chapter 8

Behind the Supply Curve: >> Inputs and Costs

Section 1: The Production Function

A **production function** is the relationship between the quantity of inputs a firm uses and the quantity of output it produces.

A *firm* is an organization that produces goods or services for sale. To do this, it must transform inputs into output. The quantity of output a firm produces depends on the quantity of inputs; this relationship is known as the firm's **production function**. As we'll see, a firm's production function underlies its *cost curves*. But as a first step, let's look at the characteristics of a hypothetical production function.

Inputs and Output

To understand the concept of a production function, let's consider a farm that we assume, for the sake of simplicity, produces only one output, wheat, and uses only two inputs, land and labor. This particular farm is owned by a couple named George and Martha. They hire workers to do the actual physical labor on the farm. Moreover, we will assume that all potential workers are of the same quality—they are all equally knowledgeable and capable of performing farmwork.

George and Martha's farm sits on 10 acres of land; no more acres are available to them, and they are currently unable to either increase or decrease the size of their farm by selling, buying, or leasing acreage. Land here is what economists call

A **fixed input** is an input whose quantity is fixed and cannot be varied.

A **variable input** is an input whose quantity the firm can vary.

The **long run** is the time period in which all inputs can be varied.

The **short run** is the time period in which at least one input is fixed.

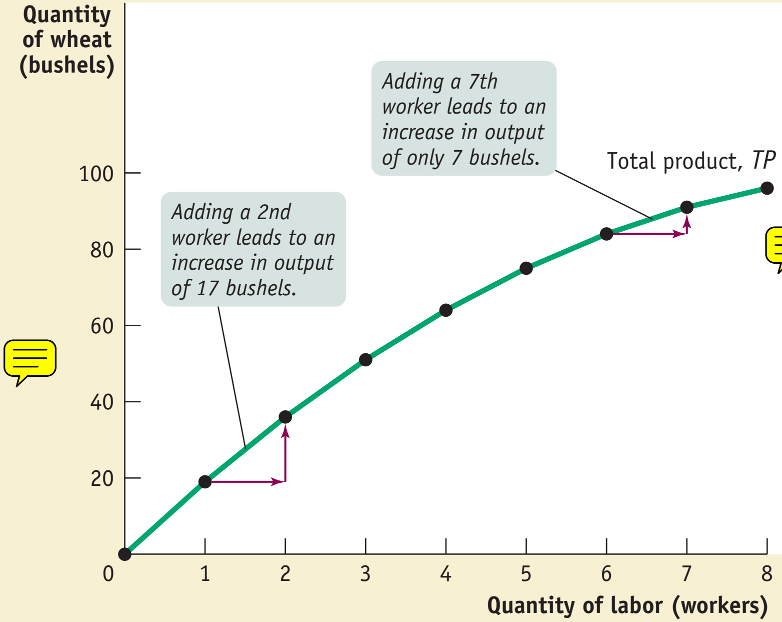
A **fixed input**—an input whose quantity is fixed and cannot be varied. On the other hand, George and Martha are free to decide how many workers to hire. The labor provided by these workers is called a **variable input**—an input whose quantity the firm can vary. (In Chapter 7, when we considered the example of Felix's Lawn-Mowing Service, Felix's fixed input was his lawn mower and his variable input was his own labor.)

In reality, whether or not the quantity of an input is really fixed depends on the time horizon. In the **long run**—that is, given that a long enough period of time has elapsed—firms can adjust the quantity of any input. So there are no fixed inputs in the long run, only in the **short run**. Later in this chapter we'll look more carefully at the distinction between the short run and the long run. But for now, we will restrict our attention to the short run and assume that at least one input is fixed.

George and Martha know that the quantity of wheat they produce depends on the number of workers they hire. Given modern farming techniques, one worker can cultivate the 10-acre farm, albeit not very intensively. When an additional worker is added, the land is divided equally among all the workers: each worker has 5 acres to cultivate when 2 workers are employed, each cultivates $3\frac{1}{3}$ acres when 3 are employed, and so on. So as additional workers are employed, the 10 acres of land are cultivated more intensively and more bushels of wheat are produced. The relationship between the quantity of labor and the quantity of output, for a given amount of the fixed input, constitutes the farm's production function. The production function for George and Martha's farm is given in the first two columns of the table in Figure 8-1; the diagram there shows the same information graphically. The curve in Figure 8-1 shows how the quantity of output depends on the quantity of the variable input, for a given amount of the fixed input; it is called the farm's **total product curve**. The physical quantity of output, bushels of wheat, is measured on the vertical axis, while the quantity of the variable input, labor, that is, the number of workers employed, is measured on the horizontal axis. The total product curve here is upward sloping, reflecting the fact that more bushels of wheat are produced as more workers are employed.

The **total product curve** shows how the quantity of output depends on the quantity of the variable input, for a given amount of the fixed input.

Figure 8-1 Production Function and Total Product Curve for George and Martha's Farm



Quantity of labor *L* (workers)	Quantity of wheat *Q* (bushels)	Marginal product of labor $MPL = \Delta Q/\Delta L$ (bushels per worker)
0	0	
		19
1	19	
		17
2	36	
		15
3	51	
		13
4	64	
		11
5	75	
		9
6	84	
		7
7	91	
		5
8	96	

The table shows the production function, the relationship between the quantity of the variable input (labor, measured in number of workers) and the quantity of output (bushels of wheat). It also calculates the marginal product of labor on George and Martha's farm. The total product curve shows the production function graphically. It slopes upward because more wheat is produced as more workers are employed. It also becomes flatter because the marginal product of labor declines as more and more workers are employed.

The **marginal product** of an input is the additional quantity of output that is produced by using one more unit of that input.

Although the total product curve in Figure 8-1 slopes upward along its entire length, the slope isn't constant: as you move up the curve to the right, it flattens out. To understand this changing slope, look at the third column of the table in Figure 8-1, which shows the *change in the quantity of output* that is generated by adding one more worker. That is, it shows the **marginal product** of labor: the additional quantity of output from using one more unit of labor (that is, one more worker).

In this case, we have data at intervals of 1 worker—that is, we have information on the quantity of output when there are 3 workers, 4 workers, and so on. Sometimes data aren't available in increments of 1 unit—for example, you might have information only on the quantity of output when there are 40 workers and when there are 50 workers. In this case, you can use the following equation to figure out the marginal product of labor:

$$\textbf{(8-1)}\quad \text{Marginal product of labor} = \frac{\text{Change in quantity of output}}{\text{Change in quantity of labor}} = \text{Change in quantity of output generated by one additional unit of labor}$$

or

$$MPL = \Delta Q / \Delta L$$

In this equation, Δ, the Greek capital delta, represents the change in a variable.

Now we can explain the significance of the slope of the total product curve: it is equal to the marginal product of labor. Remember from the Chapter 2 Appendix that the slope of a line is equal to "rise" over "run." This implies that the slope of the total product curve is the change in the quantity of output (the "rise") divided by the change in the quantity of labor (the "run"). And this, as we can see from Equation 8-1, is simply the marginal product of labor. So the fact that the marginal product of the first worker is 19 also means that the slope of the total product curve in going from 0 to 1 worker is 19. Similarly, the slope of the total product curve in going from 1 to 2 workers is the same as the marginal product of the second worker, 17, and so on.

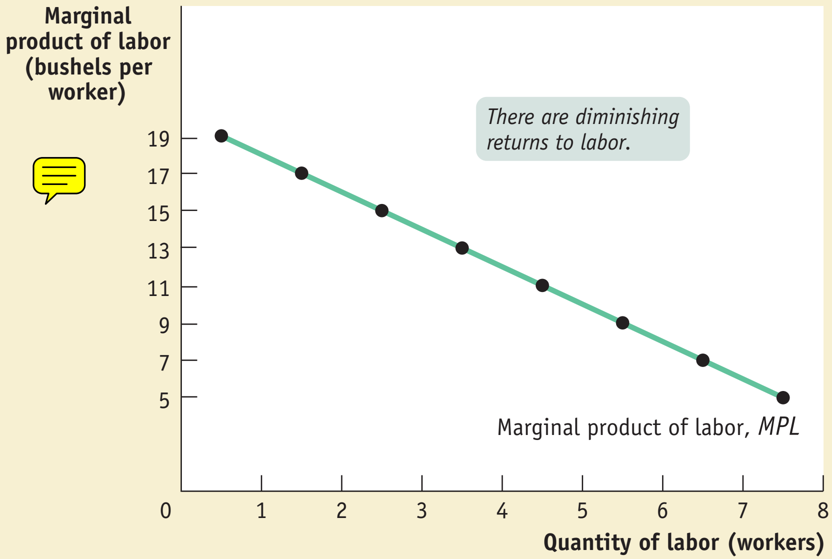
In this example, the marginal product of labor steadily declines as more workers are hired—that is, each successive worker adds less to output than the previous worker. So as employment increases, the total product curve gets flatter.

Figure 8-2 shows how the marginal product of labor depends on the number of workers employed on the farm. The marginal product of labor, *MPL*, is measured on

Figure 8-2

Marginal Product of Labor Curve for George and Martha's Farm

The marginal product of labor curve plots each worker's marginal product, the increase in the quantity of output generated by each additional worker. The change in the quantity of output is measured on the vertical axis and the number of workers employed on the horizontal axis. The first worker employed generates an increase in output of 19 bushels, the second worker generates an increase of 17 bushels, and so on. The curve slopes downward due to diminishing returns. >web...



There are **diminishing returns to an input** when an increase in the quantity of that input, holding the levels of all other inputs fixed, leads to a decline in the marginal product of that input.

PITFALLS

WHAT'S A UNIT?

The marginal product of labor (or any other input) is defined as the increase in the quantity of output when you increase the quantity of that input by one unit. But what do we mean by a "unit" of labor? Is it an additional hour of labor, an additional week, or a person-year?

The answer is that it doesn't matter, *as long as you are consistent*. One common source of error in economics is getting units confused—say, comparing the output added by an additional *hour* of labor with the cost of employing a worker for a *week*. Whatever units you use, always be careful that you use the same units throughout your analysis of any problem.

the vertical axis in units of physical output—bushels of wheat—produced per additional worker, and the number of workers employed is measured on the horizontal axis. You can see from the table in Figure 8-1 that if 5 workers are employed instead of 4, output rises from 64 to 75 bushels; so in this case the marginal product of labor is 11 bushels—the same number found in Figure 8-2. To indicate that 11 bushels is the marginal product when employment rises from 4 to 5, we place the point corresponding to that information halfway between 4 and 5 workers.

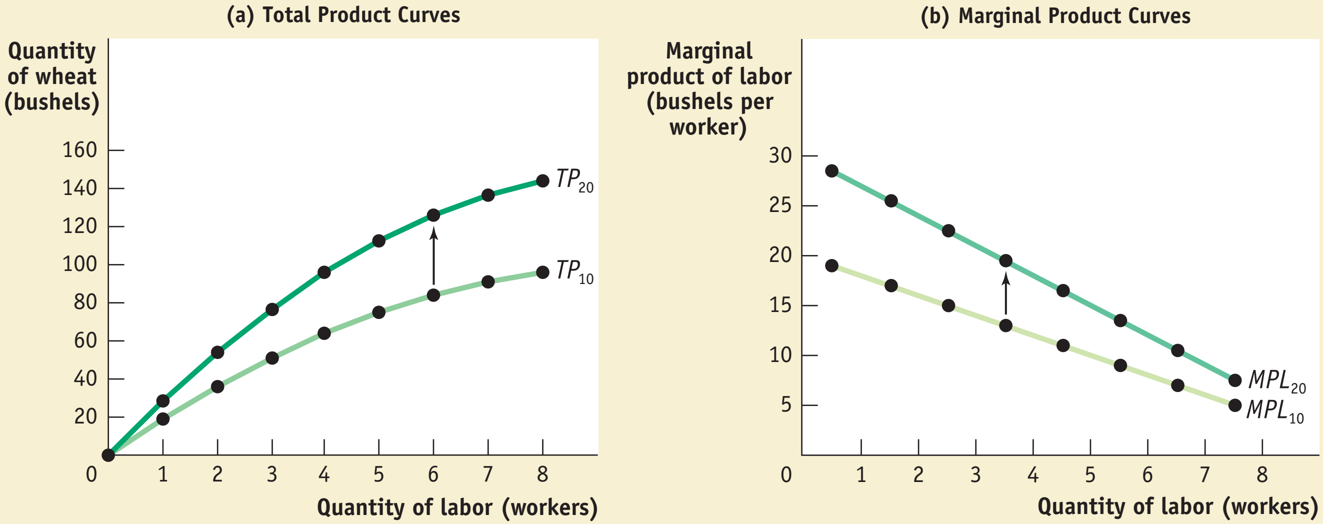
In this example the marginal product of labor falls as the number of workers increases. That is, there are *diminishing returns to labor* on George and Martha's farm. In general, there are **diminishing returns to an input** when an increase in the quantity of that input, holding the quantity of all other inputs fixed, reduces that input's marginal product.

To grasp why diminishing returns can occur, think about what happens as George and Martha add more and more workers, without increasing the number of acres. As the number of workers increases, the land is farmed more intensively and the number of bushels increases. But each additional worker is working with a smaller share of the 10 acres—the fixed input—than the previous worker. As a result, the additional worker cannot produce as much output as the previous worker. So it's not surprising that the marginal product of the additional worker falls.

The crucial thing to emphasize about diminishing returns is that, like many propositions in economics, it is an "other things equal" proposition: each successive unit of an input will raise production by less than the last *if the quantity of all other inputs is held fixed*.

What would happen if the levels of other inputs were allowed to change? You can see the answer in Figure 8-3. Panel (a) shows two total product curves, TP_{10} and TP_{20}. TP_{10} is the farm's total product curve when its total area is 10 acres (the same curve as in Figure 8-1). TP_{20} is the total product curve when the farm has increased to 20 acres. Except when no workers are employed, TP_{20} lies everywhere above TP_{10} because

Figure 8-3 Total Product, Marginal Product, and the Fixed Input



This figure shows how the quantity of output—illustrated by the total product curve—and marginal product depend on the level of the fixed input. Panel (a) shows two total product curves for George and Martha's farm, TP_{10} when their farm is 10 acres and TP_{20} when it is 20 acres. Panel (b) shows the corresponding marginal product of labor curves. With more land, each worker can produce more wheat. So an increase in the fixed input shifts the total product curve up from TP_{10} to TP_{20}. This also implies that the marginal product of each worker is higher when the farm is 20 acres than when it is 10 acres. As a result, an increase in acreage also shifts the marginal product of labor curve up from MPL_{10} to MPL_{20}. Note that both marginal product of labor curves still slope downward due to diminishing returns.

with more acres available, any given number of workers produces more output. Panel (b) shows the corresponding marginal product of labor curves. MPL_{10} is the marginal product of labor curve given 10 acres to cultivate (the same curve as in Figure 8-2) and MPL_{20} is the marginal product of labor curve given 20 acres. Both curves slope downward because, in each case, the amount of land is fixed, albeit at different levels. But MPL_{20} lies everywhere above MPL_{10}, reflecting the fact that the marginal product of the same worker is higher when he or she has more of the fixed input to work with.

Figure 8-3 demonstrates a general result: the position of the total product curve depends on the quantities of other inputs. If you change the quantity of the other inputs, both the total product curve and the marginal product curve of the remaining input will shift.

From the Production Function to Cost Curves

Once George and Martha know their production function, they know the relationship between inputs of labor and land and output of wheat. But if they want to maximize their profits, they need to translate this knowledge into information about the relationship between the quantity of output and cost. Let's see how they can do this.

To translate information about a firm's production function into information about its costs, we need to know how much the firm must pay for its inputs. We will assume that George and Martha face either an explicit or an implicit cost of $400 for the use of the land. As we learned in Chapter 7, it is irrelevant whether George and Martha must rent the land for $400 from someone else, or they own the land themselves and forgo earning $400 by renting it to someone else. Either way, they pay an opportunity cost of $400 by using the land to grow wheat. Moreover, since the land is a fixed input, the $400 George and Martha pay for it is a **fixed cost**, denoted by *FC*—a cost that does not depend on the quantity of output produced. In business, fixed cost is often referred to as "overhead cost."

A **fixed cost** is a cost that does not depend on the quantity of output produced. It is the cost of the fixed input.

A **variable cost** is a cost that depends on the quantity of output produced. It is the cost of the variable input.

The **total cost** of producing a given quantity of output is the sum of the fixed cost and the variable cost of producing that quantity of output.

We also assume that George and Martha must pay each worker $200. Using their production function, George and Martha know that the number of workers they must hire depends on the amount of wheat they intend to produce. So the cost of labor, which is equal to the number of workers multiplied by $200, is a **variable cost**, denoted by *VC*—a cost that depends on the quantity of output produced. Adding the fixed cost and the variable cost of a given quantity of output gives the **total cost**, or *TC*, of that quantity of output. We can express the relationship among fixed cost, variable cost, and total cost as an equation:

(8-2) Total cost = Fixed cost + Variable cost

or

$$TC = FC + VC$$

The table in Figure 8-4 shows how total cost is calculated for George and Martha's farm. The second column shows the number of workers employed. The third column shows the corresponding level of output, taken from the table in Figure 8-1. The fourth column shows the variable cost, equal to the number of workers multiplied by $200. The fifth column shows the fixed cost, which is $400 regardless of how many workers are employed. The sixth column shows the total cost of output, which is the variable cost plus the fixed cost.

The first column labels each row of the table with a letter, from *A* to *I*. These labels will be helpful in understanding our next step: drawing the **total cost curve**, a curve that shows how total cost depends on the quantity of output.

The **total cost curve** shows how total cost depends on the quantity of output.

George and Martha's total cost curve is shown in the diagram in Figure 8-4, where the horizontal axis measures the quantity of output in bushels of wheat and the vertical axis measures total cost in dollars. Each point on the curve corresponds to one row of the table in Figure 8-4. For example, point *A* shows the situation when 0 workers are employed: output is zero, and total cost is equal to fixed cost, $400. Similarly, point *B* shows the situation when 1 worker is employed: output is 19 bushels, and total cost is $600, equal to the sum of $400 in fixed cost and $200 in variable cost.

Like the total product curve, the total cost curve is upward sloping: due to the variable cost, the more output produced, the higher the farm's total cost. But unlike the total product curve, which gets flatter as employment rises, the total cost curve gets *steeper*. That is, the slope of the total cost curve is greater as the amount of output

Figure 8-4

Total Cost Curve for George and Martha's Farm

The table shows the variable cost, fixed cost, and total cost for various output quantities on George and Martha's 10-acre farm. The total cost curve shows how total cost (measured on the vertical axis) depends on the quantity of output (measured on the horizontal axis). The labeled points on the curve correspond to the rows of the table. The total cost curve slopes upward because the number of workers employed, and hence total cost, increases as the quantity of output increases. The curve gets steeper as output increases due to the diminishing returns to additional workers.

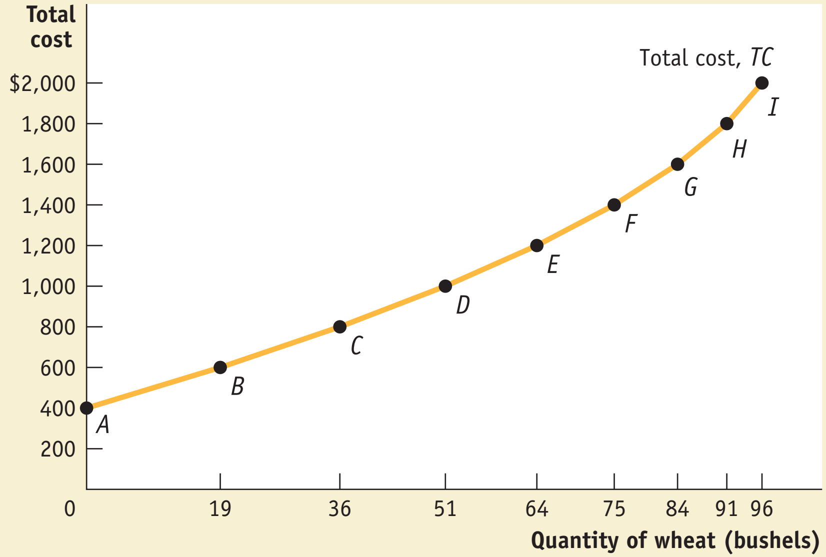


Figure 8-4 (continued)

Point on graph	Quantity of labor *L* (workers)	Quantity of wheat *Q* (bushels)	Variable cost *VC*	Fixed cost *FC*	Total cost *TC* = *FC* + *VC*
A	0	0	$0	$400	$400
B	1	19	200	400	600
C	2	36	400	400	800
D	3	51	600	400	1,000
E	4	64	800	400	1,200
F	5	75	1,000	400	1,400
G	6	84	1,200	400	1,600
H	7	91	1,400	400	1,800
I	8	96	1,600	400	2,000

produced increases. And as we will soon see, the steepening of the total cost curve is also due to diminishing returns to the variable input. Before we can understand this, we must first look at the relationships, among several useful measures of cost. ■