Inquiry-Based Learning in College Mathematics

Assessment & Evaluation Center for Inquiry-Based Learning in Mathematics

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ES 1.0 Introduction

We report findings from a comprehensive, mixed-methods study of inquiry-based learning (IBL) in college mathematics as implemented at four university IBL Mathematics Centers. Six sub-studies were collectively designed to examine the following questions:

- What are the student outcomes of IBL mathematics courses?
- How do these outcomes vary among student groups, and how do they compare with other types of courses?
- How do these outcomes come about?
- What are the costs and benefits of IBL teaching for instructors and departments?

Data analyzed for this report include approximately 300 hours of classroom observation, 1100 surveys, 220 tests, 3200 student transcripts, and 110 interviews with students, faculty, and graduate teaching assistants (TAs), gathered from over 100 course sections at four campuses across two academic years.

In the main report,[1] we introduce the IBL Mathematics Centers, the study and its context (Chapter 1), then document findings from the six sub-studies (Chapters 2-8). Our summary (Chapter 9) gathers key findings, emphasizing corroborating evidence from multiple lines of inquiry. The appendices (A1-A7) document the study methods and samples for each sub-study. In the summary below, section numbers are provided to reference discussion of specific findings.

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[1] http://www.colorado.edu/eer/research/steminquiry.html#Reports

ES 2.0 Setting for the Study

The study sites were four university mathematics departments that host IBL Mathematics Centers supported by the Educational Advancement Foundation. All four Centers are at research universities (three public, one private) with selective undergraduate enrollment and highly ranked graduate programs in mathematics. They are geographically dispersed and vary in size. Because the IBL Mathematics Centers were established in 2004, before this study was commissioned, many study parameters were pre-determined. The study used an observational, not experimental, design, reflecting realistically variable implementations of inquiry-based learning and not an idealized laboratory situation. That circumstance imposed some constraints on what we could learn, but also makes our findings relevant to other real-world reform efforts in STEM higher education.

ES 2.1 Courses Studied

Starting in 2004, each Center independently chose certain courses in which to develop and teach IBL mathematics curricula. These courses were well established by the time our study began, and in most cases multiple instructors had participated in addition to the course originators. Varying levels of professional development and support were available to instructors new to IBL. Most commonly, they visited colleagues' classes, drew upon others' course materials, and exchanged ideas informally or at occasional IBL events.

The courses included in the study addressed a range of student audiences and mathematical topics. Courses for pre-service K-12 teachers focused on deep understanding of the mathematical concepts needed to teach elementary, middle, or secondary schoolchildren. Courses such as cryptology and calculus sought to provide talented first-year students with a stimulating mathematics experience that might draw them into the major. More advanced courses in number theory, analysis, geometry, and discrete math served "mainstream" mathematics majors as well as science and engineering majors.

Some IBL courses were offered with parallel non-IBL sections also available. Others—including all of the courses targeted to pre-service K-12 teachers—were offered only in IBL format. Thus the availability of comparison groups for the study was uneven. In the analyses discussed below, data from first-year and advanced courses were combined in describing "math-track" courses, while data from courses for pre-service teachers were analyzed separately.

ES 2.2 Inquiry-Based Learning as Implemented at the IBL Centers

We labeled course sections as "IBL" or "non-IBL" following the campus Centers' own designations, which generally indicated support by the Center's grant. To establish that the IBL label indeed differentiated teaching and learning approaches, we observed classrooms to directly establish the nature of instruction actually occurring in IBL and non-IBL courses.

Students in IBL classes spent class time giving and listening to student presentations, working in small groups, discussing ideas that generally arose from a group problem or student-presented solution, and, in a few courses, working with peers on computer activities such as modeling or simulations (2.2.1). On average, over 60% of IBL class time was spent on such student-centered activities, while students in non-IBL courses spent 87% of their time listening to their instructor talk. While there was substantial variation in practice, overall the IBL courses offered students very different experiences from typical lecture-based courses. That is, as a group, instruction was clearly distinct between “reformed” IBL courses and comparative non-IBL sections.

Drawing on both the classroom observation data (Ch. 2) and student and instructor interviews (7.3; 8.2.6, 8.3), we identify key features of IBL courses in this study by both their statistical frequency and their importance to students and instructors. In IBL courses typical of this project:

- The main work of the course was problem-solving: students solved challenging problems alone or in groups, in and out of class. In class they shared, evaluated and refined their own and each others’ solutions.
- The course was driven by a carefully built sequence of problems or proofs, rather than a textbook. The pace of the course was set by students’ movement through this sequence, rather than pegged to a pre-set schedule.
- Course goals tended to emphasize development of skills such as problem-solving, communication, and mathematical habits of mind, not just covering specific content.
- Most of class time was spent on student-centered instructional activities. Students or groups of students played a leading role in guiding these activities. Most activities were conducted for just a few minutes at a time: class work tended to change gears often and switched frequently between activities.
- Instructors’ main role was not lecturing. They might give mini-lectures to set up or sum up the day’s work, introduce a group activity, or provide context for a set of theorems. Instructors (as well as other students) might offer impromptu explanations to respond to a comment or question. They might ask questions to clarify student thinking, refine a proposed solution, give feedback, or elicit such comments from other students.
- Student voices were heard in the classroom: presenting, explaining, arguing, asking questions. Their active participation meant that students themselves had considerable influence on how class time was spent and how fast the class moved through the material.
- This joint responsibility for the depth and progress of the course fostered a collegial atmosphere that placed value on respectful listening and critique and that invited every class member to contribute fruitfully to the mutual development of mathematical ideas. Instructors made efforts to set and maintain this atmosphere.

From instructor[2] and student interviews, we also know something about the behind-the-scenes structure typical of an IBL course. Outside class, much of students’ work time—which was

[2] The term instructor refers to faculty and TAs together.

substantial—was spent in preparing for class: solving problems or deriving proofs to present or discuss in groups. Because work was due nearly every class, the workload was steady rather than test-driven (3.3.2). Instructors invested substantial time in constructing the "script," or sequence of problems, or in understanding and fine-tuning scripts shared by other instructors. Checking homework took on greater importance for IBL courses, because students' work improved most rapidly when they got timely feedback on their solutions or proofs. Faculty and TAs held many office hours outside class, and students made much use of them, because timely help could be important to making progress and separating "fruitful struggle" from wasted time.

ES 3.0 Student Learning Outcomes

Findings on student learning outcomes were derived from several independent lines of evidence:

- surveys of students' self-assessed learning gains (Ch. 3), which include both numerical ratings and open-ended comments;
- tests given to subsets of math-track and pre-service students (Ch. 5);
- interviews with students (Ch. 7) and their instructors (Ch. 8); and,
- for math-track students, analysis of their grades and course-taking patterns subsequent to an IBL or comparative course (Ch. 6).

On surveys of their self-assessed learning gains, math-track students who took IBL courses reported greater gains than their peers who took non-IBL sections of the same courses (3.2.1). These gains were higher across several areas: cognitive gains, including understanding of mathematical concepts and improved thinking and problem-solving skills; affective gains, including confidence, positive attitude, and persistence; and social gains, including collaboration and comfort in teaching mathematical ideas to others.

For pre-service teachers who took IBL courses, the self-reported learning gains were different (3.2.1). In general, their cognitive and affective gains were lower than those of math-track IBL students. However, their gains in applying mathematical knowledge, collaboration, and comfort in teaching mathematical ideas to others were as strong or stronger than those of math-track IBL students, and clearly higher than those of math-track non-IBL students. These differences likely reflect pre-service teachers' lower general interest in mathematics (3.4.3), but also emphasize their gains in areas that are especially significant for their future work in teaching K-12 students.

Students' write-in comments on surveys reinforce their survey ratings about the breadth and depth of learning they experienced (3.2.2). From both math-track and pre-service courses, twice as many IBL students wrote about learning gains as did non-IBL math-track students, and IBL students also wrote much more often of multiple gains from their course. Their comments emphasized cognitive gains—especially learning more deeply because they had figured things out for themselves. They also described changes in how they learned mathematics and solved problems, including improved learning on their own and from others. They described affective benefits and new communication skills—less often than they noted cognitive and learning changes, but more often than either type was noted by non-IBL students.

Student interview data further corroborate these findings. Again, students most emphasized cognitive gains (7.2.1), especially the deep and lasting learning that came from working through ideas for themselves. They saw their gains in thinking and problem-solving skills as transferable to other courses and to life in general. Changes in learning, also commonly reported, were of two types: personal learning changes such as self-awareness, persistence and independence, and greater appreciation for the benefits of collaborative work (7.2.3). Affective gains emphasized confidence, enjoyment and interest (7.2.4). They noted communication gains too: improved writing and speaking about proofs and enhanced abilities to explain and critique ideas (7.2.5).

Instructor observations of student learning aligned well with students' own reports, with some differences in perspective (8.2.1-8.2.4). Instructors could better spot gains in communication skills and understanding the nature of mathematics that indicated students' development as budding mathematicians, but less easily observed gains in students' personal learning processes.

For pre-service teachers, pre- to post-test score gains on a well-validated external test of learning mathematics for teaching (LMT) reflected real gains in understanding after an IBL course (5.2.2). In prior work, improved scores on this test have been connected to positive effects on teachers' instruction. The LMT test results thus suggest that IBL courses benefited pre-service teaching students in ways that will benefit their future work as teachers.

With a sample of math-track students, we gave a "proof test" to compare IBL and non-IBL students' ability to evaluate mathematical arguments and their reasons for judging arguments to be proofs or not. Both groups performed well on the test; there were no general differences in their scores overall or on specific problems (5.3.3). However, there was some evidence that IBL students were more skilled in recognizing valid and invalid arguments (5.3.3) and that they applied more expert-like reasoning in making such evaluations (5.3.6). The small sample size of academically strong students limits interpretation of these findings, as effects of IBL courses on performance may be greater for students whose prior academic record is less strong (see 4.2).

Academic records data from three target courses indicated that IBL students earned grades in subsequent courses that were as good or better than the grades of their non-IBL peers (6.2.1). The pattern favoring IBL students was broadly consistent across different subsets of later courses, but few of the differences were statistically significant, due to the wide scatter in the grades of both groups. Overall, taking IBL courses may benefit, and certainly does not harm, students' performance in later mathematics courses.

Among students who took mid-level and advanced courses, there were no general differences in pursuit of additional mathematics courses (6.2.2). However, students who took an IBL honors course early in their college career did appear to take more subsequent math courses than did a matched sample of peers from the large lecture-based section. Most of these differences are suggestive rather than definitive because of small sample sizes, but the IBL students did complete further IBL courses at a statistically much higher rate. These results imply that IBL honors courses can draw strong students into further study, especially additional IBL courses. Later IBL experiences appeared to neither spur nor deter further study of mathematics.

Overall, several lines of evidence suggest that students who had a college IBL course grew as mathematicians and as learners in ways that their peers taking non-IBL courses typically did not. The nature and types of the observed cognitive, affective and social gains were very consistent across multiple data sources. Some of IBL students' cognitive gains in reasoning and problem-solving were detectable on tests; and there is some evidence that these gains carry over to benefit students' work in later courses. Of all the learning outcome measures that we compared between IBL and non-IBL students, very few pointed to deficits for IBL students.

ES 4.0 Group Differences in Student Learning Outcomes

Our findings on group differences in student learning outcomes are based on subdividing the same data sets listed in ES 3.0: student surveys, tests, interviews, and academic records.

ES 4.1 Group Differences by Gender

Women's share of undergraduate degrees in mathematics has declined in the past two decades, unlike most other STEM fields. Thus we examined differences in student outcomes by gender.

In survey items on self-assessed learning gains, women in IBL classes reported as high or higher gains than their male classmates across all cognitive, affective and social gains areas (3.2.3). But women in non-IBL classes reported statistically much lower gains than their male classmates in several important domains: understanding concepts, thinking and problem-solving, confidence, and positive attitude toward mathematics. Overall, both men and women reported higher learning gains from IBL courses than from non-IBL courses, while traditional teaching approaches did disservice to women in particular, inhibiting their learning and reducing their confidence. Women's spontaneous write-in comments echoed this finding: IBL women wrote over four times as many comments about their cognitive gains, in particular, compared to their non-IBL peers, and also more comments about gains in confidence. IBL approaches appeared to level the playing field for women, compared to traditional lecture-based approaches.

The academic records data offer perspective on how well these short-term gains may last for women in later courses. IBL women outperformed their non-IBL counterparts on several measures of subsequent grades (6.3.1), as did IBL men (6.3.2). IBL women also took as many or more courses than non-IBL women (6.5.1), though generally fewer than their male classmates (Figures 6.7-6.9). These patterns were fairly consistent but mostly not statistically significant.

How women's grades compared to their male classmates seemed to depend somewhat on the course level (6.3.3.). Women in the most advanced course held their ground versus their male classmates, but women in the first-year course underperformed both their female non-IBL peers and their male IBL classmates. In the mid-level course, results for women fell roughly between the other two. First-year IBL women also pursued further IBL courses at lower rates than their male classmates. Perhaps women taking advanced courses are survivors who have learned to thrive independent of the challenges they encounter, while women early in their college careers may be more sensitive to stereotype threat and other differences in real or perceived classroom climate. While the short-term results indicate that IBL experiences particularly benefit women, over the long haul, a single IBL experience may help to close but not erase the gender gap.

There were no gender differences in the gains reported by the students we interviewed (Table 7.2, 7.4), and very few reports of gender-based differences in their experiences. The fact that gender is such a non-issue in the interview data explains the survey data rather well: from students' perspective, IBL classrooms offered equitable environments where all could succeed.

Overall, IBL experiences appear to be powerful for women, leveling the playing field by eliminating discouraging experiences that impede learning in traditionally taught courses.

ES 4.2 Group Differences by Prior Achievement

Instructors commonly hypothesized that IBL experiences were most beneficial for students who were not the most academically qualified—students who were good, perhaps, but not great (8.2.5). Thus we examined the data for differences in student outcomes by prior achievement.[3]

On the LMT test given to IBL pre-service teachers, test score gains were anti-correlated with the initial score (5.2.3). That is, students who had the lowest scores on the pre-test improved the most on the post-test. This finding matches students' self-report of learning gains, where IBL pre-service teachers with lower overall GPAs reported higher gains than their classmates with higher grades (3.2.6), mirroring the pattern seen on the LMT test.

For math-track students, the pattern among IBL students was less striking, but similar. Students with the lowest GPAs reported higher gains than the middle GPA group (3.2.6). However, in non-IBL classes, students who had the highest GPAs reported the highest gains, and low-GPA students reported the lowest gains.

Among students who entered with low math GPAs (<2.5), IBL students generally earned better grades in later classes than did their non-IBL peers (6.4.1). IBL students who entered with higher math grades also did as well or better than non-IBL peers in later courses (6.4.2, 6.4.3), but the improvement for previously low-achieving IBL students was striking (6.4.4.).

Overall, it appeared that non-IBL courses tended to reinforce prior achievement patterns, helping the "rich" to get "richer." In contrast, IBL courses seemed to offer an extra boost to lower-achieving students, especially among pre-service teachers. Yet there was no evidence of harm done to the strongest students. Indeed, high-achieving students may be encouraged by an IBL experience to take more mathematics courses, especially more IBL courses (6.6.2)—again, consistent with instructor observations that strong students found the IBL approach stimulating

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[3] Different types of data were necessarily used to distinguish prior achievement in each sub-study.

(8.2.5). Instructors did have reservations about the benefits of IBL for the "weakest" students; and our analyses may not have detected a small subset of students who genuinely struggled.

ES 4.3 Group Differences by Experience Level

In the pilot study, first-year students were particularly enthusiastic about how IBL courses had enhanced their learning in other courses. Thus we investigated differences in student outcomes for students taking IBL courses earlier or later in their college careers.

On surveys, first-year math-track students who took IBL courses reported higher gains than did IBL students later in their careers, across several areas: mathematical thinking, persistence, and collaboration (3.2.4). Moreover, both first-year and mid-career (sophomore/junior) students reported higher gains in these areas, as well as confidence and positive attitude about mathematics, compared to their non-IBL peers. Even late-stage (senior and graduate) students reported higher affective and collaboration gains than did their non-IBL peers.

A similar pattern was found when survey data were differentiated by the number of prior college mathematics courses taken (3.2.5). Among IBL students, less experienced students reported higher gains than more experienced colleagues, but this was not the case for non-IBL students. Gains enhanced by IBL experience included cognitive, affective, and social gains for novice math students, but only affective and social gains for students with more math background.

In interviews, first-year math-track students reported more gains overall than advanced math-track and pre-service teaching students (Table 7.3; 7.4). Consistent with the survey results, they particularly emphasized cognitive gains, changes in their understanding of the nature of mathematics, and affective gains including confidence and enjoyment.

In sum, several lines of evidence indicate that IBL experiences were more powerful for students earlier in their college career. This finding is also consistent with data from an IBL first-year honors course on students' later course-taking patterns (Figure 6.9). A first-year IBL experience may contrast strikingly with students' high school work, and changes in students' approaches to learning or studying may influence their work in later courses in mathematics and other fields.

ES 4.4 Other Group Differences

When considering other student sub-groups, we found very few other differences, and no systematic patterns of difference. We detected no meaningful differences in the outcomes of IBL courses for students of varying race, ethnicity, or academic major. However, these study sites did not provide good tests of these issues. In general, we have no evidence that IBL methods did not work equally well for students of different personal and academic backgrounds.

ES 5.0 Student Attitudinal Outcomes

Student surveys explored several attitudinal variables that characterized students' mathematical beliefs, motivation and strategies for learning and problem-solving.

ES 5.1 Characterization of Students' Beliefs, Motivations, and Learning Strategies

Based on their pre-course survey responses, math-track students in both IBL and non-IBL classes had strong interest in mathematics, and high motivation in both intrinsic (internal) and extrinsic (grades, future plans) dimensions (3.4.2, Table 3.5). They held fairly sophisticated views of mathematical problem-solving as a constructive and logic-based process. While IBL and non-IBL students were alike in many ways, IBL students more often rated mathematics as a personal, not just academic interest. They were also more confident and had greater preference for group work. These differences confirm that there is some preferential selection (by advising and/or self-selection) of certain type of students into IBL courses, as students and faculty also told us.

Compared with math-track students, pre-service teachers were less interested in mathematics, less likely to enjoy it, and more extrinsically motivated (3.4.3). Their beliefs about learning and problem-solving were somewhat more novice-like, viewing learning as more instructor-driven and problem-solving as more about confirming truths and practicing procedures than about discovering ideas. However, they believed more in the value of group work, made more use of it in their own studying, and were more interested in teaching and communicating mathematics.

ES 5.2 Attitudinal Changes Following an IBL Course

In general, the changes in these attitudinal variables from pre- to post-course survey were modest for all groups (3.4.4). They were also modestly but positively correlated with student learning gains (3.4.13), showing that attitudinal changes and learning outcomes are indeed related.

For non-IBL math-track students, attitudinal changes were mixed, but more negative than positive (3.4.5). After their course, these students reported lower confidence and enjoyment, less willingness to study hard for a math course, and less strong beliefs in rigorous reasoning as a general problem-solving approach. But for IBL students, most of the changes were positive: stronger personal interest in mathematics and in communicating it, stronger beliefs in proving as a constructive and creative activity, and stronger beliefs in and use of collaborative learning.

Among pre-service teachers, attitudinal changes were mixed (3.4.6). Following an IBL course, they placed less emphasis on extrinsic goals and instructor-driven instruction, suggesting some maturation of their approach to learning mathematics. However, they did not gain in confidence, and lost some ground in their use of self-regulatory learning strategies that are used by successful learners.

For women, there were small positive changes in confidence and motivation following an IBL course, contrasting with larger negative changes in confidence, collaboration, and use of effective learning strategies for women who took traditionally taught courses (3.4.7, 3.4.8). These findings align well with the learning gains observed for women (ES 4.1): again, IBL approaches appeared to remedy problems with traditional lecture-based teaching that were particularly detrimental to college women's interest and confidence in mathematics.

Among first-year students, there were enhancements to students' strategies for learning and problem-solving, while these did not change for older students (3.4.9-3.4.11). But among older students, positive changes in interest, motivation and confidence were observed, which were instead modestly negative for the first-year students. We suggest that early IBL experiences have an influence on students' approach to learning that may be powerful if it carries over to other college work. Later IBL experiences may not shift students' well-established study habits and beliefs, but may revive their interest in mathematics, as some interviewees suggested (7.2.4).

Overall, IBL math courses tended to promote slightly more sophisticated and expert-like views of mathematics and more interactive approaches to learning. In contrast, traditional mathematics courses appeared to weaken students' confidence and enjoyment, and did not help them to develop expert-like views or skillful practices for studying college mathematics.

ES 6.0 Teaching and Learning Processes

Several lines of evidence clarify the teaching and learning processes important in IBL courses. Clear differences in IBL and non-IBL student outcomes (ES 3.0) mirrored the clear differences between IBL and non-IBL course practices (ES 2.2). More importantly, we can directly link student outcomes to course practices: student learning gains correlated statistically significantly with the fraction of class time spent doing student-centered activities (small group work, student presentation, computer work, and discussion), and anti-correlated with the fraction of time spent listening to instructors talk (4.3.1). Similar correlations were found for the relation of learning outcomes to the proportion of class time that was student- or instructor-led (4.3.1), and for variables that reflect how students and instructors interact and share responsibility for the course (4.4). Moreover, statistical modeling shows that the degree of student-centered class time was the strongest predictor of student learning as measured by our broadest learning indicator, survey learning gains. When observation data were not available, the binary IBL/non-IBL label was a good predictor of learning.

Students themselves reported in some detail on how particular course practices supported their learning. On surveys, IBL math-track students cited several practices as "helping me learn" to statistically higher degrees than cited by their non-IBL peers: the overall approach; their own active participation; and interactions with the instructor, the TA, and their peers (3.3.1, 3.3.2). Non-IBL students cited tests as important for their learning, while IBL students found other types of assignments more helpful. Individual effort was important to both groups' learning. IBL pre-service teachers emphasized a somewhat different mix of experiences, including their own active participation and interaction with instructors, but also tests.

Interactive and collaborative course experiences were especially important for women (3.3.3) and first-year students (3.3.4), which may help to explain the strong learning outcomes for these groups (9.4.1, 9.4.2). There were no clear patterns of difference in the experiences of other student sub-groups, consistent with the lack of clear patterns in their learning outcomes (9.4.4).

Finally, in interviews, student discussion of their learning processes emphasized the twin pillars of deep engagement with mathematical ideas and collaboration with others (7.3.1). Deep engagement fostered deep understanding; it rested on both students' individual effort and the assignment of meaningful problem-solving tasks that were not mere "busy work." Collaboration was integral to IBL courses, whether as structured small group work, whole-class discussion, or out-of-class informal group work. Students found it efficient and useful to tackle hard problems with multiple brains, and they learned from explaining their ideas and trying to understand others. The twin pillars reinforced each other: after struggling with a problem individually, students were well prepared to contribute meaningfully during class, and interested in the solutions that others proposed. Collaboration in turn motivated them to complete the individual work. It also made class enjoyable, encouraged clear thinking, and built communication skills.

Instructors also described the twin pillars and linked them to students' positive cognitive and affective outcomes (8.2.6). Descriptions of instructors' teaching practices enable us to identify critical teaching decisions that can sensitively influence the success of an IBL course. Their choices about course materials, assessment, classroom dynamics, and other factors may aid, abet, or interfere with the central learning processes of deep engagement and collaboration (8.3).

Overall, surveys and interviews provided strong and consistent evidence about the dual importance of individual engagement and collaborative learning processes in IBL courses. The significant role of collaborative learning reflects a deliberate shift in modern instructional practices that yielded enriched student learning, growth in collaboration and communication skills, and an enjoyable experience for students and instructors alike.

ES 7.0 Outcomes for IBL Instructors

Instructors reported numerous professional and personal benefits of teaching with IBL methods, which outnumbered their costs (8.4.1). The chief benefit was enhancements to their teaching: deeper understanding of students and learning; stronger beliefs in the value of student-centered learning; and a larger and more nuanced portfolio of teaching skills. They also gained intellectual stimulation, interest, enjoyment, and pride in their students' progress. The main costs were time and effort (8.4.2).

Early-career mathematicians felt that their career preparation and prospects were enhanced by IBL teaching experience (8.4.3). The more common career influence was professional development that they felt prepared them for future teaching roles. All who had gone on the job market reported that their IBL background had been an asset rather than a liability.

Of the instructors interviewed, at least 85% wanted to teach with IBL methods again (8.4.4). They described profound and permanent changes in their teaching styles and beliefs about what and how students learned. These influences on their teaching approach carried beyond the original courses where they had learned IBL methods, as they took "IBL principles" to other courses. Most of their learning came from on-the-job experience, although in most departments a loose-knit IBL community had emerged that could offer support and ideas.

Overall, IBL teaching experiences were rewarding for instructors. As professional development experiences, they permanently shifted instructors' beliefs and practices toward student-centered approaches known to improve student learning. The most important legacy of the project may be this cadre of young instructors who gained IBL teaching experience at the Centers and are now moving on to teach at institutions across the United States and in other countries.

ES 8.0 Outcomes for the Project as a Reform Effort

Outcomes for students and teachers are one way to view the overall impact of the IBL Centers. We can also view the Centers' work collectively and examine the magnitude of its impact as a single reform effort: How big an impact does this project have?

About 425 unique, non-repeating students had an IBL experience each year during our study period. Over the same period, these four departments graduated about 500 mathematics majors each year.[4] On the order of 40% of all mathematics majors graduating from the four Centers may have had an IBL experience. The two institutions with IBL programs that targeted pre-service teachers graduated about 160 pre-service teachers per year.[4] Essentially all students preparing for elementary/middle school teaching at these two campuses had an IBL experience.

Another measure of impact is the sustainability of the IBL Centers' effort over time. All the Centers had taken measures to protect aspects of their program from budget cuts. Some aspects of the IBL programs were seen to be "here to stay," yet leaders also testified to aspects that were fragile and dependent on grant funding. In times of economic retrenchment and falling support for U.S. higher education, the jury is still out as to the sustainability of these reform efforts.

ES 9.0 Issues for Future Research

Our literature reviews identify fairly modest bodies of evidence on student outcomes of IBL for mathematics majors and for pre-service teachers. Findings from this study will thus contribute meaningfully to these literatures, as well as add to important conversations on teaching practices in college mathematics and on change in STEM higher education. We identify several issues for further analysis within our data sets and propose several areas for future investigation (9.9).

ES 10.0 Conclusion

The approaches implemented at the IBL Mathematics Centers benefited students in multiple, profound, and perhaps lasting ways. Learning gains and attitudinal changes were especially positive for groups that are often under-served by traditional lecture-based approaches, including women and lower-achieving students. First-year and less mathematically experienced students also benefited particularly. Yet there was no evidence of negative consequences of IBL for men, high-achieving students, older and more experienced students: these groups too made gains greater than their non-IBL peers.

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[4] National Center for Education Statistics (NCES) (2011). Integrated Postsecondary Education Data System (IPEDS). Department of Education. Office of Educational Research and Improvement.

The positive outcomes for students were linked to classroom practices that emphasized deep engagement with mathematical ideas and collaborative exploration of these ideas. IBL classrooms offered equal learning opportunities for men and women and motivated students to invest their own effort to advance class progress. Instructors also benefited from their IBL teaching experiences and made lasting changes to their teaching practice. On the whole, the teaching and learning methods implemented at the IBL Mathematics Centers were broadly consistent with evidence and best practices from research on the learning sciences. Our results augment that body of evidence with good support for student-centered approaches to undergraduate mathematics education.

ES 11.0 Acknowledgments

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