Appendix A3: Study Methods for Student Surveys

A3.1 Introduction

We used two survey instruments to measure student outcomes from inquiry-based learning in undergraduate mathematics and to compare these outcomes between various student groups, in particular, between IBL and non-IBL students. The attitudinal survey was designed to detect the quality of and changes in students' mathematical beliefs, affect, learning goals, and mathematical problem-solving strategies. The learning gains survey (SALG-M) measured students' experiences of class activities and their cognitive, affective and social gains from a college mathematics class. The surveys addressed the following questions

- What learning gains do students report from an IBL mathematics class?
- How do students experience IBL class activities? How do students' class experiences account for their gains?
- What kind of beliefs, affect, goals and strategies do IBL students report at the start of a mathematics course?
- How do these approaches change during a college mathematics course? How do these changes relate to or explain students' learning gains?
- For each of these outcomes—learning gains, experiences, attitudinal measures, and changes—how do the outcomes for IBL students differ from those of non-IBL students, and among IBL student sub-groups?

The survey instruments provided us with large student data sets from four campuses, gathered during the two academic years 2008-2010. They offered us a comprehensive picture of students' approaches to learning college mathematics as well as of their experiences and gains from IBL classes. Moreover, the survey data could be used to analyze differences in reported learning approaches, classroom experiences and learning outcomes among various student groups. In addition to structured questions, students also could write about their experiences and gains in the open-ended survey questions. Both the open-ended survey answers and student interview data were used to validate, confirm, and fill in the picture of student outcomes obtained from the structured survey responses.

A3.2 Study sample

The data were gathered on all four campuses in a variety of undergraduate courses. These included courses entitled:

- (Honors) Analysis 1-3,
- (Honors) Calculus 1-3,
- Cryptology
- Discrete mathematics,

- Explorations in mathematics,
- Exploratory calculus,
- Group theory,
- Introduction to proofs,
- Introduction to real analysis,
- Multivariate calculus 1-2,
- Number theory,
- Probability,
- Real analysis 1.

They covered the full range of introductory to advanced mathematics courses.

Mathematics courses specifically developed for elementary and middle school or secondary school pre-service teachers represented another type of course in the sample. This kind of survey data was obtained from two campuses. Additional smaller data sets came from a geometry course designed (but not required) for prospective high school mathematics teachers at one campus.

In all, we collected surveys from 82 college mathematics sections, of which 65 were IBL sections and 17 non-IBL sections. Data obtained with our surveys consisted of an attitudinal pre-survey, a learning gains post-survey, and a combined post-survey including both the attitudinal and the learning gains questions. We received pre-surveys from 1245 students, learning gains post-surveys from 200 students, and combined post-surveys from1165 students. Combining the pre-survey data with the post-survey data produced us information from 800 individually matched surveys. These surveys included responses from 412 IBL math track students (i.e., students who studied mathematics as their major or minor subject), 156 non-IBL math track students, 208 IBL pre-service teachers, and 25 non-IBL pre-service teachers.

Tables A3.1-A3.4 display features of our sample based on the personal information from the pre-survey responses.

A3.2.1 Survey Sample by Gender

Students reported their gender both in the pre- and post-survey. Even though these were not always the same students, the percentages of women and men were rather consistent in the two surveys. About 60% of all the students were men. This varied along with student groups. Typically, nearly 70% of the math-track students were men, whereas most of the IBL pre-service teachers (84% pre; 86% post) were women.

Table A3.1: Survey Respondents by Gender and Course Type

	IBL math-track		Non-IBL math-track		IBL pre-service		Non-IBL pre-service		Total	
Gender	Count	%	Count	%	Count	%	Count	%	Count	%
Pre-Survey										
Women	194	*33.6*	104	*30.4*	190	*83.7*	12	*48.0*	500	***42.7***
Men	383	*66.4*	194	*69.6*	37	*16.3*	13	*52.0*	671	***57.3***
TOTAL	577	*100%*	342	*100%*	227	*100%*	25	*100%*	1171	***100%***
Learning Gains Survey										
Women	169	*32.3*	92	*28.5*	190	*85.6*	17	*53.1*	468	***42.5***
Men	354	*67.7*	231	*71.5*	32	*14.4*	15	*46.9*	632	***57.5***
TOTAL	523	*100%*	323	*100%*	222	*100%*	32	*100%*	1100	***100%***

A3.2.2 Survey Sample by Academic Major

We classified students by their reported main major, prioritizing their most mathematically oriented major. Accordingly, all students with a major in mathematics or applied mathematics were classified into one category, even if they had a second, non-mathematics major. Science majors included students with a major in physics, chemistry or another science, but not in mathematics or applied mathematics. Engineering and computer science majors formed another category, as did students with a major in economics. All students who reported any a non-science major were classified into one group.

Table A3.2: Survey Respondents by Academic Major

Main academic major	IBL math-track		Non-IBL math-track		IBL pre-service		Non-IBL pre-service		Total	
	Count	%	Count	%	Count	%	Count	%	Count	%
Math or applied math	337	*60.3*	163	*49.4*	86	*38.7*	19	*76.0*	605	***53.3***
Science	82	*14.7*	53	*16.1*	18	*8.1*	1	*4.0*	154	***13.6***
Engineering or computer science	63	*11.3*	57	*17.3*	4	*1.8*	2	*8.0*	126	***11.1***
Economics	28	*5.0*	47	*14.2*	3	*1.4*	0	*0.0*	78	***6.9***
Other non-science	49	*8.8*	10	*3.0*	111	*50.0*	3	*12.0*	173	***15.2***
TOTAL	559	*100%*	330	*100%*	222	*100%*	25	*100%*	1136	***100%***

More than half the students reported a mathematics or applied mathematics major. Students who had a non-science major mostly represented IBL pre-service teachers. Science majors formed the next biggest student group. Engineering or computer science majors (11.1%) and economics majors were the two other majors represented in the sample. In addition to mathematics or applied mathematics students, students from other STEM fields were also well represented in the sample.

IBL math-track students were pursuing a math major slightly more often (60.3%) than non-IBL math-track students (49.4%). The proportions of science majors was similar, but more of the non-IBL math-track students were economics majors or engineers. Fully half of the IBL pre-service teachers were non-science (e.g., education) majors. These represented mostly elementary or middle school pre-service teachers. But the sample also included many secondary pre-service teachers with a math major.

A3.2.3 Ethnicity and Race

We classified students by race into three different categories. All the students who considered themselves white and not a representative of any other race were denoted White. The category Asian consists of all the students who considered themselves only Asian, or Asian and some other race. If the students did report some other race besides White or Asian, they were classified as multiracial students. Ethnicity was a separate item; here students could choose between Hispanic or Latino, or Not Hispanic or Latino. The distributions of respondents by ethnicity and race are shown in Table A3.3.

Table A3.3: Survey Respondents by Ethnicity and Race

	IBL Math Track		**Non-IBL Math Track**		**IBL Pre-Service**		**Non-IBL Pre-Service**		**Total**	
	Count	%	Count	%	Count	%	Count	%	Count	%
Ethnicity										
Hispanic or Latino	47	*8.3*	37	*11.1*	27	*12.2*	10	*41.7*	121	***10.6***
Not Hispanic or Latino	520	*91.7*	296	*88.9*	195	*87.8*	14	*58.3*	1025	***89.4***
	567	*100%*	333	*100%*	222	*100%*	24	*100%*	1146	***100%***
Race										
Asian	138	*25.6*	118	*37.9*	22	*11.0*	6	*31.6*	284	***26.5***
Multiracial	24	*4.4*	12	*3.9*	19	*9.5*	1	*5.3*	56	***5.2***
White	378	*70.0*	181	*58.2*	159	*79.5*	12	*63.2*	730	***68.2***
TOTAL	540	*100%*	311	*100%*	200	*100%*	19	*100%*	1070	***100%***

Less variety appeared in students' ethnicity and race. Most of the students were white and not Hispanic or Latino. About a quarter of the students were Asian (26.5%), but the sample included only a few students from other races (5.2%). The sample represents a distribution that is typical for mathematics students in the large research universities that our study targeted.

A3.2.4 Academic Status

The pre-survey provided us with information about students' academic status at the beginning of their mathematics course. Table A3.4 shows the distribution of respondents' academic background by course type.

Table A3.4: Survey Respondents by Academic Status

	First year		Sophomore or Junior		Senior or more		*Total*	
Student Group	Count	%	Count	%	Count	%	Count	*%*
IBL math-track	206	*35.8*	183	*31.8*	187	*32.5*	576	*100*
Non-IBL math-track	147	*43.2*	117	*34.4*	76	*22.4*	340	*100*
IBL pre-service teachers	3	*1.3*	99	*43.8*	124	*54.9*	226	*100*
Non-IBL pre-service teachers	0	*0.0*	10	*40.0*	15	*60.0*	25	*100*
TOTAL	356	*30.5*	409	*35.0*	402	*34.4*	1167	*100%*

Our sample included students across all stages of their college studies. However, nearly one-third (30.5%) of all the students were first-year students. This applied especially to IBL (35.8%) and even more to non-IBL (43.2%) math-track students. The pre-service teachers in the sample were further along in their studies. More than half (54.9%) were seniors or even more advanced students but only three of them were first-year students. The same trend applied to the small group of non-IBL pre-service teachers.

A3.3 Survey instruments

The final survey instruments consisted of an attitudinal pre-survey, a learning gains post-survey, and a combined post-survey including both the attitudinal questions and the learning gains questions. Both the pre- and post surveys gathered personal information about students' gender, race and ethnicity, class year, academic majors, grade-point average, and plans to pursue teaching certification. We asked students to set themselves an identifier at the end of each survey. These identifiers were used to match the pre-survey responses with the post-survey responses individually.

In order to check the survey items and the structures, both the attitudinal and the learning gains survey were tested with two small samples of college mathematics students. Descriptive statistics and principal component analysis were used with these preliminary data sets to check the reliability of the questions and theoretical constructs in the surveys. Based on these analysis, we left out ill-behaving questions and shortened the attitudinal survey. In order to shorten the combined post-survey, we also left out some overlapping questions from the initial learning gains survey. The final surveys are presented as Exhibit E3.1 and E3.2.

A3.3.1 Attitudinal Survey

We wanted to study the nature of students' mathematical beliefs, affect, learning goals, and mathematical problem-solving strategies, and changes in these during a college mathematics course. We designed a structured survey to measure undergraduate students' mathematical beliefs, affect, learning goals and strategies of problem solving, to be administered at the beginning and end of a college mathematics course. The seven sections measured students' interest in and enjoyment of mathematics, preferred goals in studying mathematics, and their frequency of use of various problem-solving actions when doing mathematics, and their beliefs about learning mathematics, problem solving, and proofs.

A3.3.1.1 Theoretical basis of the attitudinal survey

The sub-sections and items were constructed on the basis of theory and previous research on mathematical beliefs, affect, learning goals and strategies of learning and problem solving. Mathematics education research on beliefs has introduced concepts such as beliefs about the nature of mathematics, about learning mathematics, about problem-solving, and beliefs about the self as a mathematics learner (Malmivuori, 2001; McLeod, 1992). All these categories of beliefs appear to have important implications for how students approach the study of mathematics and act in mathematics learning situations at various age and schooling levels. They may either significantly hinder or help student learning, performance and problem solving (Leder, Pehkonen, & Torner, 2002; Schoenfeld, 1992). Moreover, they influence the development of negative or positive attitudes toward mathematics that have longer-term impacts on students' choices of studying mathematics.

We wanted to check the quality of and changes in these types of important mathematical beliefs. In addition, we chose to study certain types of beliefs that were particularly important for studying college mathematics and that might display possible differences between students in traditional and IBL mathematics courses. For example, mathematical proving represents an important area of beliefs in college mathematics. How students see the nature of proofs significantly affects their success in college mathematics (Knuth, 2002; Selden & Selden, 2007; Sowder & Harel, 2003). Moreover, previous research has identified some differences in these beliefs between students who took traditional or student-centered IBL mathematics classes students (Ju & Kwon, 2007; Yoo & Smith, 2007).

Based on these criteria, testing and revisions of the attitudinal survey, we measured students' beliefs about:

- learning of mathematics (instructor-driven, group work, exchange of ideas)
- mathematical problem-solving (practice vs. reasoning)
- mathematical proving (proving as a constructive activity or as confirming truths; Yoo & Smith, 2007),
- beliefs about the self (confidence in their own math ability, in teaching mathematics)

Studies on affect have a long tradition in mathematics education research. Attitudes about mathematics, confidence, motivation and anxiety are the most-studied factors, found to essentially strengthen or diminish students' willingness and ability to learn mathematics (Frost, Hyde, & Fennema, 1994; Goldin, 2000; Malmivuori, 2001, 2007; McLeod, 1992). Interest in and enjoyment of mathematics learning represent central features of affect and students' motivation. Interest is suggested to facilitate deep rather than surface-level processing, and the use of more efficient learning strategies (Entwistle, 1988; Schiefele, 1991). In turn, students who enjoy learning tend to exert more effort and persist longer when they are challenged (Stipek, 2002). Both interest and enjoyment indicate students' strong positive relationship to mathematics and willingness to spend time and effort in studying mathematics. This relationship is also importantly weakened or strengthened by students' confidence in their own ability to do and learn mathematics. This applies to female students in particular (Fennema, Seegers & Boekaerts, 1996). But confidence as related to enhanced self-efficacy is found to essentially promote all students' engagement and cognitive performance (Bandura, 1993; Malmivuori, 2001; Zimmerman, 2000).

Recent education psychological literatures suggest that the types of learning goals pursued by students also profoundly impact the quality of their learning. They direct students' level of achievement, self-regulation and problem-solving strategies (Pintrich, 2000). Closely related to personal interest, we studied students' learning goals, categorizing them broadly as intrinsic vs. external. For example, students who pursue high grades, seek particular degrees, and display high competence and self-concept express external or performance goals that are related to superficial learning. In contrast, intrinsic or mastery-focused goals such as a focus on one's own effort, pursuit of knowledge, and desire to understand the learned material are seen to result in independence, responsibility and deeper learning (Ames & Archer, 1988). In contrast to externally motivated students, intrinsically motivated students show higher interest, excitement, and confidence that enhance their performance, persistence, and creativity (Ryan & Deci, 2000).

IBL teaching practices often involve group work and collaboration that both require and develop communication skills (Gillies, 2007; Duch, Groh, & Allen, 2001). Indeed, recommendations for undergraduate programs in mathematics include development of analytical thinking, critical reasoning and problem-solving but also communication skills (Pollatsek et al., 2004). In the attitudinal survey, we wanted to study students' preferences for communicating about mathematics and any change in this during their IBL course. Items on students' goal of communicating about mathematics measured this preference.

In addition to mathematical confidence, our attitudinal survey studied students' affect and motivation in the form of:

- personal interest in mathematics,
- willingness to pursue a math major (or minor),
- plans to study more math in the future,

- interest in teaching,
- intrinsic and extrinsic learning goals,
- goal for communicating about mathematics,
- enjoyment of learning mathematics

Current understanding of mathematical knowledge as a constructive activity (Steffe & Thompson, 2000; Tall, 1991) focuses on skillful problem-solving (Schoenfeld, 1992). Competent mathematicians use strategies to make sense of new problem contexts or to make progress toward the solution of problems when they do not have ready access to solution methods for them (Schoenfeld, 2004). The nature of the problem-solving strategies they choose influence students' approaches to and success at challenging mathematical problems. For example, unlike novices, expert problem-solvers use high-level planning and qualitative analysis before attacking a problem. They also demonstrate facility in choosing appropriate strategies in various situations (Kroll & Miller, 1993; Schoenfeld, 1985). This contrasts with a lack of strategies, mechanical use of concepts, and rote memorization of previous similar problems.

IBL approaches provide opportunities for students to engage in knowledge creation and argumentation (Rasmussen & Kwon, 2007). Such activities are generally suggested to promote problem-solving skills, independent thinking and intellectual growth (Buch & Wolff, 2000; Duch, Gron & Allen, 2001). Competent problem solvers can communicate the results of their mathematical work effectively, both orally and in writing (Schoenfeld, 2004). Planning and self-monitoring the solving process also help to ensure skillful problem-solving (Schoenfeld, 1985). Moreover, mathematics students must develop persistence in the face of difficulties, tolerance for ambiguity, and willingness to try multiple approaches, and they must learn to apply the necessary amount of rigorous and judgmental reasoning (Hanna, 1991; Pollatsek et al., 2004). These skills require them to develop self-reflective and self-regulatory strategies (Burn, Appleby & Maher, 1998; De Corte, Verschaffel & Eynde, 2000).

We wanted to check what kind of learning and problem-solving strategies students report and how these change during IBL and traditional math courses. Our attitudinal survey studied students' use of:

- independent (or individual),
- collaborative, or
- self-regulatory strategies.

Items related to these strategies were intended to explore the extent to which students counted on their own thinking and creativity when solving math problems and proofs, shared their thinking and strategies with other students, and actively reflected on and regulated (planned or checked) their own thinking and actions while solving math problems.

A3.3.2 Example items from the pre/post attitudinal survey instrument

We divided the attitudinal survey into eight subsections, each consisting of 7-15 structured items. The answers varied on a 7-point Likert-scale between negative and positive responses. The subsections of the survey measured students' mathematical beliefs, motivation, learning goals, enjoyment, confidence, and strategies for leaning and problem-solving. All the attitudinal pre-survey questions and items are presented in Exhibit E3.1. We studied students'

- *Personal interest* in studying mathematics: How likely is it that you will…
  e.g., "Bring up mathematical ideas in a non-mathematical conversation?"
- *Enjoyment* of doing and discovering mathematics: How much do you enjoy…
  e.g., "Discovering a new mathematical idea?"
- *Goals in studying mathematics*: Below are some goals that students may have in studying mathematics. How important is each goal for you?
  e.g., "Memorizing the sets of facts important for doing mathematics." (extrinsic goal)
  "Learning to construct convincing mathematical arguments." (intrinsic goal)
- *Beliefs about the self*: confidence: How confident are you that you can…
  e.g., "Apply a variety of perspectives in solving problems?" (math ability)
  "Teach mathematics to high school students?" (teaching mathematics)
- *Beliefs about learning mathematics*: I learn mathematics best when…
  e.g., "The instructor lectures." (instructor-driven)
  "I work on problems in a small group." (group work)
  "I explain ideas to other students." (exchange of ideas)
- *Beliefs about problem-solving*: In order to solve a challenging math problem, I need…
  e.g., "To have lots of practice in solving similar problems." (practice)
  "To use rigorous reasoning." (reasoning)
- *Beliefs about proofs*: The following statements reflect some students' views about mathematical proof. How much do you agree or disagree with each statement?
  e.g., "Proof is a tool for understanding mathematical ideas." (constructive)
  "The main purpose of proof is to confirm the truth of a mathematical result that is already known to be true." (confirming)
- *Problem-solving strategies*: When you do math, how often do you take each action listed below?
  e.g., "Find your own ways of thinking and understanding." (independent)
  "Brainstorm with other students." (collaborative)
  "Plan a solving strategy before attacking a problem." (self-regulatory)

A3.3.3 Demographic and background information

The attitudinal survey also asked for demographic and background information about students' previous achievement, personal information, and expectation for the grade of the target course. The questions dealt with:

- achievement history: the highest level of high school mathematics taken, any AP Calculus test taken and scores received, number of college math courses taken, estimated overall GPA;
- academic background: class year, college major, pursue for a teaching certification;
- personal background: gender, ethnicity, race;
- expected grade for the course.

At the end of both the pre- and post-survey, students were asked to assign themselves an identifier. This enabled us to match between pre-survey and post-survey responses by individual student (see Exhibit E3.1).

A3.4 Learning Gains Survey

The learning gains post-survey was based on the SALG instruments (SALG, 2008) developed to enable faculty and program evaluators to gather formative and summative data on classroom practices. The questions address students' self-reported experiences of mathematics class practices and their cognitive, social and affective learning gains due to their participation in a college mathematics course. Students provide both quantitative ratings and written responses about the course focus, learning activities, content and materials. The learning gains instrument is grounded in its authors' (Seymour, Wiese, Hunter & Daffinrud, 2000) findings that:

- students can make realistic appraisals of their gains from aspects of class pedagogy and of the pedagogical approach employed, and
- this feedback allows faculty to identify course elements that support student learning and those that need improvement if specific learning needs are to be met.

The SALG instrument is easily modified to meet the needs of individual faculty in different disciplines and it has been found to be a powerful and useful tool for instructors in student feedback and course development. When first developed, data about the use of the survey showed that eighty-five percent of the instructors reported that the SALG provided qualitatively different and more useful student feedback than traditional student course evaluations. Instructors also made modifications to course design (60%) and class activities (lecture, discussion, hands-on activities) followed by student learning activities (54%) course content (43%), and the information given to students (33%) (Recommendations for using the SALG, 2008).

We adjusted the SALG items to match college mathematics situations. The final learning gains survey, which we call the SALG-M, consisted of four structured sections on course experiences and two sections on learning gains. The first four sections asked about students' experiences of instructional practices: how much particular practices helped their learning. The practices deal with overall instructional approach, classroom activities, tests and other assignments, and interactions during the course. Answers follow a five-point scale between "no help" and " great help." Two other structured sections of the questionnaire ask about students' gains in understanding, confidence, attitude, persistence, and collaboration. These answers vary on a five-point scale between "no gain" and "great gain." The final post-survey for the SALG-M is presented as Exhibit E3.2.

In addition to structured items, the learning gains post-survey included four open-ended questions. Students were provided space to write about:

- How the class changed the ways they learn mathematics
- How their understanding of mathematics changed as a result of the class
- How the way the class was taught affected their ability to remember key ideas
- What they will carry with them from the class into other classes or other aspects of their life.

Answers to these questions complemented numerical responses on students' gains from their mathematics courses, helping us to better understand these results.

The learning gains survey also gathered information on students' expected grade at the end of the course, college major, class year, gender, and whether they were pursuing teaching certification. These questions confirmed the match between pre- and post-surveys and enabled us to detect changes in students' ideas or plans. The complete post-survey consists of the structured items in the attitudinal pre-survey and all the sections of the learning gains survey.

A3.5 Data collection

Survey data were gathered from undergraduate students studying mathematics at all the four campuses during two academic years 2008-2010. We started with online survey instruments when testing the survey instruments and also gathered pre-surveys at one campus in early fall of 2008. Due to the low response rate to the online form, we gathered the rest of the survey data as a paper-and-pencil test in class, which yielded very high response rates. The paper questionnaire was administered at the beginning and end of each course. In the courses that were part of a multi-term sequence (e.g., a three-quarter calculus sequence), we administered the full post-survey only at the end of the final section, but also gave a learning gains survey at the end of each previous related section. This provided us with some longitudinal data on the evolution of students' experiences and learning gains over multiple terms of IBL or comparative instruction.

The surveys were delivered to our project collaborators at the four campuses who also mostly administered the surveys in class. In some cases, course instructors or teaching assistants administered the survey. Instructors were given instructions on how to administer the surveys and return the completed surveys to us. We also reminded them about keeping the confidentiality and anonymity of students in every step. Filling out the surveys took students about 10-20 minutes; instructors were asked to offer enough class time for completing the surveys.

A3.6 Data analysis

A3.6.1 Attitudinal survey variables

Composite variables were constructed based on the attitudinal survey design and the factors of mathematical beliefs, affect, learning goals, and strategies of learning and problem-solving presented in Section A3.3.1.1. Exploratory factor analysis, principal component analyses and item analyses on the attitudinal survey data were used to create the final composite variables: five measures of motivation, affect and confidence; three measures of learning goals; seven measures of beliefs about mathematics and learning; and three measures of strategies. For each composite variable, averages of student ratings across the items represented the score for each student. This enabled us to interpret results on the same scale as that for the original attitudinal survey items. The composite variables were then used to report results on students' attitudes and for further analysis on group differences in attitudes.

Table A3.5 displays the survey questions and items for each composite variable, the titles and descriptions of the composite variables, and the reliability scores for the composite variables (for the pre-survey data and post-survey data separately).

Table A3.5: Composite Variables Measuring Student Beliefs, Affect, Goals and Problem-Solving Strategies

Variable	**Description**	**Scale**	**Items**		**Reliability** Cronbach alpha	
			Count	Numbers	Pre	Post
Motivation						
Interest	Interest in learning and discussing mathematics	7	3	Q1: 5,6,7	0.808	0.828
Math major	Desire to graduate with a math major	7	1	Q1: 2	-	-
Math future	Desire to pursue math in future work or education	7	2	Q1: 1,4	0.439	0.615
Teaching	Desire to teach math	7	1	Q1: 8	-	-
Enjoyment	Pleasure in doing and discovering mathematics	7	6	Q2: 1-6	0.914	0.928
Confidence						
Math confidence	Confidence in own mathematical ability	7	5	Q9: 1,2,4, 5,6	0.820	0.826
Teaching confidence	Confidence in teaching math	7	2	Q9: 3,8	0.696	0.645

Table 3.5, continued…

Goals for studying math						
Intrinsic	Learning new ways to think & to apply math	7	4	Q3: 7-10	0.791	0.828
Extrinsic	Meeting requirements; degree, good grades	7	4	Q3: 1,3,4,6	0.724	0.744
Communicating	Communicating mathematical ideas to others	7	2	Q3: 2,5	0.783	0.810
Beliefs about learning						
Instructor-driven	Exams, lectures, instructor activities	7	4	Q5: 1,6,7,8	0.642	0.667
Group work	Whole-class or small group work	7	3	Q5: 2,3,5	0.685	0.719
Exchange of ideas	Active verbal interaction with other students	7	3	Q5: 9,10, 11	0.731	0.745
Beliefs about problem-solving						
Practice	Repeated practice, remembering	7	2	Q6: 2,6	0.690	0.758
Reasoning	Rigorous reasoning, flexibility in solving	7	5	Q6: 1,5,7, 8,9	0.734	0.712
Beliefs about proofs (Yoo & Smith, 2007)						
Constructive	Process view; revealing mathematical ideas	7	4	Q8: 2,6,7,8	0.637	0.675
Confirming	Product view; recall and confirming conjectures	7	3	Q8: 1,3,5	0.692	0.672
Strategies						
Independent	Finding one's own way to think & solve problems	7	4	Q4: 5,9,11, 12	0.747	0.775
Collaborative	Seeking help, actively sharing with others	7	3	Q4: 2,4,14	0.774	0.813
Self-regulatory	Planning, organizing, reviewing one's own work	7	6	Q4: 1,3,6, 7,8,10	0.747	0.747

A3.6.2 Learning gains survey variables

Similar to the treatment of attitudinal variables, composite variables were constructed on the basis of the questions and structures in the SALG-M survey. Exploratory and principal component analyses and item analysis produced five measures of instructional practices and nine measures of learning gains (see Table A3.6). The five composite variables related to instructional practices were used in reporting results on students' course experiences. Results on learning gains from the nine composite variables represented students':

- Cognitive gains: mathematical concepts, mathematical thinking, application of mathematical knowledge,
- Affective gains: positive attitude, confidence, persistence,
- Social gains: collaboration, comfort in teaching mathematics,
- Independence in learning mathematics.

Table A3.6: Composite Variables Measuring Student Experiences and Learning Gains

Variable	Description	Scale	Items		Reliability (Cronbach)
			Count	Numbers	
Experience of course practices (what helped me learn)					
Overall	Teaching approach, atmosphere, pace, workload	5	7	Q1: 1-7	0.898
Active participation	Personal engagement in discussion & group work	5	5	Q2: 3-7	0.839
Individual work	Studying & problem-solving on one's own	5	4	Q2: 2,8,9	0.695
Assignments	Nature of tests, homework, other assigned tasks	5	8	Q4: 1-8	0.764
Personal interactions	Interaction with peers & instructor, in/out of class	5	6	Q5: 1-6	0.696
Learning gains: Cognitive gains					
Math concepts	Understanding concepts	5	2	Q6: 1,2	0.921
Math thinking	Understanding how mathematicians think	5	2	Q6: 3,4	0.819
Application	Applying ideas elsewhere, understanding others' ideas	5	3	Q6: 5,6,7	0.629
Learning gains: Affective gains					
Positive attitude	Appreciation of math	5	2	Q8: 3,6	0.821
Confidence	Confidence to do math	5	4	Q8: 1,2,7,8	0.905
Persistence	Persistence, stretching	5	2	Q8: 9,14	0.852
Learning gains: Social gains					
Collaboration	Working with others, seeking help	5	3	Q8: 10,12,13	0.841
Teaching	Comfort in teaching math	5	1	Q8: 11	-
Learning gains: Independence	Work/organize on own	5	2	Q8: 4,5	0.806

We also report results on students' cognitive, affective and social gains as three main areas of learning gains. Gain in independence in learning mathematics represented a measure that is distinct from the other three main areas.

Table A3.6 displays the titles and descriptions of the learning-related composite variables, the survey items that comprise each variable, and the reliability scores for each composite variable.

A3.6.3 Analysis methods for structured survey questions

All survey data was entered by student technicians and analyzed using the SPSS computer software package. Statistical analyses included descriptive statistics of each composite variable and background variable. Correlation analysis was used to study relationships between composite variables and their relation to background information on students' overall college GPA and expected grade at the beginning and end of their course. Parametric (independent and pair-wise T-tests, ANOVA) or non-parametric (Chi-square, Mann-Whitney, Kruskas-Wallis) tests were used to explore group differences in students' attitudes, experiences, and learning gains, sorted by demographic information on students' gender, ethnicity, race, academic status, and college major. The most important of these analyses focused on differences between IBL and non-IBL students, and between math-track students and pre-service teachers. Analysis of covariance (ANCOVA) was used to check intermediate effects (GPA, expected grade, gender) on students' learning gains. Stepwise regression analysis was applied to examine the variation in students' learning gains versus changes in their attitudes and self-reported class experiences.

A3.6.4 Analysis of open-ended survey questions

The open-ended survey questions asked about students' gains or changes in their understanding of mathematics, remembering key ideas, ways to learn mathematics, and other things they carry with themselves from a math course. Most of students' written comments addressed reports of learning gains from a course or possible difficulties or negative experiences from a course. To analyze the written responses, we applied the same categories that were constructed for analyzing student interview data. The preliminary categories were fleshed out with more detailed descriptions and subdivided into several subcategories of learning gains and processes using inductive content analysis (Miles & Huberman, 1994; Strauss & Corbin, 1990). As each statement was examined, the detected gains were classified into one of the preliminary categories or a new category creating during reading and analysis. Table A3.7 summarizes the final coding scheme for learning gains reported in the open-ended answers, and the frequency with which each was reported.

Table A3.7. Counts for Reported Learning Gains from Open-Ended Survey Comments

Main category	**Subcategory**	**Number of students reporting each gain**		
	Description	once	2-3 times	≥4 times
Cognitive gains	**Subtotal**	**455**	**115**	**5**
	Better recall	82	2	-
	Better knowledge, deeper understanding of mathematical concepts and ideas	120	54	-
	Thinking and problem solving skills	119	42	4
	Transferable mathematical knowledge	23	3	-
	Transferable thinking skills	20	1	-
	Did not gain cognitively	91	13	1
Affective gains	**Subtotal**	**158**	**24**	**1**
	Positive attitude towards mathematics	16	1	-
	Confidence to do math, solve math problems, and be a mathematician	38	8	1
	Less confidence, no gain in confidence	5	-	-
	Enjoyment, liking math	28	2	-
	Negative experience, liking less	62	13	-
	Interest and motivation	8	-	-
	Less interested	1	-	-
Changes in learning	**Subtotal**	**428**	**63**	**4**
	Beliefs about learning math, deeper learning, problem solving, creativity and discovery, finding own style of learning math	133	33	3
	Independence in mathematical thinking, learning or problem solving	93	14	1
	Persistence	17	-	-
	Work ethic, learned to study hard	21	-	-
	Metacognition, self-reflection	17	1	-
	Appreciation others' thinking, learning from others	66	14	-
	No change in learning math	81	1	-

Table A3.7, continued…

Gains in communication skills	**Subtotal**	**116**	**9**	**-**
	Speaking or presenting	14	-	-
	Writing mathematics	23	4	-
	Collaboration, group work	29	3	-
	Teaching others, explaining to others	46	2	-
	Giving or receiving critique	4	-	-
Changes in understanding the nature of mathematics	**Subtotal**	**78**	**20**	**-**
	How knowledge is built; how research math is done	14	-	-
	Change in conceptualization of math	59	20	-
	No change in concepts of the nature of math	5	-	-
Total		**1235**	**231**	**10**

In all, 544 students wrote in at least one gain in cognition, affect, communication, ways of learning mathematics, and/or understanding the nature of mathematics. They reported one to as many as nine gains each. In all, 197 students reported 1-6 times each that they did not make gains or undergo changes in cognition, affect, communication, ways of learning mathematics, or understanding the nature of mathematics. The rest of survey respondents wrote no comments.

A3.7 Reliability and validity

Most of the pre- and post-surveys were administered and gathered in mathematics classes by the project coordinators of each campus, or by instructors. The surveys were completed in class, which strengthened the response rate. The coordinators were given written instructions for administering the surveys that were intended to ensure that students had enough time to answer the surveys and that their anonymity was preserved. Completed surveys were delivered to the research team by the campus coordinators, entered by trained project assistants into separate SPSS files, and checked and analyzed by the researchers. We will describe features of our survey data from the attitudinal survey and the learning gains survey separately.

A3.7.1 Attitudinal survey data

After testing the structures and items in the attitudinal survey with a small group of students, we revised the instrument accordingly. In the full data collection, we gathered a large number of completed pre-and post-surveys, which ensured high statistical power in our results. Missing answers both in the structured attitudinal and gain survey items were rare. Even though the post-survey was rather long, most students responded to all the structured items. The number of missing answers on the items varied between 0-49 for the main pre-survey items and 0-37 for the main attitudinal post-survey items. These low numbers indicate that students understood the questions and statements in the items. All these features strengthen the reliability and validity of our attitudinal survey results.

Among the combined survey responses, some students did not report specific demographic information on their:

- gender (49),
- race (148) or ethnicity (74),
- academic major (79),
- academic status (46),
- number of prior college mathematics courses (53)
- AP test score (430; many students did not take the AP test),
- prior GPA (291; many first year students had no prior GPA),
- expected grade at the beginning of a course (86).

The most common type of missing information was self-reported AP test scores and prior GPA. Thus the results reported on group differences by prior GPA largely excluded first-year students. We used AP test scores in our study of group differences only in analyzing LMT scores for pre-service teachers.

The number of responses about students' beliefs on mathematical proofs (N=877) and about confidence (N=672) were somewhat smaller than those on other topics. Only students with prior experience on proofs were asked to answer proof-related survey questions. A subsection on confidence was added to the attitudinal questionnaire after some students had completed it. For both sections, the lower number of answers implies slightly lower statistical power in comparison to the results on other survey questions.

Descriptive statistics for the composite attitudinal variables showed variation among students. Responses for the items varied between the minimum 1 and maximum 7 for most items. The minimum for only one composite variable was above 2 (Reasoning 2.2 on the post-survey). However, standard deviations for the pre-survey variables varied between 0.88 and 2.56, and for the post-survey variables between 0.86 and 2.63, indicating low or moderate variation among students in their attitudes.

Cronbach's alphas were used to study the reliabilities of the subsections and composite variables for both attitudinal and SALG-M survey instruments. The final reliability scores for attitudinal composite variables are presented in Table A3.5, for pre-survey and post-survey data separately. Reliability scores for the pre-survey varied between 0.439 and 0.914 and between 0.615 and 0.928 for the post-survey. Only five composite variables had a low reliability score (below 0.7) in the pre-/post-survey data: math future interest, teaching confidence, instructor-driven beliefs, and beliefs about proofs.

Correlational analysis showed that the composite variables had good construct validity, meaning they produced real results on students' motivation, beliefs and strategies. For example, on the post-survey data, both composite variables related to motivation (personal interest, math future) correlated highly positively with each other (r= 0.417) but also with the variables on intrinsic learning goals (r=0.552, r=0.354) and enjoyment of learning mathematics (r=0.718, r=0.413). Confidence in mathematics (r=0.507) and teaching (r=0.309) correlated highly positively with enjoyment of mathematics. Math confidence also correlated clearly with the use of independent strategies (r=0.445) and beliefs about math problem-solving as reasoning (r=0.531).

Beliefs that mathematics learning is an instructor-driven activity correlated highly positively with extrinsic learning goals (r=0.454), beliefs about problem-solving as practice (r=0.499), and beliefs about proving as confirming the truth (r=0.359). In contrast, beliefs about math problem-solving as reasoning correlated highly positively with enjoyment of mathematics (r=0.512) and intrinsic learning goal (r=0.539). Moreover, use of independent strategies correlated highly positively with use of self-regulatory strategies (r=0.606), whereas use of collaboration correlated positively with communicating goal (r=0.289) and beliefs about mathematics learning as group work (r=0.459) or exchange of ideas with other students (r=0.422).

A3.7.2 Learning gains survey data

We collected even larger numbers of responses on the SALG-M items (N=1074-1127) than on the attitudinal pre-/post-survey measures. This ensured high statistical power for results on students' self-reported class experiences and learning gains. The numbers of analyzed responses on the SALG-M items was lower than in the attitudinal data because students could choose the response "Not applicable." Open-ended survey questions on the SALG-M were optional, and students commonly chose not to respond to these. Overall, 13 to 38% of IBL math-track students and 12 to 20% of non-IBL math-track students chose to write in a comment about their learning gains (depending on the question). These low percentages are typical for open-ended survey questions.

The whole scale between 1 and 5 was used by students in their answers to questions about both course experiences and learning gains. However, descriptive statistics showed a rather strong "halo" effect. The median for all the course experience variables (except Assignments other than tests) was above 3.8, and for the learning gains variables above 3.5 (except Application). The standard deviations ranged between 0.82 and 0.95 for the course experience variables and between 0.93 and 1.16 for the learning gains variables. These scores indicate rather low variation in students' answers to the survey items.

The reliability scores for the composite variables from the SALG-M instrument are presented in Table A3.6. The scores indicated even higher reliability and internal consistency than for the attitudinal variables. For the course experience composite variables, the reliability scores varied between 0.696 and 0.898. Reliability scores for the composite learning gains variables similarly varied between 0.629 and 0.921. Only one composite variable (gains in application) had a reliability score less than 0.7.

The SALG survey instruments are based on extensive research that support the validity of self-report in situations where students have the ability to provide accurate information (Wentland & Smith, 1993) and where they have few or no obvious reasons (such as adverse consequences or social embarrassment) for providing inaccurate information (Aaker, Kumar & Day, 1998). The SALG survey instruments meet standards of good validation. The developers of the instrument confirmed that most survey items functioned adequately and that item composites formed reliable subscales (Weston, Seymour & Thiry, 2006; Weston, Seymour, Lottridge & Thiry, 2006). Moreover, latent factors underlying items conformed to the hypothesized structures of the survey.

We adjusted the original SALG survey instrument to fit college mathematics learning situations and refer to the result as the SALG-M. The constructed composite variables represented the underlying structures of the SALG instrument and factors specific for the SALG-M survey. These factors were checked for construct validity by correlational analysis. For example, Active Participation, a variable denoting participation in class discussions and group work, was most strongly (positively) related (r=0.634) to the measure of Personal Interaction (see Table A3.6). In turn, Individual Work, measuring students' studying and problem solving on their own, was the most strongly positively related to Assignment (r=0.557), in particular to Assignments other than regular tests (r=0.529). Moreover, the Overall measure of course experience correlated highly positively (0.505-0.680) with all the other composite variables on course experiences.

All students reported rather positive course experiences and high learning gains as result of a course. Correlational analysis further showed strong positive linkages between the learning gains variables (0.403-0.740) and between course experiences and learning gains (0.314-0.680). These results confirm construct validity of the results and the fact that positive class experiences were related to higher reported learning gains. On the other hand, students who reported positive experience in one area also did so on the other measured class experiences. Moreover, students who reported high learning gains in one area also did so on other measures of learning gains. This again reflects the halo effect that makes it more difficult to find real differences in students' class experiences and learning gains.

A3.7.3 Connections between the surveys and other measures of learning gains

We checked connections between the composite variables based on the structured SALG-M survey questions and numerical variables related to the open-ended answers. Correlations between the answers to structured and open-ended questions showed that higher self-assessed cognitive, affective and social learning gains were all clearly positively related to a higher number of gains each student wrote in. This was valid both for the total count of gains reported in written comments (r=0.133 to 0.239) and for the separate counts of cognitive gains, affective gains and changes in students' ways of learning math. In particular, higher self-assessed gains in math concepts and thinking were clearly positively connected (r=0.209**) to a greater number of cognitive gains written in response to the open-ended questions. Moreover, higher self-assessed gains in collaboration were clearly positively connected (r=0.209**) to a greater number of gains in ways of learning mathematics as reflected in written comments. These indicate reliability and construct validity of the results on learning gains.

We also checked how changes in attitudes related to learning gains as measured by the SALG-M instrument. The correlations (see Table A3.8) show that students who reported higher cognitive, affective, and social learning gains also showed increases in many of the attitudinal variables. This applied particularly to enhanced motivation, enjoyment, math confidence, and intrinsic and communicating goals during a math course. Pre/post increases both in enjoyment and math confidence were clearly positively related to reported affective gains in confidence, positive attitude, and persistence. This displays construct validity of these affective measures.

Table A3.8: Statistically Significant Correlations between Changes in Beliefs, Motivation and Strategies and Learning Gains

Attitudinal Variable		Correlation with Learning Gains			
		Math concepts & thinking	Application	Affective	Social
Motivation	*interest*	0.186**	0.140**	0.199**	0.143**
	math major	0.149**	0.125**	0.158**	
	math future	0.115**		0.107**	
	teaching	0.118**		0.105**	
Goals	*intrinsic*	0.170**	0.104**	0.182**	0.156**
	communicating	0.121**		0.132**	
Enjoyment		0.228**	0.204**	0.266**	0.194**
Confidence	*math ability*	0.249**	0.233**	0.245**	0.166**
Beliefs about learning	*group work*	0.142**	0.121**	0.145**	0.195**
	exchange of ideas	0.154**	0.150**	0.166**	0.144**
Beliefs about problem-solving	*reasoning*	0.205**	0.155*	0.186**	0.189**
	practice	0.205**	0.155**	0.186**	0.189**
Strategies	*independent*	0.134**	0.129**	0.121**	0.164**
	collaborative				0.188**
	self-regulatory	0.204**	0.138**	0.204**	0.183**

*** p< 0.01*

Similarly, gains in mathematical concepts and thinking were positively related to increases in most of the attitudinal variables. The strongest positive relation was to increased belief in reasoning in solving math problems, but also to enhanced enjoyment and math confidence and to increased use of self-regulatory strategies. Positive relationships between reported gains in application and attitudinal variables showed similar but somewhat weaker correlations. All these positive relations display good construct validity for the variables involved.

Moreover, students with higher reported learning gains developed strengthened beliefs in the value of group work and exchange of ideas with other students that generally contribute to learning. Increased belief in reasoning as a way to solve math problems again enhances mathematics learning and problem solving. Moreover, students' increased use of independent and self-regulatory strategies in learning were clearly positively related to their learning gains. The use of these strategies generally enhances learning. In particular, gains in collaboration were positively related to increased use of collaborative learning strategies.

Correlational analysis (Spearman) between the learning gains composite variables and other measures of learning outcomes indicated low to moderate connections. Self-reported cognitive, affective and social learning gains from a mathematics course did not correlate with self-reported GPA level at the beginning of a course. But grades generally measure student *performance*, which is not necessarily related to *learning* in any given course.

In addition, our GPA measure excluded first-year students who could not report their prior GPA at the beginning of a course but who nonetheless reported higher gains than older students. Furthermore, our results on learning gains indicated higher learning gains among students with lower prior GPA. These features are also perhaps reflected in the low correlations between learning gains and GPA level at the beginning of a course.

We also checked the correlations (Spearman) between self-reported learning gains and self-reported AP test score, for the smaller number of students who reported an AP score. However, the correlations indicated only a weak positive relation to gains in math concepts and thinking (r=0.106*). Again, AP test score is a measure of past mathematics performance, but does not determine learning in the present course.

However, correlations between learning gains and expected grade in the course were somewhat stronger, especially to the expected grade reported at the end of the course. These correlations varied between 0.10** and 0.307**. In particular, gains in mathematical concepts and thinking clearly correlated with expected grade at the end of a course (r=0.238**). The positive correlation of expected grade was even stronger to affective learning gains (r=0.307**) but weaker to social gains in collaboration (r=0.10**). These correlations display clear but moderate connections between students' self-reported learning gains and their assessment of the quality of their learning during a college math course.

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Exhibit E3.1: Attitudinal Pre-Survey

Dear student,

Our research team is studying methods of improving teaching and learning in college mathematics courses, including the methods used in the course you are taking now. Because you are enrolled in a college math course, we would like to know about your own experiences in learning mathematics. This survey asks about your views about mathematics, your strategies for learning math, and your personal reasons for studying mathematics.

Your participation is voluntary. You may skip questions you do not wish to answer, or choose not to participate. Your answers are anonymous and will not be reported in any way that may identify you individually; they will be aggregated with responses by other students from your course and other courses. Your instructor will not know how you answered.

By completing this survey, in part or in whole, you agree that we may use this data to understand and improve the quality and effectiveness of mathematics instruction.

Please, mark clearly the best answer to each question. You do NOT need to fill in the bubble completely.

Thank you for your candid responses! Please contact us with any questions.

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Your interest in mathematics

1. HOW LIKELY is it that you will...

	Not at all likely						Extremely likely
Take additional math courses after this course?	○	○	○	○	○	○	○
Graduate with a college math major?	○	○	○	○	○	○	○
Graduate with a college math minor?	○	○	○	○	○	○	○
Study hard for a college math course?	○	○	○	○	○	○	○
Read magazine or newspaper articles related to math?	○	○	○	○	○	○	○
Bring up mathematical ideas in a non-mathematical conversation?	○	○	○	○	○	○	○
Participate in a club or organization related to math?	○	○	○	○	○	○	○
Teach math in the future?	○	○	○	○	○	○	○

Exhibit E3.1: Attitudinal Pre-Survey

Your enjoyment of mathematics

2. HOW MUCH do you ENJOY...

	No enjoyment						Extreme enjoyment
Working on a challenging mathematical problem?	○	○	○	○	○	○	○
Discovering a new mathematical idea?	○	○	○	○	○	○	○
Seeing mathematics in everyday life?	○	○	○	○	○	○	○
Perceiving beauty in mathematical ideas?	○	○	○	○	○	○	○
Using rigorous reasoning in a math problem?	○	○	○	○	○	○	○
Thinking about abstract concepts?	○	○	○	○	○	○	○
Teaching mathematics to other people?	○	○	○	○	○	○	○

Your goals in studying mathematics

3. Below are some goals that students may have in studying mathematics. HOW IMPORTANT is each goal for YOU?

	Not at all important						Extremely important
Learning specific procedures for solving math problems	○	○	○	○	○	○	○
Improving your ability to communicate mathematical ideas to others	○	○	○	○	○	○	○
Getting a good grade in college mathematics courses	○	○	○	○	○	○	○
Memorizing the sets of facts important for doing mathematics	○	○	○	○	○	○	○
Making mathematics understandable for other people	○	○	○	○	○	○	○
Meeting the requirements for your degree	○	○	○	○	○	○	○
Learning to construct convincing mathematical arguments	○	○	○	○	○	○	○
Using mathematics as a tool to study other fields	○	○	○	○	○	○	○
Learning new ways of thinking	○	○	○	○	○	○	○
Applying mathematical thinking outside the university context	○	○	○	○	○	○	○

Other goals (please specify)

Exhibit E3.1: Attitudinal Pre-Survey

Your strategies for learning mathematics

4. When you DO MATH, how often do you take each action listed below?

	Very seldom						Very often
Study on your own.	○	○	○	○	○	○	○
Brainstorm with other students.	○	○	○	○	○	○	○
Try to organize or summarize your own ideas.	○	○	○	○	○	○	○
Share problem-solving strategies with other students.	○	○	○	○	○	○	○
Find your own ways of thinking and understanding.	○	○	○	○	○	○	○
Review your work for mistakes or misconceptions.	○	○	○	○	○	○	○
Read the assigned readings.	○	○	○	○	○	○	○
Plan a solving strategy before attacking a problem.	○	○	○	○	○	○	○
Try to find your own way to solve a problem.	○	○	○	○	○	○	○
Check your understanding of what the problem is asking.	○	○	○	○	○	○	○
Use your intuition about what the answer should be.	○	○	○	○	○	○	○
Look for an alternate strategy to solve the problem.	○	○	○	○	○	○	○
Give up when you get stuck.	○	○	○	○	○	○	○
Ask another student for help.	○	○	○	○	○	○	○
Ask the instructor or TA for help.	○	○	○	○	○	○	○

Your preferences for learning mathematics

5. Indicate how much you agree or disagree:
I learn mathematics BEST when...

	Strongly disagree						Strongly agree
The instructor lectures.	○	○	○	○	○	○	○
The class critiques other students' solutions.	○	○	○	○	○	○	○
I work on problems in a small group.	○	○	○	○	○	○	○
The exams let me prove my mathematical skills.	○	○	○	○	○	○	○
Groups present their solutions in class.	○	○	○	○	○	○	○
The instructor explains the solutions to problems.	○	○	○	○	○	○	○
The homework assignments are similar to the examples considered in class.	○	○	○	○	○	○	○
I study my class notes.	○	○	○	○	○	○	○
I can compare my math knowledge with other students.	○	○	○	○	○	○	○
I explain ideas to other students.	○	○	○	○	○	○	○
I get frequent feedback on my mathematical thinking.	○	○	○	○	○	○	○

Exhibit E3.1: Attitudinal Pre-Survey

6. Indicate how much you agree or disagree:
In order to solve a challenging math problem, I NEED...

	Not at all						Very much
To carefully analyze different possible solutions.	○	○	○	○	○	○	○
To have lots of practice in solving similar problems.	○	○	○	○	○	○	○
To understand other students' mathematical thinking.	○	○	○	○	○	○	○
To have natural talent for mathematics.	○	○	○	○	○	○	○
To try multiple approaches to constructing a solution.	○	○	○	○	○	○	○
To remember a lot of examples that I might use in constructing a solution.	○	○	○	○	○	○	○
To use rigorous reasoning.	○	○	○	○	○	○	○
To have freedom to do the problem in my own way.	○	○	○	○	○	○	○
To work hard	○	○	○	○	○	○	○

Your experience and views about mathematical proof

7. Have you had math classes that included mathematical proofs?

○ yes

○ no (Go directly to question 9)

8. The following statements reflect some students' views about mathematical proof. How much do you AGREE or DISAGREE with each statement?

	Strongly disagree						Strongly agree
The main purpose of proof is to confirm the truth of a mathematical result that is already known to be true.	○	○	○	○	○	○	○
Proof is a tool for understanding mathematical ideas.	○	○	○	○	○	○	○
Doing proofs well requires good recall of previous proofs of similar statements.	○	○	○	○	○	○	○
The main purpose of proof is to explain why a certain statement is true.	○	○	○	○	○	○	○
In math class, doing proofs means confirming conjectures that have been previously proven by an expert.	○	○	○	○	○	○	○
There are several different ways to prove a mathematical statement.	○	○	○	○	○	○	○
When evaluating a proof, the most important thing to look at is its logical structure.	○	○	○	○	○	○	○
A proof is something you have to construct based on your own understanding.	○	○	○	○	○	○	○

Exhibit E3.1: Attitudinal Pre-Survey

Your confidence in doing math

9. HOW CONFIDENT are you that you can...

	Not at all confident						Extremely confident
Get a high grade in this course?	○	○	○	○	○	○	○
Successfully work with complex mathematical ideas?	○	○	○	○	○	○	○
Teach mathematics to high school students?	○	○	○	○	○	○	○
Develop new mathematical ideas?	○	○	○	○	○	○	○
Apply a variety of perspectives in solving problems?	○	○	○	○	○	○	○
Present your work at the board in a math class?	○	○	○	○	○	○	○
Work on math problems with other students?	○	○	○	○	○	○	○
Teach math to children?	○	○	○	○	○	○	○

Your math background

10. What was the highest level of math that you took in HIGH SCHOOL?

- ○ Algebra, one year
- ○ Algebra, two years
- ○ Geometry with an algebra prerequisite
- ○ Pre-calculus or trigonometry
- ○ Calculus
- ○ Other (please specify)

11. Did you take the AP Calculus test?

- ○ Yes
- ○ No (go directly to question 14)

12. Which of the AP Calculus tests did your take?

- ○ A/B
- ○ B/C

13. What was your score in the AP Calculus test?

- ○ 1
- ○ 2
- ○ 3
- ○ 4
- ○ 5

14. How many COLLEGE math courses have you taken prior to this course? Please count the total number of semesters or quarters.

- ◯ 0
- ◯ 1
- ◯ 2
- ◯ 3
- ◯ 4
- ◯ 5
- ◯ 6
- ◯ 7 or more

15. What grade do you expect to receive in this course?

- ◯ A
- ◯ A-
- ◯ B+
- ◯ B
- ◯ B-
- ◯ C+
- ◯ C
- ◯ C-
- ◯ D
- ◯ F

Your academic background

16. What is your overall UNDERGRADUATE GPA? (estimated)

- ◯ 3.8 or higher
- ◯ 3.5 - 3.79
- ◯ 3.0 - 3.49
- ◯ 2.5 - 2.99
- ◯ 2.0 - 2.49
- ◯ below 2.0
- ◯ Not applicable

17. What is your class year?

- ◯ First-year
- ◯ Sophomore
- ◯ Junior
- ◯ Senior
- ◯ Graduate student
- ◯ Other (please specify) ____________

Exhibit E3.1: Attitudinal Pre-Survey

Your academic interests

18. What is your college major? (Check ALL that apply)

- ☐ Math or Applied Math
- ☐ Physics
- ☐ Chemistry
- ☐ Engineering
- ☐ Computer science
- ☐ Other science or technical field
- ☐ Economics
- ☐ Other non-science field

19. Are you pursuing a teaching certification?

- ○ no
- ○ yes, elementary (grades K-6 or K-8)
- ○ yes, secondary math (grades 6-12, 8-12, or 9-12)
- ○ yes, secondary in a field other than math

Other (please specify)

____

Your personal background

Our funding agency requires us to gather data on the gender, race and ethnicity of study participants. Please choose the answers that best apply.

20. What is your gender?

- ○ male
- ○ female

21. What is your ethnicity?

- ○ Hispanic or Latino
- ○ Not Hispanic or Latino

22. What is your race? (please check ALL that apply)

- ☐ American Indian or Alaskan Native
- ☐ Asian
- ☐ Black or African American
- ☐ Native Hawaiian or other Pacific Islander
- ☐ White

Exhibit E3.1: Attitudinal Pre-Survey

Assign yourself an identifier

On this page, we ask for some information that will enable us to match your survey responses with those in other surveys. The information will be unique to you but will not identify you individually.

* 23. Enter the following data. Please, print neatly.

FIRST two letters of your FIRST NAME

Two-digit DAY of your BIRTHDAY (01 through 31)

FIRST two letters of your MOTHER'S FIRST NAME

FIRST two letters of the TOWN where you were BORN

Course information

* 24. What is this math course?

Number of the course

Section of the course

Name of the instructor

Survey completed

Thank you for completing the survey! Your input is important to us, and will help us to help math instructors improve teaching and learning in their courses.

If you have any questions, please contact us:

Sandra Laursen, project director
sandra.laursen@colorado.edu
Marja-Liisa Hassi, research associate
hassi@colorado.edu

Exhibit E3.2: Learning Gains Post-Survey (SALG-M)

Dear student,

Our research team is studying methods of improving teaching and learning in college mathematics courses, including the methods used in the course you are taking now. Because you are enrolled in a college math course, we would like to know about your own experiences in learning mathematics. This survey asks about your experiences in this course.

Your participation is voluntary. You may skip questions you do not wish to answer, or choose not to participate. Your answers are anonymous and will not be reported in any way that may identify you individually; they will be aggregated with responses by other students from your course and other courses. Your instructor will not know how you answered.

By completing this survey, in part or in whole, you agree that we may use this data to understand and improve the quality and effectiveness of mathematics instruction. We may compare your responses with your gains from the course, assessed by your instructor. This will be done anonymously by using the identifiers. We will not know your individual grades and your instructor will not know how you answered the questions in this survey.

Please, mark clearly the best answer to each question. You do NOT need to fill in the bubble completely.

Thank you for your candid responses! Please contact us with any questions.

Sandra Laursen, study director
Marja-Liisa Hassi, research associate

Ethnography & Evaluation Research/University of Colorado at Boulder
sandra.laursen@colorado.edu
hassi@colorado.edu

The course as a whole

1. HOW MUCH did the following aspects of the class HELP YOUR LEARNING?

	No help	A little help	Moderate help	Much help	Great help	NOT APPLICABLE
The overall approach to teaching and learning in the course	○	○	○	○	○	○
How class topics, activities, & assignments fit together	○	○	○	○	○	○
The pace of the class	○	○	○	○	○	○
The workload of the class	○	○	○	○	○	○
The general atmosphere of the class	○	○	○	○	○	○
The course material	○	○	○	○	○	○
The mental stretch required of you	○	○	○	○	○	○
The information you were given about the class when it began	○	○	○	○	○	○

Other (please specify)

Exhibit E3.2: Learning Gains Post-Survey (SALG-M)

Class activities

2. HOW MUCH did the following CLASS activities HELP YOUR LEARNING?

	No help	A little help	Moderate help	Much help	Great help	DID NOT HAPPEN
Listening to lectures	○	○	○	○	○	○
Studying on your own	○	○	○	○	○	○
Participating in class discussions	○	○	○	○	○	○
Participating in group work during class	○	○	○	○	○	○
Explaining your work to other students	○	○	○	○	○	○
Hearing other students explain their work	○	○	○	○	○	○
Giving presentations in front of class	○	○	○	○	○	○
Writing solutions to problems	○	○	○	○	○	○
Checking solutions to problems	○	○	○	○	○	○
Working on a computer	○	○	○	○	○	○
Examining children's mathematical work	○	○	○	○	○	○

3. Please comment on how this class has CHANGED THE WAYS YOU LEARN mathematics?

Assignments and tests

4. HOW MUCH did the assignments and tests HELP YOUR LEARNING?

	No help	A little help	Moderate help	Much help	Great help	DID NOT HAPPEN
Taking tests	○	○	○	○	○	○
Doing other assignments	○	○	○	○	○	○
Doing homework	○	○	○	○	○	○
The fit between class content and tests	○	○	○	○	○	○
The match between the grading system and what you needed to work on	○	○	○	○	○	○
The mental stretch required on tests	○	○	○	○	○	○
Preparing class presentations	○	○	○	○	○	○
The feedback you received on your written work	○	○	○	○	○	○

Exhibit E3.2: Learning Gains Post-Survey (SALG-M)

Support for you as a learner

5. HOW MUCH did each of the following HELP YOUR LEARNING?

	No help	A little help	Moderate help	Much help	Great help	DID NOT HAPPEN
Interacting with the instructor DURING class	○	○	○	○	○	○
Interacting with the instructor OUTSIDE class	○	○	○	○	○	○
Interacting with teaching assistants DURING class	○	○	○	○	○	○
Interacting with teaching assistants OUTSIDE class	○	○	○	○	○	○
Working with peers DURING class	○	○	○	○	○	○
Working with peers OUTSIDE class	○	○	○	○	○	○

Your understanding of class content

6. As a result of your work in this class, what GAINS did you make in your UNDERSTANDING of each of the following?

	No gain	A little gain	Moderate gain	Good gain	Great gain	NOT APPLICABLE
The main concepts explored in this class	○	○	○	○	○	○
The relationships among the main concepts	○	○	○	○	○	○
Your own ways of mathematical thinking	○	○	○	○	○	○
How mathematicians think and work	○	○	○	○	○	○
How ideas from this class relate to ideas outside mathematics	○	○	○	○	○	○
How children solve mathematical problems	○	○	○	○	○	○
How to make mathematics understandable for other people	○	○	○	○	○	○

Please comment on how YOUR UNDERSTANDING OF MATHEMATICS has changed as a result of this class.

Exhibit E3.2: Learning Gains Post-Survey (SALG-M)

7. Please comment on how THE WAY THIS CLASS WAS TAUGHT affects your ability to REMEMBER key ideas.

Confidence, attitudes and abilities

8. As a result of your work in this class, what GAINS did you make in the following?

	No gain	A little gain	Moderate gain	Good gain	Great gain	NOT APPLICABLE
Confidence that you can do mathematics	○	○	○	○	○	○
Comfort in working with complex mathematical ideas	○	○	○	○	○	○
Development of a positive attitude about learning mathematics	○	○	○	○	○	○
Ability to work on your own	○	○	○	○	○	○
Ability to organize your work and time	○	○	○	○	○	○
Appreciation of mathematical thinking	○	○	○	○	○	○
Comfort in communicating about mathematics	○	○	○	○	○	○
Confidence that you will remember what you have learned in this class	○	○	○	○	○	○
Persistence in solving problems	○	○	○	○	○	○
Willingness to seek help from others	○	○	○	○	○	○
Comfort in teaching mathematics	○	○	○	○	○	○
Ability to work well with others	○	○	○	○	○	○
Appreciation of different perspectives	○	○	○	○	○	○
Ability to stretch your own mathematical capacity	○	○	○	○	○	○

9. What will you CARRY WITH YOU from this class into other classes or other aspects of your life?

Exhibit E3.2: Learning Gains Post-Survey (SALG-M)

Your expectation

10. What grade do you expect to receive in this course?

- ◯ A
- ◯ A-
- ◯ B+
- ◯ B
- ◯ B-
- ◯ C+
- ◯ C
- ◯ C-
- ◯ D
- ◯ F

Your background

11. What is your college major? (Check ALL that apply)

- ☐ Math or Applied Math
- ☐ Physics
- ☐ Chemistry
- ☐ Engineering
- ☐ Computer science
- ☐ Other science or technical field
- ☐ Economics
- ☐ Other non-science field

12. Are you pursuing a teaching certification?

- ◯ no
- ◯ yes, elementary (grades K-6 or K-8)
- ◯ yes, secondary math (grades 6-12, 8-12, or 9-12)
- ◯ yes, secondary in a field other than math

Other (please specify)

[ ]

13. What is your gender?

- ◯ male
- ◯ female

Exhibit E3.2: Learning Gains Post-Survey (SALG-M)

14. What is your class year?

- ◯ First-year
- ◯ Sophomore
- ◯ Junior
- ◯ Senior
- ◯ Graduate student
- ◯ Other (please specify)

Assign yourself an identifier

On this page, we ask for some information that will enable us to match your survey responses at the beginning and end of your math classes. The information will be unique to you but will not identify you individually.

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Course information

* 16. What is this math course?

Number of the course

Section of the course

Name of the instructor

Survey completed

Thank you for completing the survey! Your input is important to us, and will help us to help math instructors improve teaching and learning in their courses.

If you have any questions, please contact us:

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