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Supporting the Transition from Arithmetic to Algebraic Reasoning

The Problem-Solving Cycle: Professional Development for Middle School Mathematics Teachers

The Facilitator's Guide

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Contents

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Chapter 1

Introduction to the Problem-Solving Cycle

"The improvement of students' opportunities to learn mathematics depends fundamentally on teachers' skill and knowledge. No curriculum or framework is self-enacting, no students self-teaching. Moreover, teachers are often expected to teach mathematical topics and skills in ways substantially different from the ways in which they themselves learned that content.... Hence, if students' learning is to improve, teachers' professional learning opportunities are key." (Boaler & Humphreys, 2005)

As is aptly described in the quotation at left, there is currently a great deal of pressure on mathematics teachers. Teachers all across the United States are being asked to implement new curriculum, interact with their students in different ways, and have a much deeper knowledge of mathematics than ever before. The key, in our view (and in the view of many others), is to professionalize the field of mathematics teaching. We propose that teachers should be treated as professionals and, as such, have access to sustainable, long-term opportunities to expand and share their knowledge.

The professional development model described in this guide presents one such opportunity. As they engage in this professional development experience, which we call the Problem-Solving Cycle (PSC), teachers become part of a collaborative and supportive team. In the Problem-Solving Cycle, teachers think deeply about mathematics content and mathematics instruction. Together with a knowledgeable facilitator, teachers reflect on their areas of expertise and areas in which they want to improve. Teachers and the facilitator work together to select goals for the professional development workshops, including goals for learning mathematics content and exploring new pedagogical practices. The Problem-Solving Cycle model provides a structure for helping teachers to meet these goals, and enables the development of a professional learning community. By electing to participate in the Problem-Solving Cycle, teachers demonstrate their commitment to broadening their knowledge base and, consequently, to becoming better able to support their students' learning of mathematics.

Throughout the remainder of this facilitator's guide, we will discuss the specific nature of the Problem-Solving Cycle. First, we briefly address a few key questions that interested teachers and facilitators typically ask when we introduce this model.

What exactly is the Problem-Solving Cycle?

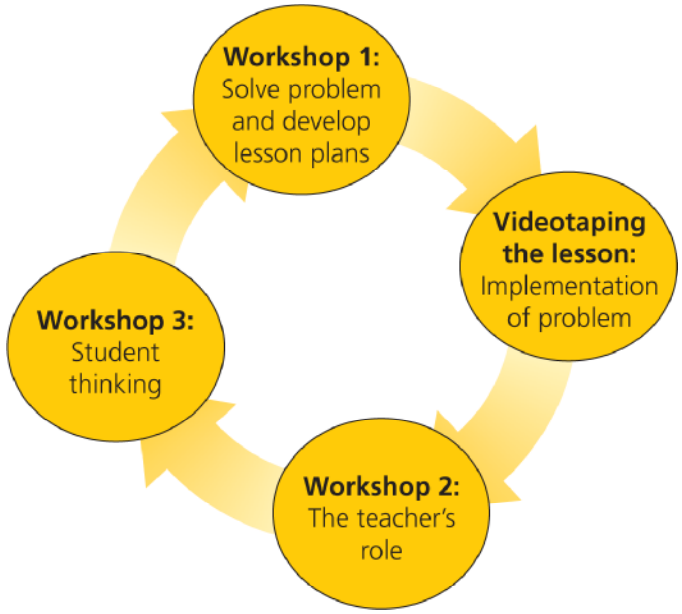
The Problem-Solving Cycle is a series of three interconnected professional development workshops for mathematics teachers, organized around a rich mathematical task. This task enables teachers to share a common learning, planning and teaching experience (see figure 1).

Each cycle (of three workshops) focuses on a different mathematical task and highlights specific topics related to teaching and learning.

During Workshop 1, teachers collaboratively solve the mathematical task and develop plans for teaching it to their own students. The goals of this workshop are to help teachers develop strong planning skills and deeper knowledge of the subject matter.

After the first workshop, teachers implement the problem with their own students and their lessons are videotaped. The facilitators then carefully select video clips for use in Workshops 2 and 3 which highlight key moments in the instruction and students' thinking about the problem.

Figure 1: The Problem-Solving Cycle model of mathematics professional development



Workshops 2 and 3 focus on the teachers' classroom experiences and rely heavily on the selected video clips. The goals of these two workshops are to help teachers learn more about the mathematical concepts and skills entailed in the problem, to explore a variety of instructional strategies for teaching the problem, and to learn how to build on student thinking.

In Workshop 2 teachers explore the role they played in implementing the problem. In Workshop 3 teachers critically examine students' mathematical reasoning. Specific topics addressed during these workshops depend on the needs and interests of the teachers, facilitators, schools, and districts. They might include ways to introduce the task, questions to pose as students work on the task, unusual or unexpected methods that students used to solve the task, or interesting conversations they had with one another in small problem-solving groups.

Who should participate in the Problem-Solving Cycle?

Ideally, all K-12 mathematics teachers will find participating in the Problem-Solving Cycle model of professional development relevant and rewarding. At the present time, we have only worked with middle school mathematics teachers in our development and implementation of the model. Therefore, the images we draw from as we present this model are of middle school teachers. However, the model is intentionally designed to be flexibly implemented and responsive to the needs of facilitators, teachers, and school district personnel. We expect it will be suitable for mathematics teachers at all levels.

In developing this model, we worked with a group of teachers from several different schools and districts. They were all middle school mathematics teachers; however, they used different textbooks, taught different grade levels (ranging from 6 to 8), had a wide range of experience (from 1 to 27 years), and had different student populations. We could envision the Problem-Solving Cycle working equally well with teachers across or within schools, teaching students at the same grade level or at different grade levels, etc.

How many teachers should be in a group?

Forming a tightly knit community is central to the effectiveness of the professional development; community development is thus a strong focus of this manual.

Our best guess is that the Problem-Solving Cycle would work best with a group of teachers that is neither too small nor too large. We suggest including 4 to 15 teachers.

Does there need to be a facilitator?

Yes. The Problem-Solving Cycle is designed to have a facilitator. Facilitators of the Problem-Solving Cycle might be mathematics coaches, department chairs, teacher leaders, district-level leaders, or other teacher educators who are knowledgeable about the model and who are enthusiastic about using it to work with teachers. In our implementation of the model, we have always used co-facilitators. However, we imagine that the PSC could be implemented equally well with one facilitator or two facilitators working together.

The PSC model is built on the premise that teachers can and should provide input into each “cycle” and each workshop within a cycle. At the same time, it is the facilitator’s role to gather this input and to respond to the participants’ developing needs and interests. Carefully planning each workshop and providing structured guidance during the workshops is essential to the success of the PSC model. For example, in advance of each workshop facilitators need to consider what topics the teachers have been struggling with or seem particularly interested in discussing (including issues related to mathematics content, pedagogy, and students), and what aspects of the community need to be addressed. In addition, facilitators select the mathematical task for each PSC and choose the video clips for the teachers to watch and discuss.

Can teachers participate in the Problem-Solving Cycle along with other PD activities?

Yes. The Problem-Solving Cycle is designed to complement other professional development activities that teachers might be involved in, not compete with them. Although there is a clear guiding philosophy for the Problem-Solving Cycle model, which we will describe in the next chapter, we expect that it will be consistent with most other professional development activities offered to teachers. Because the Problem-Solving Cycle contains so much flexibility, it can be used to complement and reinforce “standards” that are designated by individual schools, districts, and states, and it can be used to support a wide variety of mathematics curricula.

People often ask us to compare the Problem-Solving Cycle to other models of professional development. We typically respond that, in many ways, the Problem-Solving Cycle is most similar to “lesson study,” a type of teacher professional development with which Americans are becoming increasingly familiar. Like lesson study, the Problem-Solving Cycle provides a structure for a group of teachers to work together for a considerable length of time and carefully study issues around teaching and learning that are of interest to them. A key difference between these two models is that lesson study focuses on designing, carrying out, and reflecting on a designated lesson, whereas the PSC focuses on solving, teaching, and learning from a designated mathematics problem. Another important distinction is that a knowledgeable facilitator plays a prominent role in carrying out the PSC, whereas in lesson study there typically is not a facilitator to plan or guide the professional development sessions. Therefore, this manual and related training materials are critical elements in preparing facilitators to take on this role.

For how long should teachers participate in the Problem-Solving Cycle?

Similar to regularly scheduled “department meetings,” we imagine that participation in the Problem-Solving Cycle could become part of teachers’ professional routines that continue over the duration of their career. The Problem-Solving Cycle model attempts to delicately balance the realistic schedules of teachers with the delivery of workshops that are long and frequent enough to have an impact. Each “cycle” of three workshops roughly corresponds to an academic semester, so that teachers would participate in two cycles per school year (six workshops per school year). Each workshop lasts from 3 to 6 hours, depending on teachers’ availability. With longer sessions, it is possible to delve deeper into issues and to share more examples from the teachers’ videotaped lessons.

Successive iterations of the Problem-Solving Cycle build on one another and capitalize on teachers’ expanding knowledge, skills, and interests. Each cycle is focused on a unique mathematical task and specific issues related to mathematics teaching and learning. By regularly organizing workshops that are focused, productive, and collegial, facilitators play a critical role in ensuring the long-term participation of their teachers. As they become part of a learning community, we have found that teachers take genuine pleasure in gaining and sharing knowledge and working to improve their craft.

What does this Facilitator's Guide provide?

This guide is intended to help facilitators learn about the Problem-Solving Cycle and prepare them to successfully implement it. The guide offers facilitators a description of and rationale for the types of activities involved in the PSC, describes the various decisions facilitators will need to make as they prepare for and conduct each workshop, and includes examples from our experiences in developing and conducting this model of professional development.

As we have noted, the PSC model is designed to be open to a range of variations depending on the specific needs and interests of the facilitators and teacher participants. This facilitator's guide is written to help facilitators understand what components of the Problem-Solving Cycle are most central to its successful implementation, based on the goals and theoretical perspective of the research and development team. In addition, the facilitators' guide is meant be part of a package of materials facilitators will need if they decide to carry out the Problem-Solving Cycle with a group of teachers.

Contact information

For more information about the Problem-Solving Cycle, please refer to our Web site:

http://colorado.edu/education/staar

Chapter 2

Theory and Rationale Behind the Problem-Solving Cycle

In order to gain a thorough understanding of the Problem-Solving Cycle model of mathematics professional development, and to successfully implement it with a group a teachers, facilitators should have some understanding about why we developed this model. The Problem-Solving Cycle is built on a strong theoretical foundation, and draws from the most current research on how people learn. Here we provide a brief review of this research and discuss what the implications are for facilitators.

Our view of the nature of mathematics and mathematics teaching

In our view, all students can and should learn challenging mathematics. In order to be productive citizens, and to compete in today's increasingly global economy, children need a sound knowledge of basic and advanced mathematics. Knowledge of mathematics has been widely coined "the gatekeeper" to future educational and employment opportunities. But there is clear reason for concern. Several decades of research confirms that American children have an inadequate understanding of fundamental mathematical topics.

Children need to learn mathematics better than they ever have before. This means that teachers must be better prepared than ever before. In turn, professional development for teachers is critical and should be of the highest priority. The debate about what and how mathematics ought to be taught in American schools has been ongoing for several years. As we developed the Problem-Solving Cycle model, we considered what our ideal of teaching and learning mathematics would look like. We agreed that it would be consistent with the vision portrayed in the *Principles and Standards for School Mathematics* (NCTM, 2000):

> *Imagine a classroom [where] ... students confidently engage in complex mathematical tasks chosen carefully by teachers. They draw on knowledge from a wide variety of mathematical topics, sometimes approaching the same problem from different mathematical*

> *perspectives or representing the mathematics in different ways until they find methods that enable them to make progress. Teachers help students make, refine, and explore conjectures on the basis of evidence and use a variety of reasoning and proof techniques to confirm or disprove those conjectures. Students are flexible and resourceful problem solvers. Alone or in groups and with access to technology, they work productively and reflectively, with the skilled guidance of their teachers. (p. 3)*

This vision is based largely on the ideas of **constructivist learning theory**. The core idea behind constructivism is that learners actively construct their own understandings, rather than absorb what they are told or copy what someone shows them. In the field of mathematics, constructivist theory suggests that people will learn best when they solve mathematical tasks, create and examine solution strategies and representations, and generalize these strategies to new problem situations. Mathematics classrooms should provide opportunities for students to explore, justify, prove, critique, and generalize important mathematical concepts and ideas.

Many teachers agree with this vision but find it abstract. They are not exactly sure what pedagogical decisions to make at any given moment in order to best help their students "construct" knowledge. Teachers often asks questions such as, What exactly constitutes best practice from a constructivist point of view? How can I incorporate these ideas within the confines of my students, my designated curriculum, and my school and district policies? Questions such as these lie at the heart of the Problem-Solving Cycle model. Answers come from the teachers themselves, over time, as they work on mathematical tasks, reflect on their teaching and their students' thinking, and hold focused discussions with their colleagues.

When teachers have some commitment to constructivist ideas about teaching and learning, and when they share in the desire to improve their students' learning opportunities, they will likely find participation in the Problem-Solving Cycle rewarding and fulfilling. At the same time, they will be challenged to push their thinking forward, as they encounter rich mathematical content and new instructional situations, and reflect deeply on their own teaching. But what about teachers who do not feel strongly about constructivist principles or who are not actively seeking to change their instructional practices? We suggest that by becoming part of a collegial network, all teachers will find some benefit in this professional development model, with its heavy focus on developing and maintaining a supportive, nonjudgmental environment. We do not propose to "force" teachers to hold any particular view, or to discourage them if they have a vision of effective mathematics teaching and learning that differs from the constructivist ideas we have described. We simply would encourage all of the participating teachers to be open-minded and to try experimenting with new teaching and learning strategies in order to expand their knowledge base.

Our view of the knowledge mathematics teachers need

Broadly speaking, mathematics teachers draw on two types of knowledge: content knowledge (sometimes called subject matter knowledge) and pedagogical content knowledge. We want to briefly define and discuss these terms here because the Problem-Solving Cycle is intentionally designed to foster both types of knowledge, and we will refer to content and pedagogical content knowledge throughout this guide as we discuss the specifics of the PSC workshops and how to facilitate them.

Content knowledge, sometimes called subject matter knowledge, refers to knowledge of mathematical skills, procedures, and concepts. Content knowledge can be broken down into two components: *common content knowledge* and *specialized content knowledge*. Most educated adults have a basic understanding of mathematical skills, procedures, and concepts, called common content knowledge. Teachers, however, need a deeper understanding of the mathematical skills, procedures and concepts than do most other educated adults, called specialized content knowledge. Specialized content knowledge is what teachers draw on when they help their students to understand specific mathematics content in developmentally appropriate ways. For example, teachers need to know more than what the mathematical algorithm needed to solve a problem is; they need to know why one algorithm works and another does not, and what variations on that algorithm might look like. Teachers also need to understand how students are likely to approach a mathematical problem, and follow their logic as they talk—correctly or incorrectly—about a solution strategy.

Pedagogical content knowledge refers to knowledge about appropriate instructional practices within a given content area. Like specialized content knowledge, pedagogical content knowledge is unique to teachers and is developed over time as teachers gain expertise in their fields. Pedagogical content knowledge can be broken down into two components: *knowledge of content and teaching* and *knowledge of content and students*. Knowledge of content and teaching combines teachers' knowledge about mathematical content with their knowledge about appropriate pedagogy. Teachers draw on this type of knowledge, for example, to select tasks and sequence classroom activities in order to facilitate student learning of a specific mathematical topic. Knowledge of content and students combines teachers' knowledge about mathematical content with their knowledge of students. Teachers draw on this type of knowledge, for example, to respond to students' novel approaches, build on their thinking, and determine whether students have a complete and accurate understanding of a particular concept.

All teachers use content and pedagogical content knowledge every time they conduct a lesson. Therefore, all teachers have a basic foundation of knowledge in these areas. It is our contention that professional development programs should provide teachers with the opportunity to continually share and expand their knowledge base, including beginners and those with vast teaching experience. Helping to support the development of teachers' mathematical content knowledge and pedagogical content knowledge is the major goal of the Problem-Solving Cycle.

Our view of how to support teachers' learning

Our view of how to best support teachers' learning, as they seek to add to their base of content and pedagogical content knowledge, is strongly informed by two theories: **constructivist learning theory** and **situative theory**. As we described in the beginning of this chapter, constructivist learning theory is based on the premise that people actively construct their own knowledge. That is, they do not learn best by passively receiving information from others, but rather by actively working to understand and make sense of new information. Situative theory complements constructivist learning theory, and helps to provide a focused direction for designing professional development that supports teachers' learning.

Two concepts lie at the core of the situative perspective: (1) learning is social in nature, and (2) people learn the most in contexts that are personally relevant and meaningful. In terms of teacher professional development, we believe these concepts translate into (1) the importance of a strong professional learning community and (2) the potential of classroom video as a tool. We discuss each of these ideas in turn.

The Problem-Solving Cycle emphasizes creating and maintaining a professional learning community in which teachers are comfortable working together to expand their mathematical knowledge and to critically examine their instructional practices. Participating in the Problem-Solving Cycle provides teachers with the much needed, but rarely provided, opportunity to work together as professionals, share their knowledge, and support one another. The Problem-Solving Cycle provides a structure for helping teachers to establish trust and develop communication skills that enable constructive yet respectful discussions about teaching and learning.

Another idea that lies at the heart of the Problem-Solving Cycle is the use of **classroom video** as a tool for professional learning. Video captures the everyday experiences of teachers and students, and allows the professional development to be anchored in specific classroom events. Watching video from one's own classroom can be highly motivating and has the potential to be a powerful catalyst for change and improvement. The Problem-Solving Cycle capitalizes on the power of video to help teachers inquire into and deeply investigate issues around teaching and learning a specific mathematics problem. In particular, video is used to help teachers learn important mathematical content, investigate their own instructional practices, and study their students' work.

A strong community is vitally important when teachers are asked to share video clips from their classrooms with their colleagues. Sharing classroom video could be seen as more threatening than sharing written classroom documents, such as student work and lesson plans. To be willing to take such a risk, teachers must feel part of a safe and supportive professional environment. They should feel confident that showing their videos will provide learning opportunities for themselves and their colleagues, and that the atmosphere in the professional development setting will be one of respectful, productive dialogue.

When used thoughtfully and skillfully, video from teachers' own classrooms in professional development can help to foster a more tight-knit and supportive learning community. By making their own actions and voices the core component of the professional development, teachers can feel empowered and take ownership of their own learning. As teachers share authentic, descriptive records of their teaching with colleagues, they create an atmosphere of openness and bonding that is rare in professional learning settings.

Additional literature about the Problem-Solving Cycle

We refer interested readers to several published (or soon-to-be published) papers, which present theoretical foundations of the Problem-Solving Cycle and findings from our research in developing and implementing this model.

- Borko, H. (2004). **Professional development and teacher learning: Mapping the terrain**. *Educational Researcher, 33(8),* 3-15.

  In this paper, we briefly describe the Problem-Solving Cycle model and present it as an "existence proof" of effective professional development. A short case study traces one teacher's learning and instructional changes throughout his participation in the professional development program.

- Borko, H., Frykholm, J., Pittman, M., Eiteljorg, E., Nelson, M., Jacobs, J., Clark, K. K., & Schneider, C. (2005). **Preparing teachers to foster algebraic thinking.** *Zentralblatt für Didaktik der Mathematik: In-*

ternational Reviews on Mathematical Education, 37(1), 43-52.

This paper describes the conceptual framework of the PSC, outlines the structure and goals of the professional development, shares initial findings, and discusses some implications of our research.

- Borko, H., Jacobs, J., Eiteljorg, E., & Pittman, M.E. (in press). **Video as a tool for fostering productive discourse in mathematics professional development.** *Teaching and Teacher Education.*

  In this paper we focus on our use of video in the PSC to foster productive conversations about teaching and learning. We examine the nature of teachers' discussions around video and how these discussions changed over time.

- Clark, K. K., Jacobs, J. Pittman, M., & Borko, H. (2005). **Strategies for building mathematical communication in the middle school classroom: Modeled in professional development, implemented in the classroom.** *Current Issues in Middle Level Education, 11(2),* 1-12.

  In this paper we describe specific strategies for fostering mathematical communication that were modeled in our professional development program and then implemented by one of the participating teachers in her middle school classroom.

- Clark, K.K. & Jacobs, J. (2005). **Using video to support teacher learning: Theory and Practice response.** *AMTE Connections, 14(3),* 9-11.

  This paper is a short response to the question "How can video support inservice and preservice teacher learning?" The paper describes how we used video of teachers' classrooms in our professional development program as a tool for promoting analysis, discussion, and change in instructional practice.

- Koellner, K., Borko, H., Frykholm, J., Jacobs, J., Schneider, C., Eiteljorg, E., Bunning, C., & Pittman, M. (in preparation). **The problem-solving cycle: A model to support the development of teachers' professional knowledge.** *Mathematical Thinking and Learning.*

  This paper describes the general focus and goals of the Problem-Solving Cycle workshops, details of their enactment, and the opportunities participants have to expand their content and pedagogical content knowledge in each of the three workshops.

Online Resources

Please see our Web site for links to some of these papers:
http://algecolorado.edu/education/staar

Chapter 3

Getting Ready for the Problem-Solving Cycle: Workshop 0

Objectives

Workshop 0 Objective:

To develop a professional learning community in which teachers are comfortable working on mathematics problems together and watching and discussing video of mathematics lessons.

If teachers have never before participated in a Problem-Solving Cycle, it will be helpful to conduct an initial, preparatory workshop, which we call Workshop 0. In this chapter we will describe how to conduct and plan for Workshop 0.

The major objectives of Workshop 0 are to prepare teachers to take part in the Problem-Solving Cycle, and to help ensure that their experiences will be productive and rewarding. Even teachers who have worked together, established a professional relationship, and are comfortable engaging in professional development, are unlikely to have formed the kind of community that is at the heart of the Problem-Solving Cycle. In the Problem-Solving Cycle, teachers will be solving rich and challenging mathematics problems together, presenting these problems to their students, and sharing and discussing video from their lessons. Development a community in which teachers will be comfortable engaging in these types of activities is the goal of Workshop 0.

Conducting Workshop 0

Overview

Workshop 0 is composed of two broad phases: introducing the Problem-Solving Cycle and analyzing video of classroom teaching. The “analyzing video” phase includes three activities: reading an article related to watching video, doing the math task shown in the selected video clip, and analyzing the events in the selected video clip. We describe what is involved in conducting each of these phases and activities.

Phase 1. Introducing the Problem-Solving Cycle

During this initial phase of the workshop, describe the design and goals of the Problem-Solving Cycle model. give an overview of the three workshops that comprise the PSC and briefly describe the main goals of each workshop.

One option for introducing the Problem-Solving Cycle is to show the accompanying PowerPoint slides that explain the model, highlight the goals, and provide a brief overview of the experiences participating teachers can expect to have. Feel free to elaborate on selected points of the slides, as you see fit. After presenting the slides, ask the teachers if they have any questions, comments, or concerns that you can address.

Phase 2. Analyzing Video of Classroom Teaching

During Phase 2, which comprises the bulk of Workshop 0 time, teachers begin to develop a community around watching and analyzing video. Viewing video of classroom teaching is likely to be a new experience for most of the teachers. The three activities described in this phase are all intended to help teachers become comfortable discussing classroom video, and to prepare them to share their own video when they begin the first iteration of the Problem-Solving Cycle.

Throughout Phase 2 of Workshop 0, the focus is on developing community and establishing norms (for analyzing video and working on mathematics problem together). The community, and its norms and expectations, will continue to develop and evolve as the teachers become accustomed to participating in the Problem-Solving Cycle.

❑ Activity One: Reading an Article

Have the teachers read an article about the importance of video in professional development. We have come across several articles written for teachers that highlight the benefits of watching video in a professional development context. Two examples are

- *Video Volunteers* (Sherin, 2004). In this article the author describes “video clubs,” in which teachers share video from their own classrooms with one another and discuss different aspects of teaching. The article outlines several important norms that should be established in order for teachers to have a successful experience examining video in a professional development setting.

- *The New Heroes of the Teaching Profession* (Hiebert, Gallimore, & Stigler, 2003). This article details the ways that watching and analyzing video can enable professional growth. The authors suggest that the “heroes” of our current classrooms are those teachers who are willing to risk not only being videotaped but also sharing their videos with other teachers.

Distribute copies of the selected article to the teachers and have them read it to themselves. If desired, distribute copies of several articles, and have the teachers read one during the workshop and the other(s) after the workshop at their leisure.

Pose questions for the teachers to consider as they read the article. Some examples include:

- What are the ideas the author(s) have about using video to examine teaching practices?
- What value do you see in these ideas?
- What concerns do you have?

After they have read the article, **ask teachers to discuss their thoughts** about questions you posed. One format for this discussion would be for teachers to talk first in pairs, and then share their ideas with the whole group.

In addition to responding to questions such as those listed above, we suggest that the discussion **include a consideration of the norms for analyzing video** that the group will need to establish. Below are some key ideas about watching and analyzing video that are integral to the Problem-Solving Cycle model. It will be helpful if the facilitator could make sure these ideas are brought into the conversation.

- Video clips shown in the PSC are not intended to be examples of "best practice" or of how all in the group should teach their students. Rather, the group will be watching interesting moments in classroom lessons that can serve as a springboard for discussions about mathematical or pedagogical issues.
- Discussion of the video clips during the PSC should not be about evaluating the teacher or the students. Instead, the discussions should be about mathematics teaching and students' mathematical reasoning.
- In order for participants to learn from one another, they need to be willing to share their ideas and respect the ideas shared by others.

❑ Activity Two: Doing the Math Task

Turning now to the video clip selected for viewing in Workshop 0, have the teachers **do the math task** that will be shown in the video prior to actually viewing the clip. Solving a math task together in Workshop 0 serves a variety of purposes including fostering a community of mathematics learners and preparing teachers to actively engage in watching and understanding the forthcoming video clip. In our experience, participants need to have sufficient time and experience with the math problem at the center of the video clip in order to have a thorough understanding of the substance of the clip and therefore a meaningful, post-viewing discussion.

The facilitator's main role in this part of the workshop is to encourage the teachers to work together, and to help them to move past any discomfort they might have regarding their mathematical knowledge and working with colleagues of varying abilities. The facilitator should encourage the teachers to explore the mathematics deeply enough so that they can carefully analyze the teaching and learning they will see in the video.

Here we provide a suggested protocol for working on the task. We have used this protocol and found it to be helpful in promoting mathematical communication and supporting multiple solution strategies. However, other protocols may be equally (or more) valuable. Even if you do chose to follow some or all of the suggestions below, consider them as a guide rather than a script.

- **Distribute a handout of the problem** to each teacher and, if desired, read the problem aloud. Explain to the group that this is the same math task that they will see in the video clip that they will be watching later in the workshop. Ask the teachers if they have any questions about what the problem is asking them to do, and address questions or concerns that are raised.

- Have the **teachers work on the problem individually** for a few minutes (e.g., 3-5 minutes). Working individually for a short period of time will allow each teacher to develop some initial ideas about how to approach the problem. Ask the teachers to keep track of how they are solving the problem.

- Next have the **teachers share their ideas in small groups** (e.g., 3-4 teachers per group). Explain that each teacher within the group should have a chance to share ideas and strategies for solving the problem. In addition, the group should discuss any connections they notice between their individual ideas.

- During small group time, **the facilitator should circulate among the groups** to hear how the participants are approaching the problem. Consider the degree of sophistication of each approach, privately noting which solution strategies are less sophisticated and which are more advanced or further developed. This information could help you to organize the whole group debriefing.

- **Initiate a concluding whole-group discussion** so that the teachers can share their ideas and approaches. The discussion could be organized simply by asking for volunteers to explain their strategies. After the first presentation, ask for another volunteer who solved the problem in a different way. You can pose the question, "Did anyone see it differently?" in order to prompt the teachers to consider alternative solutions or solution strategies. Alternatively, you could organize the discussion by arranging teacher (or group) presentations so that they move from least to most sophisticated. Help the group to make connections between the strategies by asking them to compare and contrast the different approaches. If possible, help the teachers to see how the strategies build on one another. NOTE: If you choose this approach, it is best not to publicly identify one teacher (or group) as having a less sophisticated strategy compared to another teacher (or group).

- When necessary, **pose questions** that elicit teachers' mathematical thinking. Questions such as, "Can you show us how you thought about it?" invite the teachers to describe their reasoning, rather than simply identifying the procedures they used or stating their final answer.

- Throughout the whole group discussion, make sure that there is **a written record of each strategy**, so that the approaches are clear to everyone and can be compared to one another.

We hope that two principles stand out in the protocol described above, and that these principles will help to guide facilitators even if they choose to follow a different protocol. The Problem-Solving Cycle model rests on the principle that the participating **teachers should form a productive mathematical discourse community**. In other words, the teachers should feel comfortable making their thinking public. Teachers need to express their ideas, and to negotiate their meaning and agreement within the group. Facilitators should carefully consider what protocol to use to support the development of such a community, and how to encourage mathematical communication, participation, and learning.

A second guiding principle, related to the first, is to **encourage alternative solution strategies**. The Problem-Solving Cycle model is most effective when the selected mathematical problems can be solved using a variety of approaches. Starting in Workshop 0, facilitators should highlight the idea that teachers can (and should) use any approach they want to solve a problem, and that there is mathematical value in examining the connections among different approaches.

❑ Activity Three: Watching and Analyzing Video

Now the teachers will **watch video of a mathematics lesson** together. As noted in Activity Two, this video clip will incorporate the mathematics problem that the teachers just completed.

Briefly introduce the selected video clip by providing a short, objective description of it. For example, you might tell the teachers the length of the video, the grade level of the students, a general description of the math content (e.g., "this is a lesson on linear functions”), where it was filmed, the degree to which students work independently or have a whole group conversation, and so on. Try not to provide any value judgments regarding the teacher, the students, or the mathematical content.

When the teachers take part in later PSC workshops, they will have their own lessons videotaped, and those lessons will be the focus of their future discussions. However, in Workshop 0, teachers will not yet have video from their lessons. Therefore, the video shown in Workshop 0 likely will come from an unfamiliar teacher. In the planning section later in this chapter, we provide a number of suggestions for locating appropriate video sources.

Provide guiding questions for the teachers to consider as they watch the clip. The questions should help focus the teachers’ attention on selected aspects of the lesson. You might have teachers think about the three questions listed below. Notice that these three questions reflect the main themes of the three workshops that comprise the Problem-Solving Cycle.

- What do you find interesting about the video from a mathematical perspective?
- What do you find interesting about the video from a teaching perspective?
- What do you find interesting about the video from the perspective of students’ mathematical thinking?

We have provided a handout of these questions, “Things to Think about While Watching the Video Clip” that you can choose to give to the teachers as they watch the video.

Other possible, more specific guiding questions include the following:

- What do you see as strengths of this teacher’s instruction? What do you see as limitations?
- What did you notice about the students and their mathematical reasoning on the task?
- What would you do next if you were the teacher, and why?

Play the clip at least twice. Be sure to tell the teachers in advance that they will have the opportunity to watch the clip more than once. We have found that teachers almost always want to watch a video clip two or three times before they feel comfortable embarking on a detailed analysis.

Ask the teachers to **discuss the clip in small groups**, using the guiding questions to focus their conversation. You might suggest that the teachers take notes during their small-group discussions.

After you have determined the groups have had enough time to discuss the video clip, **initiate a concluding whole group discussion**. This concluding discussion could obviously take many forms. Here we offer some suggestions of techniques that we have found helpful to facilitate conversations of this sort. Again, consider these suggestions as guidelines, not prescriptions.

- Have each group summarize their conversation. You may want to ask teachers to support their ideas by providing evidence (specific examples) from the video clip. For example, if a group suggests that the teacher in the video clip did a good job of asking questions, you might ask, "Can you provide a time in the video where you saw this occurring?" It might even be useful to replay a portion of the video clip.

- If teachers become overly critical in their analyses of the video (as sometimes happens, especially when the videotaped teacher is unfamiliar), the facilitator might use this as a "teachable moment" to help establish norms for conversations about video of teaching. For example, the facilitator could gently remind the group about the risk teachers take when they share their video with other teachers, and help move the group toward a more productive analysis of the clip.

- You might suggest that groups look for themes or connections among their ideas. Writing down the general themes of each group's conversation on a large chart paper could be helpful in this regard, so that the teachers can keep track of and reference these themes.

- End the discussion by synthesizing the themes that were presented. The facilitator might highlight the power of watching classroom video, and point out the ways in which a careful analysis of video can extend teachers' learning.

Planning Workshop 0

Overview

In this section, we describe three key facets of planning Workshop 0. Specifically, we discuss how to prepare to introduce the Problem Solving Cycle to the participants, how to select an appropriate article, and how to select a video clip. Of these, selection of the video clip is likely to be the most time consuming.

Preparing to Introduce the Problem-Solving Cycle

The best way to prepare for introducing the PSC to the participants is to carefully review Chapters One and Two of this Facilitator's Guide. These chapters lay out the goals and structure of the PSC model and describe the experiences teachers can expect to have throughout the workshops. In addition, it will be helpful to go over the PowerPoint slide presentation included with this guide. Feel free to edit it if you would like to prepare it for use during the introduction phase of Workshop 0. Think about how much time you want to spend covering this information, and be sure that you allow plenty of time for the other activities, especially doing the math task and analyzing video.

Selecting an Article

What article should I select?

The purpose of reading an article about the importance of video in professional development is to help set the stage for the PSC and to begin to establish norms for participation. We consider both of the articles listed in the *Conducting Workshop 0* section to be good choices (and have included both in the Facilitator's Guide materials).

As we noted in the *Conducting* section, "Video Volunteers" discusses specific norms for watching video. "The New Heroes of the Teaching Profession" highlights the idea that watching video of teaching is beneficial for professional growth.

Read over these articles, and decide if you would like to select one for your teachers to read. Other options include having teachers read both articles, or having teachers read one article during the workshop and the other one on their own. Alternatively, you could select a different article for teachers to read.

What questions should I prepare?

After selecting the article(s), consider what questions you feel would be most appropriate for the participants in your workshop to address. A few possibilities are listed in the *Conducting Workshop 0* section. In addition, consider your own responses to those questions (you may want to write them down), because generating your own responses prior the workshop will help you facilitate the whole-group discussion of the article.

In addition to preparing questions, identify the norms for watching video of classroom practice that you would like to establish. In both of the articles we listed, particularly in the "Video Volunteers" article, the authors discuss general norms that would be appropriate for teachers participating in this type of professional development. In addition, we highlighted several norms that we believe are important for the Problem-Solving Cycle model in the *Conducting Workshop 0* section. There may be additional norms that you feel are important and relevant to your group of teachers. Consider what these norms are, and how to incorporate them into a group discussion.

Selecting Video

The selection of video to use in Workshop 0 requires careful decision making and considerable time on the part of the facilitator. Having an appropriate video clip is critical for the success of this workshop, and for starting off 'on the right foot' with the teachers. Here we provide guidance on several key planning decisions related to selecting video.

What are the features of an appropriate video clip?

Matches the grade level of the participants. When selecting a video clip for Workshop 0, it is best to find a clip of a classroom lesson that closely matches the grade level and/or content area of the participants in your workshop. For example, middle school teachers will have more to say about a middle school lesson than a primary grade lesson.

Has an appropriate math task. Recall that a significant portion of Workshop 0 will be devoted to solving the math task that is in the video clip. For this reason, it is important to find a video clip with a math task that is appropriate for the participating teachers. Ideally, the selected video clip will center on a single appropriate math task. (If there are multiple tasks in the clip, it is most likely not a good candidate.)

The math task should be one that is likely to promote mathematically relevant and interesting dialogue among the participating teachers, and be seen as (at least somewhat) challenging to them. A good gauge is the extent to which this problem promotes relevant and interesting dialogue among the students and teacher in the videotaped classroom. An ideal problem will offer the possibility of multiple solution strategies. Another important consideration is whether solving the math task, and debriefing teachers' work, can be done in the amount of time available in this workshop.

We also strongly recommend doing the problem yourself (or, even better, with a colleague). Working on the problem not only will help you to determine how much time is required and whether it is appropriate for your group of teachers, but will give you the opportunity to test out various solution strategies. For example, you can try to predict the most common solution strategy, and then search for as many variations as possible. This type of exercise will prepare you to effectively facilitate a discussion about the problem because you will have a sense of how teachers are likely to approach the problem, what some of the connections are between various solution strategies, and how (or whether) to order teachers' presentations of their solution strategies.

Is short. Our experiences showing video in professional development workshops suggest that shorter is better. We have shown teachers video as long as 30 minutes, and as short as 3 minutes. In general, video clips in the 3- to 5-minute range work best because there usually is too much for viewers to digest in video clips that are much longer. If you find a longer video clip that you like, see if there are less relevant sections of the clip that you can skip through (perhaps verbally describe them). Alternatively, you can break up the viewing of a longer clip into two short clips of 3 to 5 minutes each. Ultimately, we strongly suggest that the total length of video (the portions that the teachers actually watch) be no longer than 7 to 10 minutes.

Addresses relevant mathematical and pedagogical topics. In Workshop 0, the facilitator has a great deal of leeway in determining what mathematical and pedagogical topics are "relevant" for the participating teachers. This determination may be based on a long history with the teachers and a detailed knowledge of their interests and experiences, or the facilitator may rely more on his or her own experiences and interests. With respect to the Problem-Solving Cycle model, we can provide some degree of guidance as to which topics would be more or less appropriate. We provide several recommendations related to the following questions:

- Does this clip seem to be an exemplar of how to teach a particular concept? Or is it a more typical teaching episode with interesting moments to discuss?

  Our recommendation is that the clip be relatively typical and seem familiar to teachers. Video is not used in the Problem-Solving Cycle model to showcase "best practices."

- Are there moments in the clip that show the teacher engaging in mathematical discussion with the students?

  Our recommendation is that the clip show some sort of mathematical discussion, either teacher-student or student-student.

- Are there moments in the clip that show students' mathematical reasoning?

  Our recommendation is that the mathematical reasoning of one of more students should be visible (either verbally or in writing).

Where can I look for video clips?

Below are a few sources of classroom video that are freely available over the Internet, and are good places to begin your search for appropriate video clips for your participants. Some of these organizations provide additional resources to go along with the video clips, such as examples of student work, transcripts of the video clips, and suggested discussion questions. Most of these Web sites organize their clips according to grade level and content area, which makes them easy to navigate.

- http://my.nctm.org/eresources/reflections/index.htm
- http://www.learner.org/channel/courses/learningmath/algebra/overview/videoindex.html
- http://www.mmmproject.org/
- http://www.schoolsmovingup.net/cs/vcmpd/print/htdocs/vcmpd.htm

What questions should I prepare?

Consider what aspects of the video clip you would like the participants to focus on during the workshop. Your determination will depend, at least partially, on the nature of the video clip. However, the questions listed in the *Conducting Workshop 0* section are sufficiently broad that they would work for most video clips. Whether you choose to use the handout provided, a modified version of these questions, or a different set of questions, it is helpful for teachers to have a way to reference the questions during the viewing of the clip. An alternative to a handout is to write the questions you want teachers to consider on a large poster paper, or to project them from your computer screen. Also carefully consider how many questions your teachers will have time to actively think about and discuss during the workshop.

Once you have selected a few questions, think about how you would respond to them and, if possible, write down your thoughts and bring them with you to the workshop. See if you can find evidence for your ideas in the form of specific statements heard or events seen in the video.

We also suggest that you watch the selected clip with an eye for moments in the video that could spark overly critical comments or responses to your questions. For example, are there moments in the video that could tempt the participants to evaluate the teacher (rather than the *teaching*) and/or the students (rather than the students' *thinking and learning*)? Be prepared to remind the teachers about the article they read and the norms for analyzing video in the PSC model. Think about how you could refocus their attention on the teaching and learning that is (or is not) occurring in the video, rather than evaluating the teacher or the students.

Chapter 4

An In-depth Look at Workshop 1

Objectives

Workshop 1 Objective:

To foster teachers' content knowledge, and secondarily to enhance teachers' pedagogical content knowledge.

As we have described, the Problem-Solving Cycle (PSC) consists of three interconnected professional development workshops, organized around a rich mathematical task. In this chapter, we provide an in-depth look at the first workshop. We include a vignette drawn from our experiences carrying out this workshop, a detailed guide to conducting the workshop, and information useful in planning for this workshop.

The major objective of Workshop 1 is to help teachers develop their math content knowledge. The majority of the time in Workshop 1 is spent by teachers doing the selected PSC mathematics task and debriefing their solution strategies.

"Working through the problems, it was very beneficial to have two or three minds, instead of just my mind, looking at the lesson and trying to figure out where it might go.... Probably the most helpful thing was the problem solving—doing a problem together in small groups and then coming back together as a big group and sharing out our information. It was amazing just to see how many [solution strategies] there were."
—Linda, final interview

Additionally, teachers spend a significant portion of Workshop 1 developing unique lesson plans that they will each implement prior to Workshop 2. Therefore, another aim of Workshop 1 is to enhance teachers' *pedagogical content knowledge,* which can develop from discussions about different methods of teaching the selected task. We call the framework for this workshop "Doing for Planning" to highlight the dual focus on teachers' problem solving and instructional planning.

Vignette from Workshop 1

Here we present a vignette that illustrates the Workshop 1 experiences of a sixth-grade teacher, Peter, and two of his colleagues as they work through the math task selected for this Problem-Solving Cycle. This particular workshop was part of a PSC that our team carried out during the winter of 2005. The vignette focuses on activities conducted as part of Phases 1 and 2 in carrying out the workshop: introducing and working on the math task.

As you read the vignette, pay particular attention to how the facilitators interjected a whole-group conversation between periods of small-group work. The intention behind this whole-group conversation was to help make several key mathematical ideas public for the benefit of all the teachers. Notice how this whole group discussion helped Peter's group move forward and how working on the task influenced Peter's initial lesson plan ideas.

The Math Task: Washing the Skyscraper Windows

> *A building is 12 stories tall with windows on all four sides. Each floor has 38 windows. Once a year, all the windows are washed. The cost for washing the windows is $2.00 for each 1st-floor window, $2.50 for each 2nd-floor window, $3.00 for each 3rd-floor window, etc. How much will it cost to wash the windows of this building? What if it were 30 stories? What if it were "*n*" stories?*
>
> Adapted from Mark Driscoll's (1999) book "Fostering Algebraic Thinking"

Vignette

The facilitators introduced the "Skyscraper Windows" problem by distributing printed handouts and asking the teachers to consider the mathematical concepts they noticed in the task. After giving them a few minutes to read the problem and write down some initial ideas, the facilitators led a brief discussion about the math concepts entailed in the task and listed the teachers' comments on a large piece of poster paper taped onto the chalkboard.

Next, the facilitator asked the teachers to begin working on the problem in small groups. The facilitators encouraged the teachers to consider how their own students might approach the Skyscraper Windows problem and, if possible, to write down these thoughts. They stressed how valuable these ideas would become when the teachers formally engage in planning lessons around the problem later in the workshop.

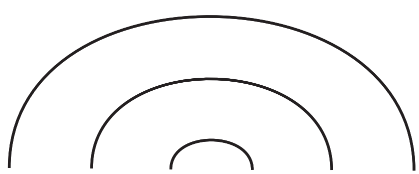
Once the teachers rearranged themselves into small groups, Peter commented to the two other teachers in his group that he did not think the Skyscraper Windows problem appeared very difficult. However, within a few minutes of working on it, Peter laughingly admitted, "Boy, this problem looked so simple when I first got it—but it's actually pretty hard." Both of his group members nodded and smiled in agreement.

After approximately thirty minutes of small-group work, the facilitators gathered all of the teachers together to discuss the mathematics they had generated thus far. The facilitators purposefully used this opportunity to expose and then to build on mathematical similarities that were emerging across the groups. First, several teachers helped to complete a table that indicated how much it would cost to wash the windows on each floor of the Skyscraper, writing on poster paper at the front of the room. Prodded by the facilitators, they began describing patterns they saw in the table. Building on these patterns, the teachers derived an expression to represent the cost of washing the windows on any individual floor (n): $19 \cdot n + 57$. The teachers realized they still needed to find the *total* cost of washing an entire building's windows. This last step proved to be the most difficult. The teachers knew that they needed to add all the individual floor costs but they struggled to derive an expression to do so.

The facilitators had anticipated that this aspect of the problem was likely to present a challenge. When the facilitators planned the workshop and worked on the Skyscraper Windows task themselves, they saw firsthand how difficult the final step was. Therefore, they planned to guide and scaffold this component of the problem more explicitly than the prior components. In particular, the facilitators expected that they would ask questions that helped the teachers to find an expression for adding together the individual floor costs of any building level. First they asked, "Is there a way to generalize the patterns you see in the table to find the total cost of buildings of any height?" The facilitators then encouraged a discussion about how to arrange strings of numbers in order to add them together more easily. They presented a short string of small numbers for the teachers to add, incorporating some of the numbers in the Skyscraper Windows problem.

Peter then offered the following idea:

Peter's method of addition by grouping:



$$\mathbf{2.00 + 2.50 + 3.00 + 3.50 + 4.00 + 4.50 = 3 \cdot (6.5) = 19.50}$$

"There was this way to add a string of numbers from 1 to 100 that works like this. It was done by a famous mathematician. I can't remember his name … [Another teacher suggests, "Gauss?"] — Yeah, it was Gauss. He did 1 + 100, which is 101. Then 2 + 99, which is also 101. And so forth for the 50 pairs of numbers in the string. In other words, you multiply 101 by 50, and that's equal to the sum of the string of numbers. So with our string of numbers, you add 2.00 + 4.50 to get 6.50. And 2.50 + 4.00 is 6.50. And then 3.00 + 3.50 is 6.50. Or you can multiply that number [6.50] times 3."

Peter's contribution furthered the whole group's thinking about how to add long sequences of numbers by generating equivalent pairs within the sequence. With the help of the facilitators, the teachers discovered that they could multiply the mean value of a sequence by the total number of values in order to find the sum of the sequence. For example, in the string of six numbers shown above, the mean value is 3.25, and 3.25 times 6 is 19.50. Using this new insight, the facilitators asked the teachers to return to their small groups and continue working on the task of finding the total cost of washing the windows of an n-story building.

Although Peter had offered the "addition by grouping" method, he was somewhat hesitant to return to the task and told his group members, "I'm not sure I want to dive back into this." However, he soon began to develop increasingly sophisticated mathematical ideas, including how to use the concept of the mean to solve the problem. As Peter explained these ideas to his group members, he became increasingly animated and enthusiastic. "I think we're on to something here," he exclaimed, as the team moved closer to an expression for calculating the total cost of a building with any number of floors. Nancy, a member of the group, noted, "I see what you're doing. Peter, you're so good at this." Working together, the group arrived at a general expression and tested it by plugging in different numbers of floors.

Satisfied that their expression was accurate and happy that they had completed the math task, Peter and his group members began to talk about implementing the problem in their classrooms. In particular, they considered what would be realistic to expect of students when solving this task. The group agreed that they would not expect their students to get as far as they did. Peter threw out several comments and questions for his colleagues, "It was kinda nice to figure out the cost of the whole building. But I wouldn't have been able to do it without that whole-group discussion about the mean. Would you expect your students to get to $19 \cdot n + 57$ [the general expression for the cost of any individual floor]? Would that be a goal for you guys, for them to get there?"

Several minutes later, a facilitator came by to chat with Peter's group, and asked Peter about his initial lesson plan ideas. Peter responded by describing what his math objective would be for this lesson. He said, "My goal would be to get them to that expression, $19 \cdot n + 57$. And to make the table and recognize the pattern of jumping by 19." The facilitator suggested that some students might be able to understand the concept of mean and therefore Peter could consider bringing in that idea. Peter said, "They would find that interesting. But whether they'd be able to make that jump, I don't know. For us as teachers that took a lot of maturity with numbers—a lot of experience with numbers. I think it can be done, but…" Peter continued to ponder the mathematical reasoning his sixth-grade students might use to solve this problem and he surveyed other teachers about their goals and expectations.

Vignette Discussion

As depicted in this vignette, the facilitators did several things to help Peter and the rest of the teachers navigate the Skyscraper Windows problem. First, the facilitators introduced the Skyscraper Windows problem by asking teachers to give their initial impressions of the mathematics concepts involved in this problem. Second, after working on the problem for a while, the facilitators brought the teachers back together to discuss a mathematical obstacle the groups encountered.

During the planning of the workshop the facilitators predicted some of the challenging portions of the task and came prepared with strategies to use with the teachers. The facilitators encouraged the teachers to share and build on one-another's understandings of the problem, and helped them to move forward in gaining new mathematical insights. Finally, the facilitators asked the teachers to consider how their own students might approach the problem, both before the teachers began working on the task and at the conclusion of the small group work, helping them to think about the task of planning a lesson for their students. As we illustrate later in this chapter, these specific facilitator moves are characteristic of the role of the facilitator in Workshop 1.

Conducting Workshop 1

Overview

Now we turn to a more complete description of how to conduct Workshop 1 of the Problem-Solving Cycle. As we have noted, a "Doing for Planning" framework guides the structure of this workshop. The phrase "Doing for Planning" is meant to suggest that the workshop should help teachers understand the importance of doing the mathematics task as part of their preparation for lesson planning. In particular, we hope that teachers will come out of this workshop with the sense that being familiar with the task and its embedded mathematical concepts is critical when designing lessons.

The workshop consists of five phases in which teachers (1) consider the mathematical concepts and skills in the selected PSC task, (2) develop and discuss their mathematical insights as they work on the problem, (3) address how the task connects to relevant mathematics standards documents, (4) design a lesson plan for using the task in their classrooms, and (5) determine when each teacher expects to teach the lesson. In this section, we describe the processes involved in conducting each of these phases.

Phase 1: Introducing the Task

*Phase 1 should take **about 10%** of the total workshop time.*

This first phase of the workshop is intended to provide teachers with a motivation for working on the math task. Below we describe the steps that we typically used to introduce the task. However, in the PSC model, facilitators have the freedom to introduce the math task in an alternative way that takes less workshop time or that seems better suited to the particular group of teachers with whom they are working. In other words, these steps are not meant to be prescriptive, but rather to give a general sense of the flow throughout Phase 1. (We use this approach of describing our typical steps in each phase of a workshop throughout the Facilitator's Guide. In all cases, the steps are meant to give a general sense of the workshop, rather than to be prescriptive.)

- ❑ Introduce the math task to the teachers by asking them to **read the problem and consider the mathematical ideas** embedded in it. We usually provided the text of the problem on a handout, which we distributed to all of the teachers. We found that many teachers wanted to start solving the task right away; however, we discouraged them from doing so. Instead, we asked the teachers to just read and think about the task.

  Ask the teachers to **individually write down a response to the question "What mathematical concepts and skills are involved in this task?"** We wanted the teachers to form some initial impressions of the mathematics involved in the problem by just reading the task. Later in the workshop we asked the teachers to compare these initial impressions to their ideas after solving the task. One rationale for engaging in this process is that teachers sometimes decide whether a problem is mathematically appropriate for their students by only reading, rather than doing, the problem. We have found it to be very powerful to highlight the dramatic effect that doing a problem can have on teachers' thoughts regarding the mathematics contained within it.

- ❑ After teachers have thought and written about the problem, have them **share their ideas.** We typically created a whole-group list, on chart paper, of the mathematical concepts and skills the teachers came up with. We kept this chart paper available and in sight throughout the remainder of the workshop so that we could reference it. The facilitator may want to clarify some of the mathematical ideas, and at the end of the discussion provide a brief summary.

Phase 2: Working on the Task

*Phase 2 should take **about 50%** of the total workshop time.*

In the second phase of Workshop 1, the teachers work collaboratively on the selected PSC math task. Our experience suggests that having teachers work together while doing the problem enables them to share their mathematical ideas and facilitates a deeper understanding of the concepts in the task. This is particularly true for math tasks that have the characteristics of a "rich" math task (see *How to Select a PSC Math Task,* in the planning section below).

- ❑ Have the teachers **sit in groups** of three or four. If desired, purposefully arrange these groups to include teachers with mixed math ability levels, same or mixed grade levels, or other characteristics. **Distribute manipulatives and mathematical tools** (such as rulers, graph paper, calculators, etc.) that are appropriate for completing the problem. Of course, the selection of such tools will depend on the math problem.

- ❑ Ask the teachers to **work on the math task.** Encourage the teachers to think, as they are solving the task, about how their students might approach this task. In addition, ask the teachers to start thinking about how to design a lesson incorporating the problem for their own students. Suggest to teachers that they keep track in writing of any ideas they have that would be important to consider when teaching this math problem to their students. (Note: Most of the lesson design process will occur after the teachers finish working on the math task. However, the "Doing for Planning" framework of Workshop 1 stresses the importance of teachers continually considering their own students' mathematical reasoning.)

  As the teachers work on the problem, **circulate among the small groups** and note how they are approaching it. Even with all the planning that facilitators undertake before each workshop, we have found that the teachers very often develop solution strategies that were not anticipated. Thus the facilitator must pay close attention to the mathematical work that the teachers are doing as the workshop unfolds.

  The facilitator should carefully observe and, when appropriate, ask questions of the small groups or individual teachers while they work on the math task in order to gain insight into (1) where they are in the process of working on the problem, (2) which groups have mathematical ideas that could benefit other groups, and (3) when to stop the work to begin debriefing their solution strategies as a whole group.

  As we illustrated in the vignette, facilitators may decide bring the whole group together intermittently, even before the teachers have completed their solutions, to discuss various mathematical issues that emerge within the small groups. For example, a facilitator may notice that several groups are having difficulty with the same part of the math task, or may may notice that a strategy one group is developing could aid another group that is struggling. Recall the portion of the vignette describing how one of the facilitators initiated a whole group discussion about how to find the total cost of an *n*-story building by adding up the individual floor costs, which yielded Peter's method of addition by grouping.

- ❑ After determining that the teachers have had enough time working on the problem, the facilitator should **lead a concluding whole group discussion.** A key element of this discussion is comparing and contrasting various solution strategies. The length of this discussion will depend on the number of strategies that emerged among the groups, the depth in which specific mathematical concepts are covered, and the nature of the intermittent whole-group discussions that were held as teachers worked on the problem (in Step 3). Typically, we debriefed the teachers' strategies by presenting them in order of increasing sophistication. In other words, we asked teachers (or small groups) with the least sophisticated (or mathematically developed) strategies to talk about their ideas first. This manner of presentation allowed teachers to see how their strategies could build on one other, and helped bring coherence to the discussion. In addition, it can also be valuable to have groups share any strategies that proved to be unsuccessful. Sharing unsuccessful strategies can promote interesting discussions about mathematical reasoning and problem solving, and helps to demonstrate that learners are free to experiment with a variety of mathematical ideas.

❑ Have the teachers **reconsider the question "What mathematical concepts and skills are involved in this task?"** posed before they began working on the task (see Phase 1, Step 2). Ask the teachers if they can generate any new ideas and, if so, add these ideas to the original list. Again, keep the (expanded) list available as a reference for the next phase of the workshop.

We have found that after working the math task and discussing various approaches to solving it, teachers often have new insights about the mathematical content within the problem. The facilitator might want to point out to the teachers that by doing the problem they are likely to (1) have a stronger understanding of the mathematics involved and (2) be better able to design effective and appropriate lessons using the problem.

Phase 3: Connecting the Task to Relevant Standards

*Phase 3 should take **about 10%** of the total workshop time.*

In phase 3, teachers map the specific mathematics concepts and skills embedded in the PSC task onto relevant standards. Relevant standards might include the NCTM (2000) *Principles and Standards for School Mathematics* and/or state, district, or school math standards. Connecting to the standards helps teachers to see the mathematical "richness" that a single problem can entail, a richness that may not be obvious upon its initial reading. This phase also helps prepare teachers for the next phase, in which they will plan a lesson.

If short on workshop time, Phase 3 can be left out of the PSC model. Like Phase 1, this phase is less vital to the professional development model than Phases 2 and 4. However, some school districts require teachers to link their lesson plans to relevant standards. In such cases, this phase may be especially pertinent.

At the beginning of this phase, distribute photocopies of the relevant standards documents (see appendix for a condensed version of the

NCTM [2000] *Standards*) so that teachers can reference them when they do this mapping. Then ask the teachers to note which standards the selected PSC math task addresses. If they have previously responded to the question "What mathematical concepts and skills are involved in this task?" (see Phase 1, Step 2 and Phase 5, Step 5), have the teachers consider their responses in light of the relevant standards. Specifically, the teachers would refer to the chart paper that contains their ideas about the concepts and skills involved in the task, and map those entries onto the standards document(s). If they have not created such a list the teachers might elect to generate one at this point in the workshop. Alternatively, the teachers could simply consider the standards without such a list.

The teachers could work in small groups to do the mapping, or they could engage in a whole-group discussion. If they work in small groups, the facilitator might elect to lead a whole-group discussion summarizing the main ideas.

Phase 4: Developing Individual Lesson Plans

Phase 4 should take ***about 10%*** *of the total workshop time.*

In the fourth phase of Workshop 1, teachers shift from "doing the math" to planning individual lessons incorporating the selected PSC task. The discussions and activities in the next two workshops of the PSC cycle depend on teachers having had a common teaching experience. Creating initial drafts of lesson plans during Workshop 1 helps to ensure that this will occur.

❑ Have the teachers **sit in small groups or work individually** on their lesson plans. There are several alternatives for grouping teachers when they are working on their lesson plans, each with different advantages. Having teachers work in grade-level groups can be beneficial because these teachers will likely share common math content goals for their students. However, if the teachers are from different school sites, working in school site groups may encourage them to continue the planning process outside of the workshop. Another option is to have teachers plan their lessons together with their colleagues with whom they solved the math task (in Phase 2). The benefit of this option is that the group members will be familiar with the mathematical ideas they developed and can help one another bring these ideas to their students, via their lesson plans. It is also possible for teachers to work on their lesson plans individually for a while, and then talk in small groups.

  One important thing about the PSC model is that the teachers should not work together as a whole group to create a single lesson plan that everyone teaches. Instead, teachers should tailor their lessons (and the problems) to their own students and their own teaching styles. In fact, the PSC model is most effective when the teachers have diverse ideas about how to teach the problem, so that in later workshops they can compare how their various approaches worked. However, if some teachers agree on selected aspects of teaching the problem (such as how to modify it, what manipulatives to offer their students, etc.), that is fine and should not be discouraged.

❑ **Distribute a list of prepared questions** to assist the teachers in planning their lessons. The facilitator should present questions that help the teachers to consider particular mathematical and pedagogical aspects of their lessons. Some examples of questions we found helpful include:

- What are your mathematical goals for your students for your lesson?
- What is the prior knowledge your students need for your lesson?
- How would you restate the math task to make it more accessible for your students?
- How can you introduce the lesson?
- How can you scaffold the lesson so students' mathematical understanding emerges?
- What specific questions can you ask students to engage their mathematical thinking?
- How can you assess your students' mathematical reasoning while they are working on this problem?
- How can you debrief the activity at the end of the lesson, to highlight the mathematical reasoning the students developed?
- How can you help your students examine different solution strategies?

While it may not be necessary to ask teachers to address all these questions in their lesson plans, developing answers to them will help build a comprehensive PSC lesson. In subsequent workshops during the cycle, the facilitator will ask teachers to examine, for instance, how they introduced the lesson or how they assessed and debriefed students' mathematical reasoning in the lesson. Having these elements explicit in the lesson plan both provides potential substance for future workshops and increases the teachers' likelihood of effective implementation of their PSC lessons.

❑ Ask the teachers to **work on their individual lesson plans.** In the earlier phases of this workshop, teachers should have already begun thinking about the lesson. They may have even written down ideas that they can now come back to, as they begin to formally engage in lesson planning. Have the teachers use these written notes to form an initial draft of their lesson plans.

As teachers begin considering instructional options for their lessons, they may want to return to the mathematics of the task to re-explore particular ideas. The Problem-Solving Cycle model encourages this continual interaction between thinking about specific mathematical issues and making instructional decisions. As teachers become more confident in their understanding of the mathematical concepts in the selected task, they are likely to produce lesson plans that are more nuanced and specific.

The length of the lesson should be up to the individual teacher, but we suggest the PSC math task be the central part of at least one day's lesson.

❑ After determining that the teachers have had enough time developing their initial lesson plans, the facilitator should **lead a concluding whole-group discussion.** In this discussion, have the teachers share with the whole group the mathematical and pedagogical components of their lessons. Hearing about the different mathematical goals teachers have for their students, the diverse ways they plan to introduce their lessons, the various methods they will use to assess students' mathematical thinking, and so forth, will help all the teachers reflect on their initial lesson plan design.

By the end of this phase (which is also the end of the workshop), each teacher should have an individual lesson plan for teaching the selected task. However, facilitators should encourage teachers to see their plans as "first drafts," which can be refined and modified outside of the workshop setting. In fact, the teachers can continue to refine their PSC lesson plans up until the day they teach the lesson. In our experience, teachers often consulted with one another informally before teaching their lesson. Some teachers called each other on the phone, sent emails, or simply initiated conversations with their colleagues in the school hallway or over lunch. A number of teachers found it helpful to use the Internet to pursue ideas for teaching the problem. The PSC model places no constraints on how teachers plan their lessons or what resources they use to do so. It is up to individual teachers to seek out as much information that they feel is necessary to implement the problem to the best of their abilities—knowing that in the next two workshops in the cycle they will draw on and discuss their experiences.

Phase 5: Scheduling the Lessons

*Phase 5 should take **about 10%** of the total workshop time, and should take place **before teachers leave the workshop.***

For the PSC model to be successful, it is critical that all the teachers teach their PSC lessons. The discussions and activities in the next two workshops in the cycle depend on the teachers having a shared common teaching experience. The PSC model rests on the idea that teachers can expand their knowledge when they have the opportunity to examine and reflect on their own teaching practice and relate their experiences to colleagues who have had a similar experience. Facilitators should convey this rationale and expectation to the teachers, as some may question the need for everyone to implement the lesson plan.

Before the workshop ends, **schedule with the teachers the days and times when they expect to teach the PSC lesson.** Be sure that teachers know when the next PSC workshop will be held, and that they understand they will need to teach the lesson prior to that workshop. Finally, **arrange for at least some of the teachers' PSC lessons to be videotaped.** Whether videotaping is done by the facilitator, the individual teacher, or another colleague, the activities of Workshops 2 and 3 depend on having video clips available from at least *some* teachers' PSC lessons.

Planning Workshop 1

Overview

Now that we have discussed how to conduct Workshop 1, in this section we will offer guidance on several key components to planning the workshop. These planning decisions roughly correspond to particular phases of the workshop, but as the reader will soon see, decisions made throughout the planning process are inherently interconnected. For example, decisions about selecting a math problem are sure to impact all of the other planning tasks, such as preparing to support teachers as they work on the task, helping to connect the task to relevant standards, and supporting the lesson planning process.

Selecting the Task

Why is selecting a task so important and where can I look for appropriate tasks?

There are many resources available from which a facilitator can select mathematics tasks to do with teachers in a professional development setting. However, not all of these choices may be appropriate when selecting a math task for the Problem-Solving Cycle (PSC). In the PSC model, teachers will work through the math task, design a lesson incorporating the task, teach that lesson to their students, and discuss their classroom experiences in two subsequent workshops. For the PSC to be successful, the selected task needs to have enough mathematics in it to foster a productive learning environment for the teachers over the course of three workshops.

We have found that good sources of PSC math tasks come from various curricula and materials that were designed in the vision of the NCTM *Standards* documents (1989, 1991, 1995, 2000). In the 1991 version of the NCTM *Standards* there is a discussion about the importance of selecting appropriate tasks and what some of the characteristics of good tasks are. These ideas have strongly informed our thinking about the PSC model.

[Teachers] should choose and develop tasks that are likely to promote the development of students' understandings of concepts and procedures in a way that also fosters their ability to solve problems and to reason and communicate mathematically. Good tasks are ones that do not separate mathematical thinking from mathematical concepts or skills, that capture students' curiosity, and that invite them to speculate and to pursue their hunches. Many such tasks can be approached in more than one interesting and legitimate way; some have more than one reasonable solution. These tasks, consequently, facilitate significant classroom discourse, for they require that students reason about different strategies and outcomes, weigh the pros and cons of alternatives, and pursue particular paths.
(NCTM, 1991, p. 25)

What are the specific characteristics of a math task that would be appropriate for the PSC?

Most important, a math task that would be appropriate for the PSC should be "rich" in mathematical ideas. The obvious next question is, "What does it mean for a math task to be rich?" We present several characteristics of rich tasks, not as an exhaustive list but rather as a general framework to guide facilitators in making this important decision.

Addresses multiple mathematics concepts and skills. A mathematically rich task should present opportunities to address multiple mathematics concepts and skills. We purposefully selected math tasks that cut across several components of the NCTM (2000) **Standards**, including the Content Standard of Algebra and several of the Process Standards. Our intention was to provide learners (including teachers and their students) with opportunities to build their knowledge of the algebraic concepts of patterns and functions, and to increase their problem solving, reasoning, and mathematical representational fluency. For example, we considered the Skyscraper Windows problem (discussed in the vignette at the beginning of this chapter) to be "rich" because of the numerous algebraic concepts that are embedded in it. We agreed that the Skyscraper Windows problem gives learners the opportunity to "represent, analyze, and generalize a variety of patterns with tables, graphs, words, and, when possible, symbolic rules" (NCTM, 2000, p. 222) so that their understanding of patterns and functions would be broadened. When we used this problem with teachers, they ultimately uncovered both linear and quadratic relationships, and probed deeply into the importance of mean and the generalizability of the addition process.

The best way for a facilitator to know if a problem is mathematically "rich" enough, in the sense of addressing multiple mathematics concepts and skills, is to actually do the problem. Solving the problem, and determining whether it meets this criterion, can serve as an initial filter for whether to pursue the problem further as a potential PSC math task.

Is accessible to learners with different levels of mathematical knowledge. If the problem seems to do a good job addressing multiple mathematical concepts and skills, it is then relevant to consider a host of other factors, including whether it appears to be accessible to learners with different levels of mathematical knowledge and understanding. Think about whether the task has *multiple entry points.* That is, does it enable learners with different background knowledge and mathematical abilities to become engaged and begin working toward a solution? This criterion is especially important for PSC workshops that include teachers from multiple grade levels.

Along with multiple entry points, an appropriate PSC math task will have *multiple exit points,* so that learners can complete the problem with varying degrees of sophistication. When problems have multiple exit points, facilitators can help the learners to make connections between alternative solution strategies, learn important mathematical content, and develop their mathematical communication skills.

To illustrate these ideas, we again use the Skyscraper Windows problem as an example. During the workshop depicted in the vignette at the beginning of this chapter, teachers began to approach the problem using different strategies. In their initial search for patterns, some teachers relied solely on a table to keep track of the cost of each floor. Others graphed the data to see if there was a linear relationship. Noticing these different strategies, the facilitators were able to order the teachers' presentations of their approaches so that they built off one another in a coherent and meaningful way. The facilitators started by having the teachers who used a table talk about their ideas. This conversation enabled the whole group to notice various patterns in the cost, such as the fact that the cost of washing the windows increases by \$19 per floor. The group then developed an expression for the cost of washing each floor (n): $19 \cdot n + 57$. The facilitators next asked the teachers who had graphically represented the data to present. Building on these ideas, the group was able to connect the information in the table with the slope and intercept characteristics of the graphed data, which also corresponded to the linear expression $19 \cdot n + 57$.

The main point here is that an effective PSC math task should encourage a variety of solution strategies (i.e., the mathematical approaches learners can use to solve the problem). If these strategies suggest various levels of mathematical sophistication, the facilitators can strategically order the presentation of the strategies to help learners follow and connect the ideas.

Has an imaginable context. The PSC math task should have an imaginable context in order to provide learners with a foundation for their mathematical thinking and reasoning. Having an imaginable context is not necessarily the same as being situated in the "real world." What we mean is that the problem should be authentic, and contextualized in a way that is appropriate for and interesting to students. Problems with an imaginable context pique the learners' curiosity, and help them start to engage in the problem-solving process. These types of tasks build on learners' prior knowledge and intuitions and can promote meaningful mathematical conversations. The Skyscraper Windows problem is a good example of a problem with an imaginable context. Learners can recognize immediately the importance of knowing the cost of washing all the windows of a skyscraper, for both the owners of that building and the window washers.

Can foster productive mathematical communication. Another important feature of an appropriate PSC math task is that it can serve as the foundation for productive mathematical communication among learners. Problems that encourage conversations are vital to the success of the PSC, which aims to promote dialogue among teachers and between teachers and their students. Productive mathematical conversations are ones that allow learners' conceptions and misconceptions to safely surface, as they explain and justify their mathematical assertions. Facilitators can capitalize on such conversations to assess the specific nature of the learners' mathematical understandings and determine how to help them advance their understandings. For example, when we facilitated the Skyscraper Windows problem, we listened carefully to the mathematical communication that was taking place among small groups of teachers in order to decide when to bring the teachers together to discuss finding the cost of washing an *n*-story building.

Is challenging for teachers and appropriate for their students. When selecting a PSC math task, it is important to remember that two different audiences, namely the teachers in the workshops and their students in the classroom, will eventually be working on this problem. Therefore, the math task needs to be challenging for teachers who take on the role of math learners in the workshop, and at the same time it needs to be appropriate for their students.

What should I do to decide on a task?

As we have mentioned, the best approach for deciding if a math problem is appropriate to be a PSC math task is to actually work on the problem. If possible, ask other math colleagues to work on it also, and discuss the problem's strengths and limitations with them. Our group typically would select several potential problems and work them out with one another while discussing the pros and cons of each. For example, we found tasks that had appropriate mathematics content, but were not highly contextualized. On the flip side, we also came across problems that had an imaginable context and were likely to support productive mathematical conversations, but seemed to lack sufficient mathematical content to sustain teachers' conversations over three workshops.

When working on potential tasks, we often were surprised to discover that the mathematical complexity of the task was not apparent to us when we first read it. In such cases, we considered this to be evidence of the mathematical richness available in the task. We should also note that despite our lengthy conversations about the math tasks *prior* to Workshop 1, unforeseen aspects of the selected task inevitably would emerge *during* the workshop. The fact that some teachers approached the problem differently than we had done was a good sign; it helped to confirm that the task selected was, indeed, mathematically rich.

Preparing to Support Teachers' Work on the Task

During Workshop 1 the teachers' understanding of the mathematics involved in the selected task is likely to be emerging and gaining in sophistication. An important responsibility of the facilitator is to help teachers synthesize the mathematical learning that occurs throughout the workshop and achieve a deeper understanding of important mathematical concepts. Any problem that a facilitator chooses will have unique mathematical qualities, and planning should be tailored around those specific qualities. However, we offer some general suggestions of activities facilitators can engage in when planning the workshop that should prove helpful.

- ❑ **Consider your rationale for selecting the problem.** Before the workshop, it would be helpful to think about, and perhaps write down, the mathematical reasons you selected the problem. For example, you may feel that the teachers, or their students, need help in a particular content area. Alternatively (or additionally), you may have selected a problem that addresses a content area that your school or district is especially attentive to. Keep track of the large mathematical concepts that are embedded in the problem, along with your rationale for selecting the problem. Use these ideas to guide your discussions as the teachers work on the problem, and ensure that they do not miss the key math concepts.

- ❑ **Consider where the teachers are likely to have difficulties.** In the section above we talked about the importance of the facilitator doing the problem beforehand, if possible with others, to foster their own understanding of the key mathematical concepts and skills in it. In addition, we suggest that during this time the facilitator would benefit from taking notes on places in the problem where he or she expects teachers may have difficulties. Then, carefully consider how to help the teachers navigate through these challenges. For example, as we noted in the vignette at the beginning of this chapter, when preparing for Workshop 1 of the Skyscraper Windows problem, we anticipated that it would be difficult for teachers to sum the costs of all of the floors, to find the total cost of washing an *n*-story building. Therefore, we came prepared to discuss with the teachers how they could group a string of numbers to generalize the addition process.

- ❑ **Select appropriate manipulatives.** When working on the problem, pay attention to the mathematical manipulatives and/or tools (such as rulers, graph paper, calculators, etc.) you used or could have used. These manipulatives and tools are likely the same types of items you would provide to students working on the problem. Therefore, teachers in the professional development workshop will need them too.

- ❑ **Generate multiple solution strategies.** In order to anticipate how the teachers might approach the problem, try to generate as many solution strategies for the problem as possible. This exercise should also prove helpful when thinking about how to organize a discussion around the teachers' actual solution strategies. As we have mentioned, it can be beneficial to debrief teachers' solution strategies by moving from the least sophisticated to the most sophisticated. In this way, members of the group can connect and build on one another's ideas.

  If you used a particular math resource to help you select a problem, it may contain common solution strategies or important mathematical insights about the problem. Use these resources in combination with your own math work to prepare for the debriefing of the teachers' strategies.

- ❑ **Consider how to visually present teachers' ideas.** Finally, consider how the teachers will report on their solution strategies when they present them to the whole group. Will you want them to write on poster paper or overhead transparencies? We have found that when teachers visually represent their solution strategies, they more clearly convey their mathematical ideas, and their presentations better contribute to the whole group's understanding.

Preparing to Help Connect the Task to Standards

During Phase 3 of Workshop 1, teachers map the math concepts and skills in the task onto relevant math standards. As we have discussed, rich problems that are appropriate for the PSC should address multiple mathematical concepts and skills. Furthermore, the facilitator may have intentionally selected a problem with specific aspects of a standards document in mind. We offer two additional suggestions for planning this component of the workshop.

- ❑ **Prepare copies of relevant pages from the standards document(s).** It will be helpful to identify and prepare copies of relevant pages from the standards document that the teachers can use during the workshop.

- ❑ **Do the mapping yourself.** We recommend that the facilitator go through a process, in planning for Workshop 1, that is comparable to what teachers will do in the workshop. That is, before working on the problem, write down the various standards you think the problem is related to. Then, after working on the problem, update this list. Ideally, you could discuss it with several colleagues. Bring your list to the workshop and use it to guide the discussion that takes place during Phase 3. Keep in mind that the teachers may notice other connections to the standards that you did not foresee.

  For example, in selecting the Skyscraper Windows problem we agreed that it addressed the NCTM (2000) algebra content standard of understanding patterns, relations, and functions. We also felt that it connected to the standard of representing and analyzing mathematical situations and structures using algebraic symbols. In addition, we agreed that it connected to the NCTM (2000) process standards of communication, representation, and problem solving.

Preparing to Support the Lesson Planning Process

During Phase 4 of Workshop 1, the teachers design an individual PSC lesson plan. We offer two suggestions of activities that facilitators can engage in when preparing for this phase.

- ❑ **Prepare guiding questions for the teachers.** The facilitator should come to Workshop 1 with a vision of what he or she would like to see included in the teachers' lesson plans. In our discussion of Phase 4, we presented some questions that facilitators might ask teachers to help guide them during the lesson planning process. Prior to the workshop, the facilitator should prepare a written list of possible questions to ask.

  When preparing to support teachers in the lesson planning process, it is also important to consider the needs of the specific teachers (and their students) who are taking part in the PSC, as well as any requirements their schools or districts might have with respect to conducting classroom lessons. For example, some school districts require teachers to specifically note the math standards addressed in their lessons, and the teachers might look to the facilitator to guide them in this aspect of their lesson planning. Additionally, some teachers may feel that they need to modify the problem to make it more accessible for their students. It could be helpful to the teachers if the facilitator helps them to identify what aspects of the problem they could alter (such as ways to make the language more accessible to younger students or English-language learners, or modifications in the number of floors in the building for the Skyscraper Windows problem) and what aspects they should not alter.

- ❑ **Consider how to debrief the lesson planning process.** Finally, we suggest that the facilitator have in mind a strategy for debriefing the lesson planning process. Depending on time, a variety of methods are possible. For example, the facilitator may elect to have teachers share their ideas with the whole group or they may decide to have the teachers form new groups and share ideas within those groups (e.g., one or two teachers from each group move to another group). Whichever method the facilitator chooses, the goal should be to help teachers learn as much as possible from one another and to deeply consider a variety of issues surrounding the successful implementation of the selected task.

Chapter 5

Videotaping Teachers' Problem-Solving Cycle Lessons

Overview

"I feel like I've benefited the most [from] the videotaping. That was huge for me, to get just a look at myself. It was really the first time that I've ever gotten to take a close look at what I'm doing in the classroom."
—Ken, final interview

Before Workshop 2, each teacher should implement the Problem Solving Cycle lesson that they developed during Workshop 1 in at least one of their mathematics classes. Some (or all) of the teachers should be videotaped. Then, in Workshops 2 and 3, the group comes together to discuss their common teaching experience and the various mathematical and pedagogical issues that arose. One of the most important components in Workshops 2 and 3 is the analysis of teachers' pedagogical moves and students' mathematical reasoning, using video clips of the PSC lessons. Therefore, after the videotaping occurs, the facilitator will need to select short clips to serve as anchors for the discussions during Workshops 2 and 3.

Do all of the teachers need to be videotaped?

While we do consider it a requirement that all of the participating teachers plan and implement a lesson utilizing the selected mathematics task, we do not feel that it is necessary for every teacher to have his or her PSC lesson videotaped. We feel strongly that teachers will benefit from having their lessons videotaped, and we would urge everyone in the group to try to make this happen. However, we recognize that, for practical purposes, some teachers may not be arrange videotaping. If a teacher cannot be videotaped, the facilitator might ask the teacher to audiotape the lesson, collect the students' work completed during the lesson, and/or write a detailed reflection about the lesson shortly after it occurs.

Who should videotape the lessons?

There are several ways a lesson could be videotaped. One option is for the facilitator to videotape the lesson. We consider this to be the ideal option because the facilitator will be able to gain firsthand knowledge about the events in the lesson, take meaningful notes, and recall critical moments that could serve as effective clips to show during Workshops 2 and 3.

A second option is for a teacher's colleague or other school or district personnel to videotape the lesson. The third option, least desirable yet most practical, is for the teacher to set up an unmanned, stationary video camera (on a tripod) in the back of the classroom. This would, at least, allow for footage capturing a wide shot of the classroom activities throughout the PSC lesson.

"I think the very most helpful was the videotaping, to watch myself on videotape, sometimes painfully so. Wanting to say, shut up, shut up, why do you keep going on with that…. But it's so helpful to see how you come across to kids and how they are or they are not responding. In some cases they were responding and in some cases they weren't responding. And to think about how I could have done that differently and what I might have changed in that lesson or how I could have connected with kids better."
—Celia, final interview

What if the PSC lesson is taught over several days?

Again, there are no hard or fast rules imposed by the PSC professional development program. Teachers can choose to implement the PSC lesson over several classroom periods, if they feel that is appropriate. Optimally, any or all of those lessons should be videotaped, as described above.

What are the filming guidelines?

Filming guidelines will vary depending on how many cameras are used to tape the lesson, and whether the cameras are operated or not. In an ideal setting, two cameras (and two camera operators) can be used to film teachers' PSC lessons. The main camera should be set up on a tripod near the back of the room, or along a side wall of the classroom. The person operating this camera should take the perspective of an "ideal student" in the lesson. That is, the camera operator should film what an ideal student, who is paying close attention in the lesson, would be looking at. In some cases, the ideal student would be looking at the teacher. In other cases, the ideal student would be looking at another student participating in the discussion, or at what is being written on the overhead projector or blackboard. During periods of small-group work, the "main" camera should follow the teacher and capture his or her interactions with students. (Alternatively, when there is not a second camera, the main camera operator may elect to film lengthier small group, student-student interactions, including periods of time when the teacher is not working with them.)

The main camera operator should use a "medium" shot most of the time; although sometimes there may be a need to zoom in (e.g., to read what is written on the board) or get a wide-angle shot (e.g., when the teacher asks students to raise their hands). Keep in mind, however, that for the PSC professional development program, it is not a critical matter to obtain high-quality video footage. The camera operator should simply try to do the best job he or she can to get a clear picture, not to shake the camera too much, to make sure that the audio and video features are functioning properly, and to film in a way that gives the viewer a reasonable image of the important events that take place in the lesson.

If a second camera is being used, its primary function should be to capture small-group interactions. In particular, whenever the class is working in small groups, this camera should be trained on a selected

"The videotaping I think is very helpful because you never really know how you come across and you never really know about that interaction with kids. And when you see that interaction, you can correct it or you can repeat it."
—Penny, final interview

group of students who are comfortable being filmed. When possible, it is helpful to film close-up shots of what these students write on their papers. When the whole class is working together, this camera can simply be used as a backup to the main camera (e.g., following the guidelines suggested above). The second camera could be operated by a second person filming the lesson, by one person who operates both cameras, or by the teacher (who simply sets it up to focus on a selected group of students).

Keep in mind that the camera operator should try to be as inconspicuous as possible. This means not talking during the lesson—to either the teacher or the students. It also means trying to keep the camera out of the way as much as possible, such as by keeping it on a tripod in an unobtrusive spot in the classroom. Unless it is absolutely necessary, avoid moving the tripod or taking the camera off the tripod during the lesson .

What about microphones?

It is very important to get good audio when videotaping; however, microphones can be very costly! There are a couple of options we can suggest. The first is simply using the built-in microphone that comes with any video camera. If the teacher makes a concerted effort to talk in a reasonably loud voice, this can work out fine most of the time. However, we have found that it is often hard to hear students, and it can also be difficult to hear the teacher during periods of small-group work. The best advice we can give when using the built in microphone is to turn it up as high as possible, and remind the teacher to talk reasonably loudly and to ask the students to do the same.

A better option is to use an external microphone. Some microphones can be plugged into the camera, and have long cords so that they can be placed away from the camera. If you have this type of microphone, place it on a desk near in the middle of the classroom.

The best, and costliest, option is to use a wireless microphone that can be attached to the teacher's clothing. This type of microphone will ensure that the teacher's voice is clearly heard throughout the lesson, and it will pick up students' voices then the teacher talks with small groups.

Be aware that the type of microphone you have access to, as a camera operator, may affect what you choose to film, particularly when students are working in small groups. If you are just using the internal videocamera microphone and the teacher is far away from the camera talking quietly to a group of students, you may decide to film the group of students who are the closest to the camera (and thus can be heard most clearly). Think about what audio the camera is picking up, and try to film video that matches the audio.

Should the camera operator take notes while filming?

We do suggest that the camera operator take notes, if possible. However, the main job is to film the lesson. It is also fine to take notes shortly after the lesson takes place. The main purpose of these notes should be to record "interesting" things that happened during the lesson, and approximately when they occurred. These notes will be extremely useful when facilitators need to select video clips to show in Workshops 2 and 3.

Some of the things that we paid close attention to when videotaping the PSC lessons were

- How and when teachers asked questions
- Students' interesting (e.g., creative) mathematical reasoning
- A unique solution strategy for the task
- Teachable moments that were possibly missed
- The overall structure of the lesson
- The introduction of the lesson
- The conclusion of the lesson

What should I do with the video footage after it is shot?

Processes involved in watching, editing, and showing video clips during workshops will depend on the format in which lessons were filmed. For example, if a digital video camera was used, clips can be selected and burned onto a DVD or copied to a computer. During workshops, digital video clips can be shown using a data projector.

The technology that facilitators have available to them will determine exactly what they can reasonably do with the video footage. However, if at all possible, we suggest that the facilitator review each teacher's videotaped lesson in order to select video clips. We also suggest that facilitators make a copy of each teacher's lesson, shortly after it is filmed, for the teacher to keep.

Will we need permissions from the students and their parents to film?

Whether or not facilitators or teachers will need to obtain permissions, and what sort of permissions, will depend on the school and district requirements. Whatever the case may be, if there are students (or parents) who are uncomfortable being videotaped, do not film them. We have found that it is quite simple for the camera operator to avoid filming these students, especially if they sit in an area that is away from the video camera.

Chapter 6

An In-depth Look at Workshop 2

Objectives

Workshop 2 Objective:

To foster teachers' pedagogical content knowledge—by thinking deeply about the role they played in teaching the selected mathematics task to their students.

In this chapter, we provide an in-depth look at the second workshop of the Problem-Solving Cycle. Workshop 2 takes place after all of the teachers have taught the selected PSC mathematics task to their own students. In this workshop the teachers debrief their experiences teaching the selected task. The majority of the time is spent watching and discussing video excerpts from one another's lessons.

This chapter includes a vignette drawn from one of our experiences carrying out Workshop 2, a detailed guide to conducting the workshop, and information useful in planning for this workshop.

Vignette from Workshop 2

"I have learned the most about my teaching by watching *my teaching practice. Even better, though, was watching* others *teach a lesson that I also taught. My ideas have been sparked by others in this group. Having a safe place to watch ourselves and not feel like we are being criticized or evaluated was critical also."*
—Ken, final free write (emphases by the teacher)

This vignette is a sequel to the vignette from Workshop 1 in Chapter 4. The vignette in Chapter 4 illustrates how a group of teachers worked through the mathematics of the Skyscraper Windows problem, focusing largely on one sixth-grade teacher, Peter. Now we consider the teachers' experiences after they all have implemented the Skyscraper Windows problem.

In this case, the teachers are analyzing a video clip from Peter's lesson. The clip shows Peter talking with a student, Kaitlin, about her method of solving the problem. The facilitators use this clip to anchor a discussion about teacher questioning. As you read the vignette, pay particular attention to (a) the teachers' need to view the video numerous times; (b) the emergence of new insights about the role of teacher questions; and (c) the persistence of a supportive and collegial atmosphere.

Vignette

Approximately two weeks after solving the Skyscraper Windows problem in Workshop 1, Peter teaches his sixth graders the lesson he has designed around that problem. A week later he attends Workshop 2, during which a large portion of time is devoted to watching a 160-second video excerpt from his lesson. The facilitators explain that Peter's clip is intended to provide a foray into the central theme of the workshop: teacher questioning. One of the facilitators, Kim, hands out the discussion questions and the other facilitator, Craig, reads the questions aloud: (1) How did Peter's questions help him understand how Kaitlin derived her expression? (2) What additional questions would you ask Kaitlin to further understand her mathematical reasoning?

After the teachers agree that they are ready to proceed, Kim starts the video. The clip begins with Peter examining an equation written on Kaitlin's paper: $n \cdot 19 + 3 \cdot 19$. Peter immediately asks Kaitlin and her partner if they can rewrite the equation so that it begins with $19n$, and the students nod. He then asks why they wrote 3 times 19, and lacking an audible response, changes his question to, "What is 3 times 19?" They respond, "57." Peter tries again to have Kaitlin explain how she got 3 times 19. This time she uses her pencil to point to numbers in her table. She shows Peter, "It was this number. So you could add 19 three times." After a few more prods and questions, Kaitlin rewrites her equation as $19n + 57$.

The group watches the entire clip one time and then, at a teacher's request, they rewatch the last minute. Next the teachers gather in small groups, prearranged by the facilitators, to discuss the video from the perspective of the guiding questions. Each group is provided with a laptop computer that contains a copy of the video clip, so that they can view it as many times as they want. At the conclusion of his small-group conversation, Peter comments to Kim and two of his colleagues, "I can tell she knows the answer. I just didn't ask the right question." Kim responds, "What would you want to have asked?" Peter replies, "I wish I had asked her how she got the 57 in her equation." Kim nods as Craig initiates a whole-group conversation.

Focusing on the first discussion question, Craig asks, "So, what questions did Peter ask to help him understand Kaitlin's thinking?" Several teachers identify effective aspects of Peter's questions, specifically the fact that he asked "why" several times. One teacher, Kelly, brings up Kaitlin's unexpected use of the expression 3 x 19 and notes that Peter did take the time to ask about her thinking:

> Going back to your question about the 3 times 19 versus 57: that's what started it, but the answer didn't come until those very last couple of seconds in the video. I mean, all that interplay in-between didn't answer the question. It didn't get answered until the last couple of seconds. Peter could have left much earlier, and just left the fact that she has 3 times 19. [He could have said,] 'She has the 57. Good. Good to go, fine,' without going back and saying, 'Where did she get the 3 times 19?' Which I thought was very interesting thinking on her part. Very, very impressive, actually.

Kelly's comment pulls the other teachers into a conversation about Kaitlin's mathematical reasoning, including whether it is possible to know if she simply stumbled onto her equation or if she developed it intentionally. The teachers provide evidence for both arguments, but when they watch the video again, paying close attention to Kaitlin's gestures, they come to agree that Kaitlin purposefully developed her mathematical equation. In addition, the teachers discuss alternative methods for determining how Kaitlin generated the value 57 and what that value represents to her. The teachers conclude that the purpose of their questions should not be to align students' thinking with their own thinking, but rather to "unpack" students' informal reasoning.

Toward the end of their conversation, Peter becomes increasingly reflective and relates powerful insights about some aspects of his teaching that he believes could be improved. He suggests that his interaction with Kaitlin limited, rather than expanded, her thinking about the mathematics in the Skyscraper Windows problem. Peter sees that his preoccupation with conceptualizing 57 as the cost for the zero floor precluded him from understanding and encouraging Kaitlin's own mathematical reasoning. He says with a tone of disappointment, "I was trying to force her down toward that bottom thing [i.e., $19n + 57$]. And the more you guys have talked about how Kaitlin was thinking, I kind of wish I had processed that a little bit more. Or actually I kind of wish I had stopped talking for about five seconds and looked at it."

One of Peter's colleagues, Karen, points out that asking questions to unpack complex or unanticipated student reasoning is extremely challenging. She reminds the group that only by watching the video clip many times could they suggest questions that might have been more effective than Peter's. Several other teachers echo this sentiment. For example, Kelly comments that it was not until she looked carefully at Kaitlin's gestures that she finally understood her thinking. Kim and Craig wrap up this discussion on the interplay between teacher questioning and student reasoning by reiterating what the teachers have concluded and by making the point that there is not a single, correct question to ask at any given time. Instead, teachers should think about their questions as a way to build upon and scaffold student reasoning, helping students to connect their own insights to more formalized mathematical reasoning.

Later in the workshop, when given the opportunity to reflect critically on the day's conversations, Peter thinks carefully about how to change his questioning patterns. He discusses these ideas with his colleagues and puts them in writing. When Peter returns to attend Workshop 3, he speaks about his ongoing efforts to monitor his own questioning and to remain conscious of taking his students' reasoning into account rather than pushing them toward his way of thinking and his solution strategies.

Vignette Discussion

The facilitators made a number of strategic decisions that helped to make this workshop successful. First, they carefully selected a video clip that would motivate the teachers to think deeply about the role of teacher questioning, particularly when students have unexpected or unique mathematical ideas. The short, rich video clip that they chose from Peter's lesson highlights the complex interplay among the teacher's role, the students' thinking, and the mathematics. Second, the facilitators prepared structured discussion questions that encouraged the teachers to critically examine Peter's questioning techniques. The facilitators capitalized on the fact that this group of teachers had formed a tightly knit professional community and would be capable of tackling challenging issues in a positive manner. Third, the facilitators played the video during the whole-group conversations as many times as the teachers requested, and provided the teachers with an opportunity to watch the clip at their leisure in small groups while using laptop computers. By watching the video numerous times, the teachers picked up on subtle details of Peter and Kaitlin's interactions, and discovered the importance of paying close attention to students' gestures. Finally, the facilitators provided the teachers with an opportunity at the end of the workshop to continue to reflect on what they had learned, and to actively consider ways of improving their instruction.

Conducting Workshop 2

Overview

Conducting Workshop 2 takes place in three phases. During the first phase, the group debriefs their experiences teaching the common PSC mathematics task. The second phase makes up the bulk of the workshop time, when the group watches video clips and examines selected aspects of their instructional role. In the last phase the teachers reflect on their conversations and consider their personal teaching goals. We discuss each phase at length and provide descriptive examples from our experiences.

Phase 1: Debriefing the lesson

Workshop 2 begins with a brief period of time allocated for teachers to reflect on their PSC lessons and compare their experiences to those of their colleagues. The objective of this phase is for teachers to share their experiences, discover what they have in common, and reflect on the value of the "doing for planning" process that took place in Workshop 1. In addition, this phase is intended to encourage the ongoing process of community building and to prepare the teachers to comfortably enter the next phase of this workshop, when they will share video clips from their own PSC lessons.

*Phase 1 should take **about 10%** of the total workshop time.*

❑ Begin by asking the teachers to **think back on their PSC lesson**. Have the teachers consider questions such as:

- What went well in your lesson?
- What was harder than you expected?
- Did anything happen during the lesson that you had not anticipated?
- If you taught the lesson more than one time, did you make any changes to your teaching?

❑ Encourage the teachers to **write their thoughts down on paper and/or talk to a partner**. After the teachers have time to collect and organize their thoughts, **encourage a whole group conversation**. We typically wrote teachers' main ideas on chart paper in order to keep track of the similarities and differences in their lesson experiences. When teachers addressed the first two questions listed above, we would put their positive comments about the lesson on one piece of paper and their negative comments on another. The facilitator should encourage all of the teachers to participate in the conversation, and then summarize some of the salient issues that are raised and/or highlight areas of commonality.

The facilitator should remain conscious of not spending too much time on this portion of the workshop. You may want to tell teachers that your intention in this phase is to have them present an overview of their lesson or a couple of noteworthy moments. Some teachers may want to go into significant details concerning their lessons; however, the facilitator should focus the conversation so that comments are relatively brief and everyone has a chance to participate.

During one of the debriefings in our STAAR program, the teachers shared several reactions to their lessons. Here are two of the comments:

- "The way the students approached the problem was very different than how I approached the problem during Workshop 1. Their approaches were more meaningful to them."
- "I was surprised that several kids found relationships between the costs for different floors."

The facilitators viewed these comments as being closely related. Both comments refer to the students and what they were able to do with the task. The facilitators nurtured this thread throughout the course of the workshop, asking teachers at different times throughout the workshop to think about how and why student reasoning looked different than their reasoning.

- ❑ Next, ask the teachers to **reflect on the degree to which Workshop 1 prepared them to teach their PSC lessons**. The purpose of this reflection is encourage the teachers to think back to the work they did in Workshop 1, and to reflect on the value of working through the problem before teaching it—including solving the problem with colleagues, anticipating students' reasoning, and developing a lesson plan. The teachers could consider questions such as:

  - How did Workshop 1 impact the development of your lesson plan, and ultimately what occurred during your PSC lesson?
  - How did doing the math help you to develop your lesson plan and react to what your students brought up during the lesson?
  - How did anticipating your students' reasoning help you to plan and react to what your students brought up during the lesson?

- ❑ Again, have the teachers **write their thoughts down or briefly talk with a neighbor**. Then initiate a whole group discussion in which teachers share their reflections on Workshop 1 and its impact on their lessons. The facilitator might be able to highlight themes that emerged during small- or whole-group conversations. The purpose of this activity is to have teachers share their experiences, build relationships with one another, and reinforce the value of Workshop 1 activities, including "doing for planning" prior to teaching.

  During a STAAR workshop, teachers shared a variety of insights regarding the impact of Workshop 1. For example:

  - "If I hadn't fully done the problem, I would have planned "too big" and had every student aim for the final equation as a first step. Instead, I looked more at just the cost per floor. It helped me manage the complexity."
  - "Doing the problem helped me budget my time with the kids."
  - "Doing the math beforehand opened my eyes to new ideas that were helpful in dealing with the students."
  - "As I was doing the problem, I was looking for troublesome areas so that I was prepared to either deal with them or avoid them when those areas came up during class."

Phase 2: Examining the teacher's role

*Phase 2 should take **about 70%** of the total workshop time.*

During the second phase, teachers view video clips (and possibly other lesson artifacts), focusing on selected aspects of their instructional role during the PSC lesson. Video clips help to ground Workshop 2 in the teachers' common experience implementing the PSC problem. They also provide concrete instances of pedagogical decisions such as how to introduce the problem, conclude the lesson, facilitate group work, or mediate student thinking. The facilitator will need to prepare carefully for this phase by selecting a topic, choosing video clips, and writing guiding questions. (See the planning section below).

❏ We suggest beginning this phase by **introducing the selected topic of the workshop**—that is, the aspect(s) of the teacher's role that will be the focus of the workshop. It may also be helpful to **include some discussion of the guidelines for watching and discussing video clips**, especially if the teachers are new to the PSC model. Facilitators might remind the group about their conversations on this topic that took place during Workshop 0. Some of the most important points to re-iterate are the following:

- The focus is on the teaching and not on the teacher.
- Criticizing the teaching is not the primary activity.
- Praising the teaching is not the primary activity.
- The discussion should center on instructional strategies and their impact on students' mathematical reasoning.

Our intention is for the teachers to analyze the instructional practices seen in the video clip, and not to evaluate the teacher. We have found that teachers sometimes want to ask the videotaped teacher to evaluate his or her performance or that of the students. We would discourage these kinds of conversations. In addition, as they carefully unpack the details surrounding the events in the video clip, the teachers may need help from the facilitator to stay "on track" and address the intended topic of the workshop. In particular, the teachers should not become overly concerned with contextual issues (regarding the students, the teacher, or the rest of the lesson) that are not directly relevant to the events at hand.

❏ **Provide guiding questions** for the teachers to consider as they watch the clip.

In addition to the guiding questions, we typically provided, verbally or in writing, a short description of the video clip and the purpose of showing the clip. Prior to viewing the selected clip, give the teachers a few minutes to read the questions.

We have found the following types of questions to be effective:

- What evidence is there that the teacher ________?
- How is this teacher's ________ impacting students' ________?

- What prior knowledge do you think this teacher is using to ________?
- As a teacher, what would you do with ________?

❑ **Cue the clip** for the whole group's viewing. In our workshops, we connected a video projector to a laptop computer containing the digitized video clip. We then projected the video onto a pull-down screen. In order to ensure adequate sound, we plugged a pair of external speakers into the computer.

Watch the clip one or two times as a whole group. We found that teachers often wanted to watch the clip two times before they felt ready to discuss or write about it. Sometimes the teachers would ask to see the clip (or portions of the clip) a third time as a whole group.

The teachers in our workshops knew that they would also be able to watch the clip (or set of clips) in small groups using laptop computers. STAAR facilitators brought to each workshop three laptop computers, each loaded with the appropriate video clips. (Portable DVD players can also be used for this purpose.)

❑ Have the teachers **move into small groups and talk through the guiding questions**. We found that groups of about three or four teachers worked well. During the small-group time, the facilitators would circulate, listen for common themes, and consider when it would be appropriate to initiate a whole-group conversation.

Because we provided each small group with a laptop computer, the teachers could view the entire clip or relevant portions on their own, as often as necessary, as they went through the guiding questions. Facilitators sometimes needed to help the groups use this technology. If multiple laptop computers are not available, the facilitator may elect to allow the teachers access to a video clip by replaying it for the whole group or small groups, at their request.

❑ Once the small groups have had enough time, the facilitator should **lead a concluding whole group discussion** related to the video clip. Ideally, this discussion will focus on relevant issues regarding the teaching practice seen in the video clip and also incorporate teachers' reflections upon their own practices that were motivated by the clip. We sometimes found that it was helpful (or requested by the teachers) to watch portions of the video clip once again during this concluding discussion.

❑ Depending on the time allotted for the workshop and how much time the group spends processing a single video clip, **there may be time to watch additional clips**. In our full-day workshops, we typically showed three to five video clips during this phase. For each clip, we followed the process outlined above: reading the discussion questions (which may differ for each clip), showing the clip to the whole group, having teachers talk in small groups, and concluding with a whole group discussion of that clip.

We found that the classroom activities captured on some clips resonated better for the group than did others. It was hard for us to anticipate exactly which clips would promote the most conversation, and how long these conversations would last. Therefore, we typically came to Workshop 2 prepared to discuss more video clips than we expected to have time for. We generally came prepared to show and discuss seven video clips, but would make it through only three to five clips.

❑ If the group has watched multiple clips, the facilitator should conclude this second phase by helping to **summarize all of the discussions about the clips**. Keep in mind that discussions about the video clips are not an ending point, but rather a launching pad for further inquiry into important pedagogical and mathematical topics. Our goal is not to convince the teachers to express or adopt any particular pedagogical stance. Instead, we want to help teachers critically reflect on mathematics teaching in general and their own instructional practices in particular. This idea of reflection on teachers' own practice leads us to the third phase.

Phase 3: Reflecting on ways to improve the teacher's instructional role

*Phase 3 should take **about 20%** of the total workshop time.*

The objective of this third and final phase of the workshop is for teachers to reflect critically on their prior conversations and to explore ways of improving their instruction in light of the emerging ideas. In this phase, the conversation typically shifts from an abstract plane to a more concrete and practical level. Teachers sketch out new ideas to take back to their classrooms and help each other work through these ideas. The time allotted for this phase needs to be enough for teachers to carefully think and talk through (and potentially write down) their ideas.

❑ Begin this phase of the workshop by **providing a set of overarching questions that encourages teachers to reflect on their instructional role**. We used questions like these to help teachers frame their conversations:

- What would you like to do differently from what you have done in the past to __________ [insert phrase relevant to the selected workshop topic, e.g., improve your questioning techniques, elicit student reasoning]?
- What is your rationale for making this change?

- How can you facilitate this change?
- How can you monitor your efforts to make the change?

As these examples show, we wanted the teachers to think beyond simply implementing a change. We also wanted them to generate a rationale for the proposed change (i.e., why is it necessary or worthwhile), along with a concrete plan for how they would go about making such a change (or changes) and a self-assessment strategy.

❑ **Have the teachers talk in small groups**. We typically asked teachers to reflect on these questions in grade-level groups. However, another possibility is to group the teachers with their colleagues who are working on similar issues with respect to improving their teaching. (To determine what issues they are working on, the facilitator might have the teachers talk briefly as a whole group about their ideas.)

The teachers could simply talk through their ideas together, or you might ask them to put their reflections in writing (before or after talking with their colleagues). For example, you could ask the teachers to write down responses to the questions you provided. Or you could ask them to write about a specific plan of action, such as how they will prepare for an upcoming lesson. Another idea is to have teachers generate a list of articles (or article topics) related to a specific aspect of instruction that they want to learn more about.

We have found that teachers were often very creative and productive in this phase of the workshop. For example, Peter (the teacher featured in our vignette) and Laura came up with an interesting idea together. They thought that their students might benefit from watching video clips of themselves in action. Specifically, Peter and Laura hypothesized that their students might be able to engage in fruitful conversations based on the mathematical ideas they saw in the clips. They agreed to test out this idea and see if watching video did in fact help students to explore their own mathematical reasoning in greater depth. Peter and Laura shared their idea with the rest of the group, and several other teachers agreed that it sounded interesting and might be something they would like to do as well.

❑ **Lead a whole group discussion** of the different ideas that the teachers generated. This discussion is meant to be more of a sharing of ideas than a focused conversation. Just as the workshop began with all of the teachers sharing their thoughts about their individual PSC lessons, the workshop ends with the teachers sharing their thoughts about how to make improvements in their classrooms. An important consideration is to make sure that every teacher gets a chance to share his or her thoughts.

- ❏ **Conclude the workshop by setting up or reminding teachers of the date for Workshop 3**. You might also want to briefly mention what the planned activities are for this last workshop in the cycle, such as watching video and/or looking at student work in order to explore students' mathematical reasoning on the PSC task.

Planning Workshop 2

Overview

In this section we offer guidance on planning for all three phases of Workshop 2. However, the most critical planning decisions come in the second phase—selecting the topic that the group will focus on, selecting video clips, and crafting discussion questions that will frame the discussions. We address these issues in the most depth.

Planning to Support Teachers' Debriefing of the Lesson

Workshop 2 begins with a relatively short phase in which teachers share their experiences teaching the PSC lesson, reflecting back on the lesson itself as well as on their Workshop 1 experiences in preparation for the lesson. There are two central planning tasks for this phase: (1) deciding how much time to allot and (2) preparing guiding questions.

How should I plan for this phase if I have only a short amount of time?

We strongly suggest that facilitators who are leading Workshop 2 meetings that will be relatively short (e.g., half-day sessions rather than full-day sessions), do NOT skip this first phase, but rather condense it. This phase is critical in providing teachers with an opportunity to recall salient aspects of their PSC lesson and the mathematics task, and to reconnect with one another. Facilitators can condense the phase by posing only one set of two or three questions (including questions about the PSC lesson as well as teachers' Workshop 1 experiences). In addition, facilitators could leave out the small-group discussion time and simply ask the teachers to take a few minutes to respond in writing. We do encourage facilitators to hold at least one brief whole-group discussion so that the teachers can share and compare their experiences.

What kinds of questions should I prepare?

Facilitators should prepare two types of questions for this phase. Some of the questions should ask the teachers to think back on their PSC lessons. Other questions should ask them to consider again the lesson planning they did in Workshop 1. The "Conducting" section includes several examples of both types of questions. If the workshop is going to be relatively short, facilitators can pose these two types of questions at the same time. If there is more time, facilitators can ask the teachers to consider and discuss each type of question separately.

Selecting the Topic

It is important to point out that selecting a topic for the workshop and selecting video clips to view in the second phase of this workshop are interrelated decisions, and either decision might occur first. In other words, a decision about which topic to focus on will naturally impact the choice of video clips to address this topic. Conversely, facilitators might select a topic because they noticed particularly interesting moments in teachers' lessons that touched upon this issue. Whichever decision is made first, it is important to think carefully about the match between the workshop topic and the video clip. In this planning section, we first discuss the selection of a topic and then move on to discuss the selection of video clips.

The primary focus of Workshop 2 is on a specific aspect (or aspects) of the teachers' role during their PSC lessons. The decision about what aspect of the teachers' role to emphasize during a given iteration of the PSC depends on the needs and interests of the particular group of teachers with whom you are working and the overall goals of your professional development program. Documents such as the NCTM (2000) *Principles and Standards for School Mathematics* and state, district, or school standards might be useful in making this decision. Using these documents to inform your planning of the workshop also helps to ensure the relevance of the selected topic to current teaching standards.

Over the course of three iterations of the Problem-Solving Cycle, aspects of the teachers' role that we elected to focus on included:

- **Introducing the PSC task**. Various ways in which teachers activated their students' prior knowledge or otherwise prepared them to engage in the selected mathematical task.
- **Posing questions**. The types of questions teachers posed in order to elicit, challenge, build on, and extend their students' thinking about the task. Also, the impact of teachers' questions on student reasoning and how questions can build on one another.
- **Providing explanations**. Decisions teachers made about when to provide explanations, ask leading questions, or let students follow their own line of reasoning without teacher input. Also, how teachers can respond when students navigate the problem in ways they did not anticipate, and how teachers can help students connect their thinking to more formal mathematics.

- **Concluding a lesson**. How teachers worked to tie together their students' mathematical ideas and conclude the PSC the lesson.

Selecting Video Clips

A facilitator needs to think carefully about how to select video clips that match the specific topic that he or she wants teachers to explore during the workshop. (Alternatively, if the video clips are selected first, a facilitator needs to think carefully about how to narrow in on a topic that matches the video clips). The key to making sure that the connection between the selected topic and the selected video clips is clear is to write focused discussion questions, a process that we will discuss next.

First, we offer guidelines for selecting video clips. All video clips shown in Workshop 2 (and Workshop 3) of a PSC should be from the participating teachers' PSC lessons. In order to decide what clips to show, the facilitator needs to be familiar with the teachers' taped lessons. This may mean watching the lessons, or if you were the videographer of the lessons, thinking back (or reviewing your notes) on events in the lesson that stood out to you. We recognize that this is a time-intensive process, but the selection of appropriate clips is critical to the success of the Problem-Solving Cycle model of professional development.

What should I look for when selecting a video clip?

Similar to the selection of a "rich" task for Workshop 1, for Workshop 2 a facilitator needs to select "rich" video clips. At first, it might be tempting to look for video clips that demonstrate "gold standard" teaching moments, or that exemplify "expert practice." However, we have found that in the PSC model such clips are not necessary to motivate meaningful discussions. Instead, we suggest looking for video clips that relate to the selected theme of the workshop and have the features listed below. When combined with carefully framed questions, these clips can be extremely effective in engaging teachers to think deeply about important and relevant issues.

In our experience, the following are characteristics of a "rich" video clip that would work well in the PSC model. Similar to our discussion about the characteristics of a rich mathematics task, we do not intend this set of features to be an exhaustive list but rather a general framework to use as a guide.

The clip is relevant to the teachers. The video clip should clearly relate to the selected workshop topic and show a period of time during the lesson that would interest the participating teachers. Because the clips are taken from the teachers' own classrooms, and everyone in the group has experience teaching the same problem, this criterion is generally easy to fulfill. The facilitator should be conscious of selecting a clip that will be interesting, motivating, and relevant to the teachers in the group.

The clip is valuable, challenging, and accessible to the teachers. The video clip should show a period of time during a lesson from which the group can learn something valuable about pedagogy and/or mathematics content. The clip should be challenging and yet accessible to all the participating teachers. It should require the teachers to take some time to work carefully through the mathematics or think through the subtle pedagogical details. Over time, as the teachers become more comfortable with and adept at analyzing video, the facilitator can actively search for clips that are more challenging. Especially in the realm of critiquing instructional practice, it is important to make sure that the sense of community is strong, and the teachers have developed a deep level of trust, before showing video that captures more-problematic moments in the classroom.

The clip covers a relatively short time period. We suggest selecting video clips that are approximately two to five minutes in length. A clip that is too long loses the ability to anchor a productive conversation because teachers' focus may shift to activities captured on the video other than the ones that the facilitators intended. When a clip is relatively short, the teachers can focus on the small number of interactions that occur in that clip. This means, however, that the facilitator may need to provide the teachers with a brief description of the context of the clip so that they do not feel "lost" as they try to make sense of what they see. By describing who is involved in the clip and when during the lesson the clip takes place, the teachers will be in a better position to focus on the details of the interactions that the facilitators have identified as potentially valuable to examine.

The clip provides an anchor for considering new instructional strategies. The video clip should foster productive conversations by encouraging the teachers to reflect on their conceptions and misconceptions about teaching and learning. In other words, the clip should motivate and inspire teachers to consider new instructional strategies, or to reconsider their current teaching practices. Thinking about and discussing these types of issues can lay the foundation for instructional change.

Following are some examples of the types of video clips that we used in the STAAR program during Workshop 2. These clips fostered productive conversations about a variety of instructional strategies. In addition, they represented different degrees of challenge; that is, some represented more problematic teaching moments than did others.

- A set of clips in which several teachers have modified the task differently and introduced it differently, adapting it to the particular characteristics of their students.

- A clip in which the teacher begins to debrief the students' poster presentations at the end of the lesson but runs out of time and does not discuss as much of the mathematics as the teacher had hoped.

- A clip that shows an interaction between the teacher and a student in which the teacher attempts to understand a student's novel approach for solving the task. However, the teacher does not elicit a complete explanation from the student or misinterprets the student's thinking.

What if only some of the teachers' lessons were videotaped?

If only some of the participating teachers were videotaped while implementing their PSC lessons, the process of video clip selection may—on the surface—be easier for the facilitator because he or she will have less video to review. On the other hand, if only a few lessons were videotaped, there will be a smaller range of instructional practices to choose from, and the facilitator will have fewer options and less flexibility in designing the workshop.

What if some teachers are not comfortable having clips from their lessons shown?

It is possible that some teachers who were videotaped will not be comfortable having clips from their lessons shown to the group. The facilitator needs to be aware of and sensitive to these concerns. While the goal of the PSC is to have all teachers be comfortable with having their lessons videotaped and shared with their colleagues, the facilitator may need to work toward building a community in which this goal can be met. In particular, the facilitator can help to ensure that the discussions do not stray from an appropriate comfort level, especially for the videotaped teacher. In the STAAR program, *prior to the workshop*, we always checked with the teachers whose clips we wanted to use to make sure they would be willing to have their clips shown. If they were not comfortable, we would not show their clips.

Ideally, most (if not all) participating teachers' classrooms will be represented at some point during the professional development program. For those teachers who are uncomfortable sharing video, bringing in artifacts from their classroom (especially in Workshop 3) for the group to review may be another alternative. The goal is for all teachers to have a selected aspect of their PSC lesson experience discussed and analyzed by the group, in detail, at some point during the PD program.

Preparing Questions to Guide Discussions of the Video Clips

The process of selecting a workshop topic, choosing video clips, and crafting guiding questions may be quite nonlinear. The selection of a topic may depend on the video available from the PSC lessons. Alternatively, interest in a particular topic may drive the choice of the video clip(s). Guiding questions will help to make the intended connection between the video clip and the topic clear to the teachers. Thus, it is important to have questions that the teachers can focus on as they watch the clip. We have also found that preparing discussion questions can be useful to facilitators, helping them to refine the selected topic and decide where the starting and ending points of the clip should be.

The "Conducting" section of this chapter includes examples of the types of questions that we have found to be effective for guiding discussions related to the teacher's role. In addition, the vignette provided at the beginning of this chapter presents the two questions we used to guide the discussion of the video clip from Peter's lesson.

Planning to Support Teachers' Reflections on Their Instructional Role

In the third phase of Workshop 2, teachers make plans for integrating emerging ideas about the teacher's role into their own classroom practice. The fact that this phase relies on the conversations and ideas that surface throughout the workshop makes planning a bit tricky. However, we can offer some general guidelines for facilitators to support teachers in moving their ideas from an abstract level to a more concrete and practical level.

Most importantly, the facilitator should come to the workshop with general questions prepared for this phase, based on the selected topic. These questions can be generated without having to anticipate the specifics of the conversations that will take place during the workshop. The questions should ask teachers to explore a change (or changes) they would like to make relative to the selected topic, and articulate both a rationale for the change and a concrete plan that may include a way of assessing the change. (If teachers are interested in making changes that do not relate directly to the selected topic of the workshop, such awareness can be a worthwhile by-product of their workshop experience. The idea is to personalize the professional development and allow each teacher to take ownership of his or her professional growth.)

In the "Conducting" section we include examples of these questions. These four examples are meant to be posed as a set, guiding teachers toward planning for and eventually making changes in their teaching. Facilitators may elect to add to these questions, use some but not all of them, or generate different questions.

Chapter 7

An In-depth Look at Workshop 3

Objectives

Workshop 3 Objective:

To deepen teachers' understanding of students' thinking about the selected PSC task, including how to foster and support students' mathematical reasoning.

We now provide an in-depth look at the third workshop of the Problem-Solving Cycle. This is the last workshop of a single PSC iteration. During Workshop 3, the group continues to discuss and reflect on their experiences implementing the selected PSC task, focusing on student thinking about the task. The majority of time is spent watching and discussing video clips or other lesson artifacts that represent students' mathematical thinking.

As the teachers explore their students' mathematical reasoning, they gain insight into both the complexities of the mathematical concepts entailed in the problem and the students' learning of those concepts. Carefully studying students' mathematical reasoning typically leads teachers to rework some or all of the PSC problem, to consider mathematical components of the problem that they had not previously addressed, and to discuss the affordances and constraints of various solution strategies.

"Looking at student thinking was really valuable to me because I hadn't looked at problems in that fashion before. You can see how kids think and solve a problem. It's so valuable to really dig down deep and look at how they're interpreting the problem. You can see how they are solving complex problems and how you can take that thinking and build the students' knowledge. And so I loved that about the [Problem-Solving Cycle]."
—Laura, final interview

This chapter includes a vignette drawn from one of our experiences in carrying out Workshop 3, a detailed guide to conducting the workshop, and information about planning for this workshop.

Vignette from Workshop 3

In Chapters 4 and 6 (describing Workshops 1 and 2, respectively), we presented vignettes from a Problem-Solving Cycle in which a group of teachers worked on and taught the Skyscraper Windows problem. Now we consider these teachers' experiences during Workshop 3, when they continued discussing this problem and their implementation of it, this time with a focus on the students' mathematical thinking.

The vignette depicts the teachers' consideration of a clip from Laura's lesson, in which a small group of students talk about how to solve the problem. In the clip, one student explains his reasoning to three of his peers, using a solution method unanticipated by any of the teachers (or by the facilitators). As they embark on an extended conversation about the nature of these students' thinking, the teachers gain an increasingly sophisticated understanding of the constructs embedded in the problem. They also consider how they might teach the problem differently, to better support the variety of approaches students might use to solve it. The group's conversation about this clip is reflective of the depth with which teachers can continue to explore important mathematical and pedagogical concepts in Workshop 3, even after they have solved the problem in Workshop 1, taught it to their students, and debriefed their experiences in Workshop 2.

As you read the vignette, pay particular attention to (a) the questions that the facilitators posed to guide the exploration of the videotaped students' mathematical reasoning; (b) when the teachers generated new mathematical ideas; and (c) the teachers' comments about their initial reactions to the students' solution strategy.

Vignette

Almost two months after Workshop 2, the teachers come together again in Workshop 3 to continue watching and discussing one another's PSC lessons. Near the beginning of the workshop one of the facilitators, Craig, says with a smile, "The Skyscraper Windows problem. Remember this one?" The teachers laugh, and he hands everyone a printed copy of the problem. Craig tells the group that they will watch a clip from Laura's lesson. He notes that Laura modified the problem slightly for her students, using an 8-story building rather than a 12-story building. Laura explains, "I just thought 12 stories would scare them too much. Those double digits they seem to get scared over."

Craig distributes a handout with three questions for the teachers to consider as they watch the video: (1) What are the students doing mathematically? (2) What mathematical background do the students appear to draw on? (3) What new ideas does this clip give you about teaching this problem? Craig reads the questions aloud and then plays the clip. After it is over, the teachers move into small groups. Each group has access to a laptop computer with the clip loaded on it. Laura distributes copies of the written work she collected from the videotaped students in her class.

One small group includes Celia, Nancy, and Peter. Celia and Nancy begin the conversation by noting that they do not understand how the videotaped students were thinking. Peter comments that the students appear to be talking about the order of operations and noticing some patterns. A facilitator, Kim, joins the conversation and poses several questions in an attempt to help extend the teachers' discussion. She says, "So the kids took 30 times 38. Where did they get those numbers? Did their strategy work? Does it give them the right answer?" Kim's questions help to steer the group in the intended direction by encouraging a focused look at some of the specific mathematical components of the students' solution method. The three teachers then engage in some of the same calculations they imagine the students must have done, and they discuss whether and why these computations are accurate.

Figure 2:
Videotaped students' idea for determining the cost of washing an 8-story building

5.50	**floor 8**
5.00	
4.50	**($30.00)(38 windows)**
4.00	**= $1,140**
3.50	
3.00	
2.50	
+2.00	**floor 1**
$30.00	

After a short time Peter exclaims, "It does work!" referring to the numeric computation of 30 times 38 (see Figure 2). He then wonders, "Why does it work?" Celia, Nancy, and Peter continue discussing and working out the mathematics, and eventually are able to replicate and explain the videotaped students' creative use of the distributive property. Peter then suggests, "Let's try it out for a 6-story building and see if it was just a coincidence."

As the teachers work on calculating the price for a 6-story building, they begin to probe deeply into mathematical nuances of the problem. The facilitators encourage all of the small groups to extend the videotaped students' ideas by using their method to find the cost of washing a building with "n" number of floors. After giving the teachers some more time to talk in their small groups, Craig asks for a representative from each group to write their ideas on the white board at the front of the room in preparation for a full-group conversation. He then initiates the conversation by saying, "So this was what the Kristen, Laura, and Pam group wrote here. Do you want to walk us through it a little bit?"

Laura begins explaining, with input from her colleagues, that the students used the distributive property and "factored out the 38." Laura demonstrates how the students calculated the cost of one window per floor and then multiplied that sum by 38. Pam notes that although she and her colleagues were impressed by this unexpected solution method and confirmed that it does lead to the correct answer, it is somewhat inefficient mathematically. She explains, "We were first thinking, 'Wow. This is cool.' But then we realized that the kids were still stuck at that numeric level because they didn't have a formula that sums the cost of one window per floor for the entire building. So if we would have asked them, 'What about a 50 story building?' They would have to add $2, $2.50, $3, $3.50 … all the way to the 50th floor."

The other teachers agree that the videotaped students approached the Skyscraper Windows task in this numeric manner. The teachers then work to come up with a generalized expression. Their efforts lead them to two strategies for adding a large string of numbers. The teachers explore and compare these strategies, and eventually use both to arrive at a quadratic expression that works for buildings of all sizes.

At this point in the workshop, the teachers have spent several hours reconsidering their conceptual understanding of the Skyscraper Windows problem in light of the student reasoning they saw taking place in Laura's lesson. The facilitators, along with the teachers, remark on the depth of their continued exploration into the algebraic concepts in this one problem, even after two previous workshops focused on the same problem. Kim and Craig ask the teachers to reflect on their detailed investigation into the mathematical content of the Skyscraper Windows problem, particularly through the lens of student thinking, and to discuss how they would now teach a lesson involving this problem.

Kristen leads off the discussion by saying, "I am thinking instructionally now. The week before I do this lesson again, I could teach the students how to add a string of numbers using a Gaussian approach or the pairing method. Then, they would have a strategy to use for problems like this one. I think it is important to give them the tools or strategies they need to understand problems."

Ken offers, "I will be able to help kids more because I now understand these two ways to solve the problem. I understand how one connects to the other. So I would probably ask better questions."

Laura adds, "Today was the first day that really solidified my understanding of this problem. We've looked at it for three days [i.e., three workshops] and I'm finally like, 'Oh gosh. Now I get it.' I think having that knowledge will help me to understand where the kids are going next time I teach it, and to understand their thinking a little bit better."

Vignette Discussion

To prepare for this workshop, the facilitators considered which video excerpts could serve as a springboard to reexamine the Skyscraper Windows task. They selected a clip from Laura's lesson and prepared discussion questions that focused the group's attention primarily on the mathematical ideas evident in the video. Although the students' solution method confounded the teachers at first, after careful study the group devised and shared multiple strategies that built on the students' numeric calculations and represented increasingly sophisticated and generalized mathematical thinking.

Throughout the workshop, the facilitators asked questions to encourage the teachers to continue probing into the nuances surrounding the students' mathematical ideas. Ultimately, the teachers not only were able to identify the creative application of the distributive property in the students' numerical calculations but they also extended the students' reasoning to develop a direct formula for the cost of an entire building. This experience helped the teachers to gain a newfound respect for studying and building on their students' mathematical reasoning.

In addition, the facilitators asked the teachers to reconsider how they would teach the Skyscraper Windows problem given their stronger understanding of how students might approach the task. The teachers generated new ways of preparing their students for the problem and considered the importance, in general, of capitalizing on children's emerging ideas.

Conducting Workshop 3

Overview

Workshop 3 takes place in four phases in which teachers (1) reconnect to one another and to the PSC task, (2) explore and analyze students' mathematical thinking, (3) reflect on the connection between student thinking and their own teaching, and (4) reflect on what they have learned throughout this PSC. Our goal is for teachers to deeply reconsider the mathematical task, and to complete an iteration of the Problem-Solving Cycle by going "back to where they started." In other words, teachers begin the PSC by studying the mathematical concepts embedded within the selected task, and now they conclude the cycle by identifying and working with "new" mathematical concepts in the same task.

Phase 1: Reconnecting

Phase 1 should take ***about 10%*** *of the total workshop time.*

At the start of Workshop 3, it is important to give the teachers time to reconnect with one another and with the PSC task. We present two options for conducting this phase. Facilitators might, in fact, want to use both options. This decision depends on time, the perceived needs and interests of the teachers, and the facilitator's insight into what activities will work best to foster and maintain a strong community amongst their teachers.

- ❑ ***Option 1.*** **Teachers reconnect with one another by discussing any innovation they tried recently in their classrooms**. For instance, if the teachers set personal goals for themselves during Workshop 2, they may want to discuss with their fellow teachers the difficulties and triumphs they have experienced over the past month or two in relation to these goals. The community of the group can be strongly enhanced as the teachers provide suggestions and encouragement to one another.

  When using this option for Phase 1, we typically began by asking teachers to reflect, in writing, on their goals and teaching experiences since Workshop 2. Having the teachers take a few minutes to put their thoughts in writing allowed them to revisit their ideas about mathematics teaching and learning. Next, we asked the teachers to talk in small groups, and finally to share aloud any salient points of the conversation or ask any lingering questions to the full group. This strategy gave everyone an opportunity to share and to receive focused feedback from colleagues.

❑ *Option 2.* **Teachers reconnect with the PSC mathematics task by beginning with an informal examination of student work**. The teachers engage in an activity that reminds them of the PSC task and encourages them to see that they can still learn from it. Specifically, this activity involves having the teachers select and share a piece of student work from their PSC lesson.

Whereas the facilitators select the video clips for the group to view during the majority of Workshop 3, here the idea is for the teachers to choose examples from their own students' work that they want to discuss with their colleagues. We have found that this activity provides teachers with a sense of ownership and autonomy within the PSC and also helps to build community. Initially, teachers might need some guidance on how to choose student work that will promote productive discussions. However, after several iterations of the PSC, the teachers are likely to have developed a strong sense of the different ways student work can foster conversations.

Prior to the workshop, ask the teachers to bring one or two examples of student work from their PSC lesson that "show" a student's mathematical thinking. The student work should be something that the teacher finds particularly interesting, typical, or difficult to understand. For example, the student work might

- Represent an original solution strategy
- Represent a solution strategy that contains a common error or misconception
- Represent a solution strategy that is mathematically incorrect but yields the correct answer

The facilitator can ask for volunteers to share their students' work with the whole group. Alternatively, the teachers could meet in small groups and share their student work with just a few colleagues. When we had teachers talk in small groups about their students' work, we found that groups composed of teachers working with different grade levels were particularly informative. These multi-grade discussions helped the teachers see a continuum of student understanding that may not exist within a single classroom. At the same time, sharing student work in multigrade-level groups enabled the teachers to recognize similarities among students in different grades. Facilitators could encourage the small groups to consider the type of learning that needs to take place at each grade level in order to move students forward in their thinking.

Phase 2: Analyzing student thinking

*Phase 2 should take **about 70%** of the total workshop time.*

In the second phase of Workshop 3, teachers analyze examples of student thinking from their PSC lessons. This analysis can take place by watching video clips or by studying written student work, or both. We typically engaged in both of these activities in our workshops, with somewhat higher priority given to the analysis of video. Both activities are consistent with the goal of this phase, which is to promote a reexamination of the PSC task through the lens of student thinking so that the teachers can deepen their understanding of students' thinking as well as their own mathematical thinking about the task.

Analyzing video. In this phase of the workshop, the teachers watch and analyze video clips from their PSC lessons selected by the facilitator. Using video clips to anchor the conversation, the facilitator helps the teachers explore students' mathematical reasoning in relation to critical mathematical concepts embedded in the PSC task.

❑ Begin the phase by **providing guiding questions** for the teachers to consider as they watch the video clip (or clips). The facilitator may also want to provide, in writing or orally, a short description of the video clip. Prior to viewing the selected clip (or clips), give the teachers a few minutes to read the questions, or read them aloud.

   A wide range of questions may be effective in fostering a productive discussion about student thinking. Some possibilities include the following:

   - What mathematical concepts are the students grappling with?
   - What mathematical concepts are displayed in the students' thinking?
   - What mathematical ideas did the students draw upon? Where might their mathematical ideas have originated?
   - What would it mean to extend or generalize the mathematical reasoning displayed in the students' solution strategies?
   - How does the mathematical reasoning these students used to solve the problem compare to the reasoning you used when you solved it?

   We have found that many teachers begin their analysis of student thinking by deciding whether the student is right or wrong. In this professional development program, we would like to help teachers move toward a more complex analysis in which they unpack the student's reasoning rather than focusing on the correctness of the response. In many cases, students use both correct and incorrect reasoning strategies, with varying levels of sophistication. Questions such as those listed above are intended to encourage teachers to identify the different mathematical ideas generated by the student, to think deeply about what the student does and does not understand, and to consider how they might build on the student's current thinking.

❑ **Cue up the clip and watch it one or two times as a whole group.** (For a further discussion of cueing and watching video clips, see Chapter 6, Phase 2.)

❑ Have the teachers move into small groups and talk through the guiding questions. (Again, see Chapter 6, Phase 2, for more discussion on this point.)

For this phase of the workshop, we suggest grouping the teachers by grade level, if possible. We have found that the depth to which the mathematics of a problem are explored often varies across teachers of different grade levels. This variance can lead to interesting discussions when the small groups share their thoughts and ideas with the full group.

It is important for the facilitators to listen in on the conversations of each group, and to be on the lookout for particular mathematical ideas that the teachers seem to be struggling with. The facilitator can then make an informed decision about when to bring the teachers together as a whole group and what mathematical topics to explore further.

❑ When appropriate, the facilitator should **initiate a full group discussion**. We typically started the conversation by asking each small group to share their various hypotheses or assumptions about the students' reasoning. This helps to bring out what the teachers were (and were not) able to deduce about the students' understanding of the problem. Generally the discussion becomes a deep exploration of the mathematics in the PSC task.

❑ Depending on the time allotted for the workshop, and how long it takes the group to process a single video clip, **there may be time to watch additional clips**. As the teachers become more accustomed to deeply analyzing student thinking, however, they are likely to have longer conversations around a single clip. Generally speaking, when Workshop 3 is a full-day workshop, there should be time to show between two and four video clips related to student thinking. For each clip, we suggest following the process outlined above: reading the discussion questions (which may differ for each clip), showing the clip to the whole group, having teachers talk in small groups, and concluding with a whole-group discussion of that clip.

Analyzing student work. Analyzing written work is a very different process from analyzing video, but it can be equally engaging and beneficial to teachers. Students' written work provides a "snapshot" of student thinking. As such, the students' mathematical reasoning may have to be surmised by the teacher. In this respect, students' written work created during the PSC lesson is similar to students' written work provided on a formal assessment, such as an end-of-unit test. Considering students' responses on formal assessments is a frequent experience for teachers. The PSC model provides teachers with an opportunity to look at written classwork with other teachers who taught the same problem. Through these focused discussions, teachers can engage in a deeper analysis of how students were thinking, rather that simply assessing the accuracy of their work.

There are several ways to go about examining student written work. For example, the facilitator can distribute copies of the work to each teacher and then hold a discussion. Or the teachers can meet first in small groups, and then talk as a whole group. Another method that we used with a good deal of success is called a "Gallery Walk." We conducted our Gallery Walks by putting several different pieces of written student work around the room (e.g., taped on a wall). Then the teachers (typically in pairs) would have a few minutes to examine and discuss each piece of student work. We found it helpful to provide the teachers with a set of Post-It notes, so that they could write down any comments and questions they had about the student work. After a couple minutes discussing one piece of written work and posting their notes, everyone rotated to the next piece of written work and repeated the process. By moving among the pieces of written work, the teachers were able to discuss and compare various examples and see the other teachers' comments and questions. After all the teachers had a chance to examine all of the written work, the facilitator would lead a whole-group discussion, working from the teachers' comments and questions on the Post-It notes.

Phase 3: Reflecting on student thinking

*Phase 3 should take **about 10%** of the total workshop time.*

During the third phase, the teachers link their analyses of student thinking back to their own classroom practices. This linking to the classroom serves as a reminder to teachers that considering student thinking is a critically important component of effective mathematics instruction. The process of reflecting on student thinking can take several forms depending on the available workshop time and the needs and interests of the group.

❑ **Provide questions that encourage teachers to reconsider pedagogical issues based on their analyses of student thinking**. We suggest two types of questions: (1) questions that relate directly back to the PSC lesson, and (2) questions that relate more generally to mathematics instruction. Facilitators can select the types of questions they think are most appropriate for their teachers. Facilitators might want to ask both types of questions.

Reflections on their PSC lessons. These questions ask teachers to consider whether and how they would change their approach to teaching the PSC lesson based on new insights they may have gained through the analysis of student thinking about the PSC task.

If teachers address questions of this sort, the facilitator may suggest that they revisit their lesson plans from Workshop 1. Specifically, teachers could add notes or make revisions to the lesson plans for future reference when teaching this problem.

Some examples of reflection questions that relate back to the PSC lesson include the following:

- What insights about the PSC math task and your lesson plan did you gain through examining student thinking? What would you change about your teaching of this lesson?
- Revisit your mathematical goals for your students.
- How would you frame this lesson within the landscape of your curriculum? Where does it fit in your curriculum? How might you “weave” it into other topics?
- What representations would you like to see students use for solving the PSC problem?
- How might you support or scaffold student thinking for the PSC problem?
- How might you facilitate the debriefing of the PSC problem?
- How might you assess student learning?

Reflections on their mathematics instruction more generally. These questions ask teachers to move beyond their PSC lessons and consider how they might make general changes in their mathematics instruction on the basis of new insights related to student thinking.

Some examples of reflection questions in this category include

- How has looking at student thinking informed your ideas about teaching mathematics?

- What will you continue to emphasize in your mathematics classroom? What might you change or add to your teaching?
- What are your pedagogical goals for the future? How will you work to meet these goals?

❑ **Have the teachers free-write and/or talk in small groups** about their ideas. If there is time, they could share their thoughts with the whole group. Otherwise, the facilitator might decide to move directly to the additional "reflection" questions in the next phase.

Phase 4. Reflecting on the Problem-Solving Cycle

Phase 4 should take ***about 10%*** *of the total workshop time.*

This phase concludes the workshop. Additionally, it is the final phase of one iteration of the Problem-Solving Cycle. During this time, teachers have an opportunity to think carefully about their own professional growth and to give input on future iterations of the PSC.

❑ Provide questions that encourage teachers to reflect on the PSC. During this phase, teachers reflect on and discuss the impact of this PSC iteration. They might consider issues such as ways their practice has changed, their plans for future changes, and their ideas about future professional development efforts.

Following are examples of questions of this sort.

- Now that we have completed a full iteration of a PSC, what has been most valuable to you?
- Thinking back on the past three workshops, what have you learned about the mathematics in the PSC task? What you learned about how to effectively teach the mathematics in that task?
- How has the PSC helped you to examine your teaching? What new insights into your teaching have you gained?
- Are these PD workshops meeting your needs and expectations? Is there anything you would like to change or do differently?
- What would you like to focus on in the next PSC?

❑ Ideally, we suggest that the facilitator have the teachers free-write responses to the selected questions, then talk in small groups, and finally participate in a whole-group conversation. The facilitator is in the best position to decide what format is most appropriate and exactly what summation activity should be posed to the teachers. Facilitators should feel free to be creative and flexible in their approach to the conclusion of each PSC iteration.

Planning Workshop 3

Overview

In this section, we offer guidance on planning for the following tasks: (1) helping teachers reconnect, (2) selecting video clips, (3) selecting students' written work, (4) crafting discussion questions, and (5) supporting teachers' reflections. Many of these planning tasks are very similar to those discussed in Planning Workshop 2 (Chapter 6). Rather than repeat information already presented in that chapter, here we provide new information relevant specifically to Workshop 3. We also encourage facilitators to review Chapter 6 when they are planning for Workshop 3.

Planning to Help Teachers Reconnect

Workshop 3 begins with a short phase in which teachers reconnect with one another, with the PSC task, or both. We strongly suggest doing at least one of the reconnecting activities described in the "Conducting Workshop 3" section, and both if possible. For a facilitator with limited time available, select the activity that you feel best meets the needs and interests of your teachers.

If you elect to have the teachers reconnect with the PSC task by sharing their students' work, make sure that at least some of the teachers have student work available from their PSC lesson that they can bring to the workshop. This means that the facilitator will need to have a brief discussion with the teachers, in advance of the workshop, about selecting examples of their students' work to show. In the Conducting section, we provide some general guidelines regarding the type of student work that would be relevant for this phase of the workshop. The facilitator might like to share this information with the teachers—again, in advance of the workshop. In addition, it would be helpful if the teachers could make copies of the student work they would like to share, so that they can be easily distributed during the workshop

Selecting Video Clips

Central to planning Workshop 3, as in Workshop 2, is the selection of video clips. The selection process should be driven by the facilitator's consideration of the types of mathematical reasoning that took place in the teachers' PSC lessons. This means that the facilitator must be sufficiently familiar with the lessons to pick a clip (or clips) likely to promote thoughtful conversation about students' mathematical thinking.

Clip selection for Workshop 3 should follow guidelines similar to those discussed with respect to Workshop 2. As described in Chapter 6, a video clip can be characterized as rich when it (a) is **relevant to the**

teachers; (b) is **valuable, challenging, and accessible to the teachers**; and (c) **covers a relatively short time period**. For this workshop, the fourth feature (*provides an anchor for considering new instructional strategies*) would be modified to read: **provides an anchor for considering student thinking**.

The goal of Workshop 3 is for the teachers to analyze and discuss examples of students grappling with mathematics concepts in diverse ways reflecting different reasoning strategies and different levels of mathematical ability. Rich video clips that provide an anchor for considering student thinking often can be found during moments in the lesson when students explain their solution strategies to fellow students or to the teacher. These moments might occur while the students are working to solve the task or at the end of a lesson, if students are asked to share their thinking with the rest of the class.

We have had success using the following types of clips to promote discussions about student thinking:

- **Clip of a student grappling in a unique or atypical manner with a mathematical concept**. This type of clip can lead to a discussion about the relative merit and accuracy of a student's unique understanding of a specific mathematical concept and related mathematical topics.

- **Clip that shows a student's misconception**. This type of clip can help teachers to learn about children's naïve mathematical conceptions. It can lead to a discussion about the prior learning experience of the student and about the underlying mathematical reasoning that might be causing the misconception. In addition, this type of clip presents teachers with the opportunity to think of ways they might build on the student's current conception of mathematical ideas as he or she progresses from a naïve to a more formal understanding.

- **Clip related to any mathematical obstacles or difficulties teachers in your group encountered during Workshop 1.** This type of clip may encourage the teachers to revisit challenging mathematical topics with a student-thinking focus, and can help the teachers extend their content knowledge.

- **Series of video clips that show a range of student reasoning**. The facilitator may choose several clips that, together, show a possible mathematical progression of thinking about specific mathematical concepts. These clips can lead teachers to develop a framework of student thinking about these concepts. Such a framework is particularly useful for mathematical topics that are challenging to teach and difficult for students to understand.

Other issues related to selecting video that we discussed with respect to Workshop 2—such as what to do if only some teachers' lessons were videotaped and if some teachers are not comfortable having their clips shown—also are relevant for Workshop 3. Therefore, facilitators may find it helpful to review the *Selecting Video Clips* portion of Chapter 6.

Selecting Students' Written Work

During the second phase of Workshop 3, in addition to showing video clips related to student thinking, the facilitator may choose to have the teachers analyze students' written work from one or more of their PSC lessons. Using students' work in this phase will look somewhat different than in Phase 1, if the facilitator elected to incorporate student work during that time. For the second phase, we recommend that the facilitator choose—in advance—one or more examples of student work for the teachers to consider, develop guiding questions to support discussions of this work, and have some ideas in mind about where the discussion is likely to go.

What should I look for when selecting students' work?

Students' written work can take various forms, depending on what the teachers had their students do during their PSC lessons. For example, some teachers may have asked their students to work on the problem individually; therefore, these teachers would have students' individual papers to examine. Other teachers may have asked their students to work in small groups and then create group posters to present at the end the lesson. Therefore, their set of posters would be available. Still other teachers might have collected written reflections from their students. All of these forms of written work can be valuable to support an in-depth study of student thinking. It is important for the facilitator to collect (or make copies of) students' written work from the teachers *in advance* of the workshop, in order to select a few examples and prepare for the discussion around these examples.

In an analogous way to selecting a few short video clips from the teachers' PSC lessons, the facilitator should carefully select a few pieces of students' written work to use during Phase 2 of the workshop. In terms of the types of examples that can promote a good discussion among teachers, we suggest trying to find "rich" pieces of work. One piece of student work might stand on its own, if it shows a particularly interesting strategy (correct or incorrect). Alternatively, a collection of student work showing a progression of thinking or a variety of strategies might promote a thoughtful discussion. We refer the reader back to our discussion about selecting video clips—and specifically about clips that could provide an anchor for considering student thinking. The examples provided in that section pertain equally well to written work.

How many pieces of student work should I bring to the workshop?

The number of pieces of student work to select for discussion during Phase 2 of the workshop is highly dependent on the amount and nature of student work available to the facilitator, the number and type of video clips the facilitator has elected to show, and the amount of time available during the workshop. As a ballpark figure, our suggestion is to choose approximately three to seven pieces of "rich" student work to bring to the workshop.

Preparing Questions to Guide Discussions of the Video Clips and Student Work

Guiding questions are essential to focusing teachers' conversations during Phase 2 of the workshop, whether they discuss video clips or written student work. The specific nature of the selected video clip, student work, and PSC task will play a large role in determining appropriate guiding questions. In general, however, the discussion questions should encourage the teachers to explore the mathematics underlying the clip or written work. In order to generate these questions, the facilitator will need to have a good sense of the mathematical concepts that are highlighted in the clips and/or student work.

We suggest preparing three or four questions that seem most likely to focus the group on important mathematical ideas in the video clip or written work. We have included several examples of guiding question in the Conducting section. Facilitators can use any of these questions, modify them, or write their own.

It may be possible to use a single set of guiding questions to focus discussions of video clips that are connected or used in conjunction with one another. However, if several distinct and unrelated clips or examples of written student work are used, it may be more beneficial to develop a different set of guiding questions for each new examination of student thinking. With each distinct set of clips or pieces of written work, the facilitator can encourage the teachers to build on their previous discussions and look to new directions.

Planning to Support Teachers' Reflections on Student Thinking

After the teachers have spent a considerable amount of time actively discussing and analyzing specific examples of student thinking, it is important to provide them with an opportunity to connect their new understandings back to their classroom practice. This reflection component of the workshop can help to guide teachers' future decision-making processes, specifically in the area of fostering student thinking. We strongly urge the facilitator to provide teachers with some time to reflect on what they have learned about student thinking, and to generate ideas for instructional changes they might like to make.

The facilitator should prepare questions to help guide teachers in this reflection process. We have included a number of examples of such questions in the Conducting section of the chapter (see Phase 3). As we note in the Conducting section, these questions should encourage teachers to reflect on the connection between student thinking and pedagogy, either in their PSC lessons or in general. A facilitator may elect to pose both types of questions to their teachers. You will have to carefully consider the interests and needs of the teachers in your group and the amount of time available. One possibility would be to prepare questions of both sorts, and then decide which to use as the workshop unfolds.

If you would like teachers to revisit their original PSC lesson plans during this time, it would be helpful to remind them to bring those lesson plans to the workshop.

Planning to Support Teachers' Reflections on the PSC

Because Workshop 3 represents the end of one iteration of a Problem-Solving Cycle, we suggest setting aside a period of time for additional reflections. By asking the teachers to consider what they have learned from this iteration of the PSC and what they might like to learn from future iterations, the facilitator can gain a good understanding of the teachers' perceived needs and interests, and thus build a foundation for future professional development efforts—including future iterations of the PSC. In addition, this exercise can help to reinforce teachers' commitment to their own professional development and growth, and to build a community that is motivated to continue working and learning together. We have included some examples of these types of reflection questions in the Conducting section; however, a facilitator should consider what information he or she personally would find beneficial to know and adapt the questions accordingly.