A Brief Tour of Mathematica

First, consider the Cobb-Douglass production function

```
x = f(k,l) = k^a l^b

     where x is units of output, k is units of
     capital services and l is units of labor

Null
```

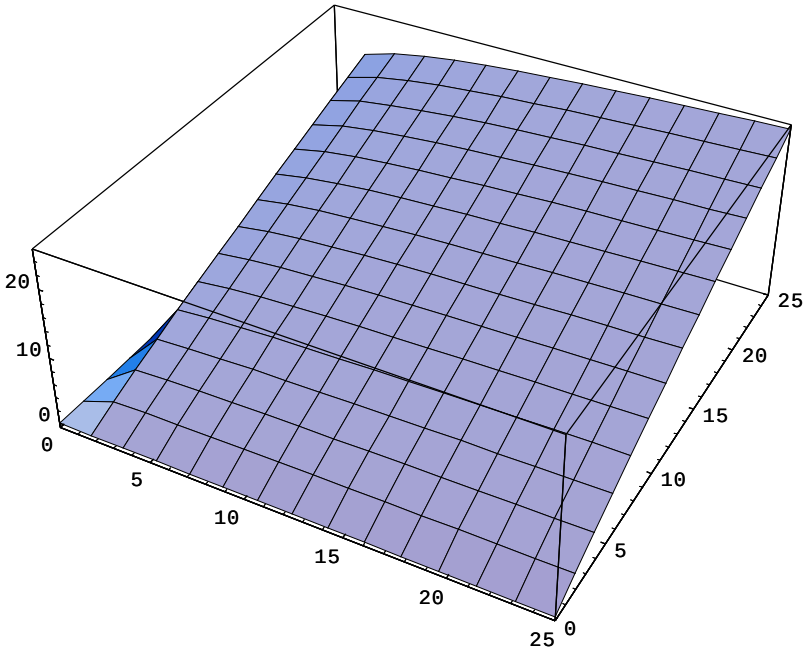
Now let's use Mathematica to graph this function for a = .1 and k = .9

```
x = k^.1 l^.9
```

$$k^{0.1}\, l^{0.9}$$

```
Note that the symbol "%" just means the previous expression
```

```
Plot3D[%, {k, 0, 25}, {l, 0, 25}]
```




```
-SurfaceGraphics-
```

Looks pretty simple doesn't it. To get a fancy looking graph, we would need a more complicated functional form for the production function

Now lets use Mathematica to find the marginal product of labor, MPl, for our general Cobb-Douglass production function

```
x = k^a l^b
```

$k^a l^b$

The Mathematica command for derivative is just "D" so to take the derivative wrt to l we just enter the command ∂_l **%**

```
mpl = ∂l x
```

$$b\, k^a\, l^{-1+b}$$

Now let's integrate mpl with respect to l and see what we get. The Mathematica command for integration is just "Integrate"

```
impl = ∫ mpl dl
```

$$k^a\, l^b$$

The answer is the production function.

Rather than continuing with our Cobb-Douglass example, let's end this tour of Mathematica with some more complicated math

First consider some algebra.

```
m = 9 (2 + s) (t + s) + (s + t)^2
```

$$9\,(2+s)\,(s+t)+(s+t)^2$$

Now let's raise it to the power of 3 and expand it

```
Expand[%^3]
```

$$5832 s^3 + 9720 s^4 + 5400 s^5 + 1000 s^6 + 17496 s^2 t + 30132 s^3 t + 17280 s^4 t + 3300 s^5 t + 17496 s t^2 + 32076 s^2 t^2 + 19494 s^3 t^2 + 3930 s^4 t^2 + 5832 t^3 + 12636 s t^3 + 8802 s^2 t^3 + 1991 s^3 t^3 + 972 t^4 + 1242 s t^4 + 393 s^2 t^4 + 54 t^5 + 33 s t^5 + t^6$$

Now let's factor it

```
Factor[%]
```

$$(s+t)^3\,(18+10\,s+t)^3$$

Now Calculus, partially differentiate wrt s

```
∂s%
```

$$30\,(s+t)^3\,(18+10\,s+t)^2+3\,(s+t)^2\,(18+10\,s+t)^3$$

maybe this partial derivative can be simplified

```
Simplify[%]
```

$$3\,(18+20\,s+11\,t)\,(18\,s+10\,s^2+18\,t+11\,s\,t+t^2)^2$$

Now consider integration of this function wrt t

```
∫% dt
```

$$24\,s^2\,(9+5\,s)^2\,(9+10\,s)\,t+6\,s\,(2916+7533\,s+5760\,s^2+1375\,s^3)\,t^2+2\,(2916+10692\,s+9747\,s^2+2620\,s^3)\,t^3+\frac{3}{4}\,(4212+5868\,s+1991\,s^2)\,t^4+\frac{6}{5}\,(207+131\,s)\,t^5+\frac{11\,t^6}{2}$$

Or integration of the more simple function

```
∫y^n dy
```

$$\frac{y^{1+n}}{1+n}$$

Mathematica will also solve equations or systems of equations. For example,

```
y^3 - 7 y^2 + 3 a y == 0
```

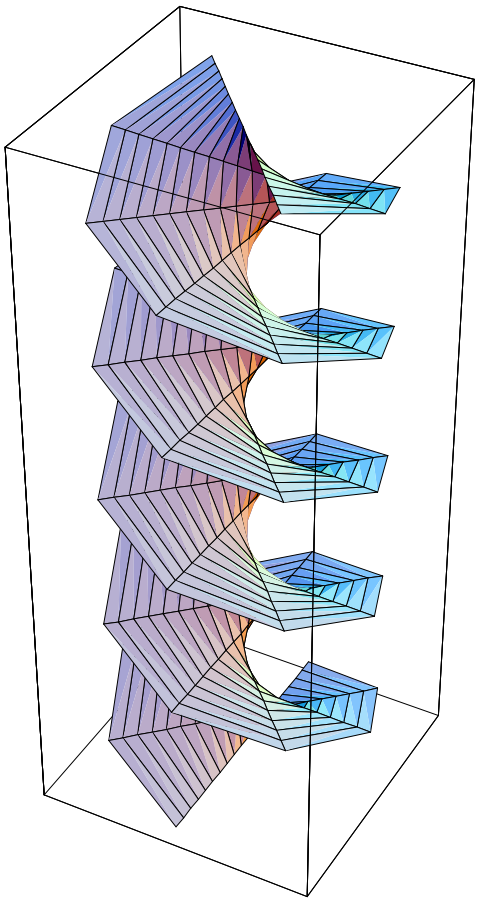
$$3\,a\,y-7\,y^2+y^3==0$$

```
Solve[%, y]
```

$$\left\{\{y \to 0\}, \left\{y \to \frac{1}{2}\left(7 + \sqrt{49 - 12\,a}\right)\right\}, \left\{y \to \frac{1}{2}\left(7 - \sqrt{49 - 12\,a}\right)\right\}\right\}$$

Lets end the tour with a 3-D Graphic using trig functions. Don't worry, in Econ 4828 we will use no trig. They do generate neat graphpics.

```
ParametricPlot3D[{u Sin[t], u Cos[t], t/3}, {t, 0, 15}, {u, -1, 1}, Ticks → None]
```



```
-Graphics3D-
```