Chapter 3

PREFERENCES, DEMAND, AND WELFARE

3.1 Optimization for a single consumer

Students will recall the standard treatment of utility and consumer theory from intermediate microeconomics. A text typically starts with the problem of a single consumer maximizing a utility function *U(X)* subject to a linear budget constraint on total expenditure. Usually, this is set up as a Langrangean function, with a Lagrangean multiplier λ on the income constraint. Let I denote income, the standard treatment is given by

$$\max U(X_1, X_2) \ + \ \lambda(I - p_1X_1 - p_2X_2) \tag{3.1}$$

$$\frac{\partial U}{\partial X_1} - \lambda p_1 = 0 \qquad \frac{\partial U}{\partial X_2} - \lambda p_2 = 0 \tag{3.2}$$

$$I - p_1X_1 - p_2X_2 = 0 \tag{3.3}$$

There are three first-order conditions in (3.2) and (3.3), which can be solved for the optimal quantities of the two goods. The utility-maximizing demand for a good is a function of prices and income, and this particular way of specifying demand is generally referred to a Marshallian demand function. As an exercise, derive the Marshallian demand function for the Cobb-Douglas function in Chapter 2, ((2.26) iii), with the two goods replacing the two factors as the function's arguments. Show that the demands are given by

$$X_1 = \frac{\beta I}{p_1} \qquad X_2 = \frac{(1 - \beta)I}{p_2} \tag{3.4}$$

The well-known graphical representation of this optimization problem is shown in Figure 3.1. The problem is to reach the highest indifference curve subject to being on the (fixed) budget constraint, and the solution is the tangency at point A.

Figure 3.1

For a lot of theoretical work, especially with numerical simulation models, it turns out to be more convenient to work with the dual problem, that of minimizing the expenditure necessary to reach a given level of utility such as $\bar{U}$. This yields demand functions that are functions of prices and the target utility level, often referred to as Hicksian demand functions.

$$\min \ p_1X_2 + p_2X_2 \ + \ \lambda(U(X_1, X_2) - \bar{U}) \tag{3.5}$$

$$p_1 - \lambda\frac{\partial U}{\partial X_1} = 0 \qquad p_2 - \lambda\frac{\partial U}{\partial X_2} = 0 \tag{3.6}$$

$$\bar{U} - U(X_1, X_2) = 0 \tag{3.7}$$

If you solve the three equation in three uknowns in (3.6) and (3.7), you should derive the Hicksian demand functions for the Cobb-Douglas case of (2.26) iii and show that they are given by

$$X_1 = \beta\left[\frac{p_2}{p_1}\right]^{1-\beta} U \qquad X_2 = (1-\beta)\left[\frac{p_1}{p_2}\right]^{\beta} U \tag{3.8}$$

It is no surprise that these are so similar to the optimal input quantities in (2.29), since the production function there and the utility function here are assumed to be the same. If you then multiply each quantity in (3.8) and add them together, we get the minimum expenditure necessary to buy level of utility U, often referred to the expenditure function.

$$e(p_1, p_2)U = (p_1^{\beta} p_2^{1-\beta})U \tag{3.9}$$

This is naturally the same as the cost function for producing goods from the same Cobb-Douglas function in (2.26). Indeed, the expenditure function is precisely a cost function: the minimum cost necessary at prices *p* to buy one unit of utility. As a final exercise, apply Shepard's lemma to (3.9) to check that you get (3.8) as the optimal input choices.

The Hicksian optimization problem is shown in Figure 3.2. Here the target utility level U is fixed, the problem is achieve this utility level for the minimum expenditure. The solution is the tangency solution at point A, which puts on the lowest possibly budget line to achieve U.

Figure 3.2

3.2 A note on homogeneous functions

Often international trade models focus on the production side of economies and differences in things like technologies and factor endowments as causes of trade. When doing so, we generally assume no differences between countries with respect to demand so that the direction of trade (which countries export and import which goods) is purely determined on the production side. More specifically, it is useful for this reason and for computational reasons as well to assume that the ratio in which consumers demand goods depends only on relative prices on not on income: in the two-good case, the ratio of goods chosen in consumption, X_2 / X_1, depends only on the ratio of prices, p_1/p_2. This is a property that characterizes all homogeneous functions. More formally, U is homogeneous of degree i if

$$U(\lambda X^0, \lambda Y^0) = \lambda^i U(X^0, Y^0) \tag{3.10}$$

where the superscript 0 denotes some specific initial values. A homogeneous function with i = 1 is, in economic terminology, constant returns to scale. Or to put it the other way around, a constant-returns-to-scale production or utility function must be homogeneous of degree 1. Differentiating both sides of (3.10), we have

$$U_1(\lambda X_1^0, \lambda X_2^0)\lambda dX_1 = \lambda^i U_1(X_1^0, X_2^0) dX_1$$

$$U_2(\lambda X_1^0, \lambda X_2^0)\lambda dX_2 = \lambda^i U_2(X_1^0, X_2^0) dX_2 \tag{3.11}$$

Diving the second equation of (3.11) by the first, we have

$$\frac{U_2(\lambda X_1^0, \lambda X_2^0)}{U_2(\lambda X_1^0, \lambda X_2^0)} = \frac{U_1(X_1^0, X_2^0)}{U_2(X_1^0, X_2^0)} \tag{3.12}$$

This result says that the marginal rate of substitution *MRS* depends only on X_2 / X_1 ratio in consumption and not on the scale. To put it the other way around, holding prices constant means that the consumer will choose the same X_2 / X_1 ratio independently of income. This result is shown in Figure 3.3: the income expansion path (sometimes called an Engels' curve) is linear from the origin holding prices constant. This is an unrealistic assumption that is contradicted by every budget study ever done, but it is a useful one. It will be examined further in a later chapter.

Figure 3.3

Second, note from (3.11) that the marginal utility of a good (or of a factor in production theory) under constant return to scale (i = 1 in (3.10) is invariant to a proportion change in the use of all goods (factors). That is, the marginal products are homogeneous or degree (i - 1), or homogeneous of degree zero in the case of constant returns to scale. From (3.10), we have

$$U_1(\lambda X^0, \lambda Y^0) = U_1(X^0, Y^0) \qquad U_2(\lambda X^0, \lambda Y^0) = U_2(X^0, Y^0) \tag{3.13}$$

We referred to this earlier in Chapter 2 but postponed discussion of the fact that the first-order conditions for maximizing profits in (2.9) and (2.10) are invariant with respect to proportional changes in inputs, and therefore the supply of a single competitive firm with constant returns is indeterminant.

While on the topic of homogeneous functions, it is interesting to note a couple of properties of the Marshallian and Hicksian functions that we derived above. Note that the Marshallian demand functions in (3.4) are homogeneous of degree zero in prices and income: demands will not change if we double all prices and income (the budget line doesn't move). The Hicksian demand functions in (3.8) are homogeneous of degree zero in prices: if we double all prices, the slope of the budget line doesn't change and so the cost-minimizing choices of X_1 and X_2 don't change. But the minimum expenditure needed to reach the target utility must change as shown in (3.9): the expenditure function is homogeneous of degree 1 in prices if *U* is homogeneous of degree 1. These are in fact very general results and by no means restricted to the Cobb-Douglas special case.

3.3 Aggregating over households to a "community" utility function

While all this should sound familiar and relatively straight forward, international trade deals with the whole economy and thus we need to aggregate up from the individual household to the full economy. It unfortunately turns out that this is not at all straight forward.

Suppose that individuals all have preferences of the type described in the preceding sections, which in turn gives rise to demand functions that depend on prices and income. Assume that there are only two goods and that the price ratio is denoted $p = p_1 / p_2$. The income of individual *J* is denoted by I_j. If all individuals in the economy face the same price ratio but generally have different incomes, then the total demand for the good *X* can be written as the function *D*:

$$X = D(p, I_1, I_2, ..., I_n) \tag{3.14}$$

where it is assumed that there are *n* individuals in the country. The question of whether or not national or "community" indifference curves exist is nearly the same as the question of whether or not the demand function such as those in equation (3.8) can be written as a function of aggregate income (the sum of the individuals' incomes); that is, whether or not the distribution of income affects total demand.[1] The intuition here is that when we draw community indifference curves, we are saying that the country has preferences over aggregate bundles of goods that depend only on prices (the slope of the price line) and total national income (the distance the price line is from the origin). Preferences and hence demands are independent of how that aggregate income is distributed. In short can we write total demand as

$$X = D(p, I) \qquad I = \sum_{j=1}^{n} I_j \tag{3.15}$$

Special assumptions are necessary for demand to be independent of the distribution of income and hence for (3.15) to be valid. One problem that arises in aggregation is shown in Figure 3.4, where we have two consumers with identical but nonhomogeneous tastes.[2] In other words, at constant relative prices, the ratio of X_2/X_1 consumed is not independent of income. Specifically, consumers desire more X_1 relative to X_2 as income increases at constant prices.

Figure 3.4

Suppose we have two individuals initially consuming at point *A* in Figure 3.4, but we take some income away from consumer 1 and give it to consumer 2 so that they adjust to consuming at points *B* and *C*, respectively. The chord *AB* is steeper than the chord *AC*, and so the changes in the consumption of the two individuals do not balance. Even though we have not changed either prices or aggregate income, there will now be a higher aggregate demand for X_1 and a lower aggregate demand for X_2. Aggregate demand depends on the distribution of income, and hence community indifference curves do not exist in this situation.

Now consider homogeneous but nonidentical tastes. This situation is shown in Figure 3.5. Consumer 2, who has a relatively strong preference for X_2, is initially at A_2, while consumer 1, who has a strong preference for X_1, is initially at A_1. Now take income away from consumer 2 and give it to consumer 1 holding prices constant. Consumer 2 moves to point B_2, while consumer 1 moves to point B_1 in Figure 3.5. Again, the changes in consumption do not cancel (more X_1 and less X_2 will be demanded) even though prices and total income are constant. Community indifference curves do not exist when preferences differ.

Figure 3.5

This analysis suggests that community indifference curves will exist if all consumers have identical and homogeneous tastes (and, of course, face the same prices) as in Figure 3.3. This is indeed true. However, it is an extremely strong assumption that is nevertheless pervasive in trade theory. A somewhat weaker assumption will do, and it is sufficient simply that all consumers have income-expansion paths (Engel's curves) that are linear and parallel. Many industrial-organization models of trade in fact go to an extreme in order to avoid general-equilibrium income effects and assume that demand in the X_1 sector depends only on prices and not on income (an example of so-called quasi-linear preferences). Much more will be said on this later, but for completeness we show an example in Figure 3.6. If all consumers have these preferences, aggregation is possible since total demand does not depend on the distribution of income.

Figure 3.6

3.4 Interpreting community indifference curves: aggregate demand versus individual welfare

There are two different interpretations of community indifference curves (assuming they exist) and it is very important to distinguish between them. One is what economists call a *positive* interpretation. Under this interpretation, the community indifference curves simply tell us what the country will demand under various price and aggregate income combinations. That is, if we pick income and prices so as to determine an aggregate budget line as in equation, we can find the quantities demanded by the intersection of the highest community indifference curve with the budget constraint. The positive interpretation of community indifference curves does not necessarily attach any welfare significance to the indifference curves.

The *normative* interpretation of community indifference curves does attach a positive welfare significance to moving from a lower indifference curve to a higher indifference curve (or even from moving along one indifference curve) in the same way that we would interpret that move for a single individual. If some trade policy can lead to such a movement, we say that the country is better off (or equally well off).

The pitfalls in the second interpretation are illustrated in Figure 3.7, where A and B are two aggregate commodity bundles and U_a and U_b two community indifference curves. Clearly under the normative interpretation of community indifference curves national welfare increases in a move from A to B. However, suppose that the country is composed of two individuals, denoted with subscripts 1 and 2, with identical homogeneous preferences. In the initial situation A, measure the consumption of individual 1 from the origin *O* and measure individual 2's consumption in the opposite direction beginning at *A* as in the Edgeworth box. As drawn in Figure 3.7, individual 1 has initial consumption *OA'* and individual 2 has consumption *A'A*. Suppose that the move from A to B somehow redistributes income from individual 2 to individual 1. This is of no consequence for the community indifference curves. However, the move from A to B clearly has the effect of greatly helping individual 1 but harming individual 2 whose consumption is reduced from *A'A* to *B'B*. The point is that in making normative interpretations of community indifference curves we must remember that *a movement to a higher community indifference curve does not mean that the welfare of all individuals in society has increased*. Similarly, moving along a single community indifference curve does not mean that the welfare of all individuals is being held constant.

Figure 3.7

In later chapters we will present cases where aggregate welfare increases but, due to income re-distribution via factor-price changes, some individuals are worse off. Indeed, this is a relatively common problem with trade policy and it is one of the points of departure in what is called the political-economy approach to trade policy. Throughout the book, we will often make normative interpretations of community indifference curves, but we must keep the caveat just mentioned in mind. One traditional way of avoiding difficulties is to say that, if preferences are indeed identical and homogeneous, then there will always exist some domestic redistribution of income or compensation that will make all individuals better off when the economy moves to a higher indifference curve. The existence of such possible compensations implies that all individuals are *potentially* better off. This is not very satisfactory, however, since in the absence of actual redistributions the individuals who are made worse off are not at all happy with the potential-improvement argument. We will explicitly deal with the income-distribution problem at a number of points in the book, but in many others we will simply attach normative significance to community indifference curves.

3.5 Summary: what you should know

This chapter has developed tools for the consumption or demand side of the economy. We begin with the standard Marshallian representation of the consumer’s optimization problem: maximizing utility subject to a budget constraint gives us demand as a function of prices and income. Then we analyze the Hicksian approach: minimizing expenditure subject to a target utility level gives us demand as a function of prices and utility. Then we turn to the question of aggregating individual demands into aggregate demand and note that extremely restrictive assumptions are needed in order to make this theoretically valid. But having made note of this, we will then follow the tradition of trade theory and more or less ignore this problem in what follows. We will not however, ignore the fact that a rise in welfare as indicated by the aggregate or community utility function generally does not imply that all individuals or households within the economy have their welfare increased in the same proportion. We will see repeatedly that liberalizations or its opposite, protectionism, and changes in world prices exogenous to our own economy not only affect aggregate welfare but also redistribution income within the society. Income redistribution through changes in factor prices or a redistribution between profit income and factor income are just two examples.

Endnotes to Chapter 3

1. This is not precisely theoretically correct; that is, the existence of community indifference curves and the ability to write demand as a function of aggregate income are not quite the same thing. In particular, there are cases in which the latter is true but the former is not (in particular when heterogeneous consumers each have a fixed share of income). For the purposes of this book, we have decided to avoid a lengthy discussion of the fine points. The analysis which follows develops the intuition and presents restrictions on preferences which guarantee both the existence of community indifference curves and aggregate demand functions. Relatively simple discussions of aggregation can be found in Green (1976) or Deaton and Muellbauer (1980).

2. In other texts, you may see the assumption that preferences are homothetic rather than homogeneous. The assumption that a function is homothetic is a somewhat weaker assumption than assuming it is homogeneous. Specifically, any monotonic transformation of a homogenous function is homothetic. Since for preferences we only require tastes to have the property that more is preferred to less, without really caring by how much more it is preferred, the weaker and more general homothetic property is sufficient for most results in consumer theory. But then again this is not really important for our purposes, so we will use the term homogeneous throughout.

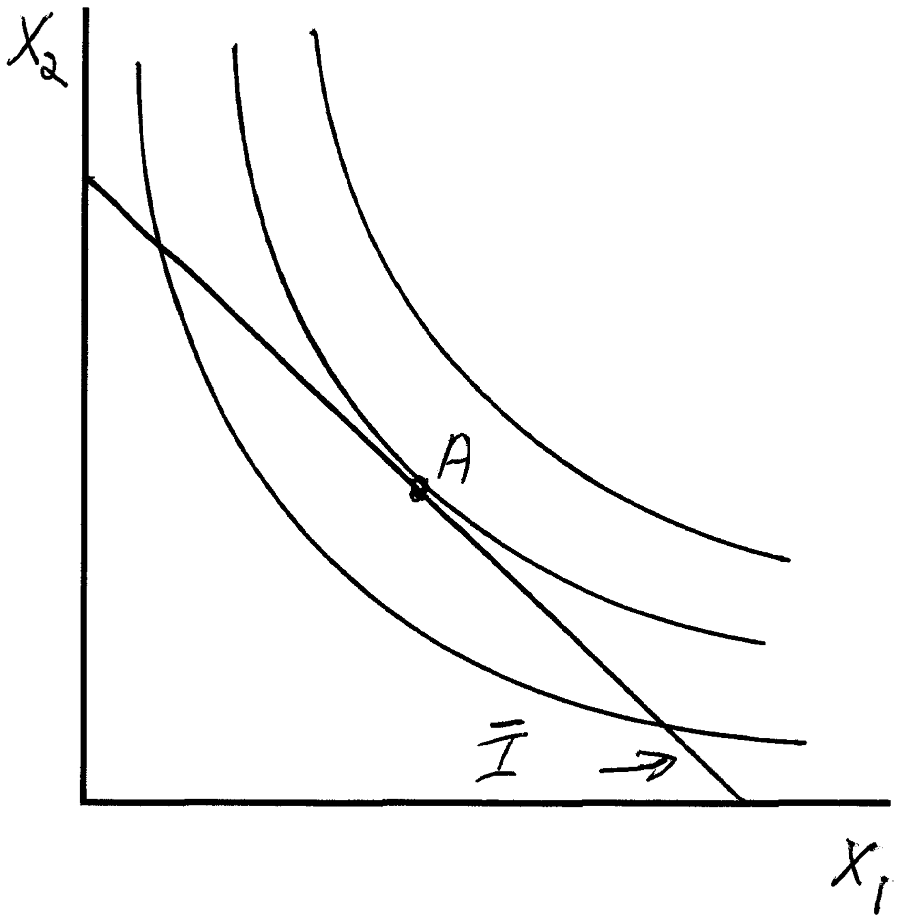

Figure 3.1

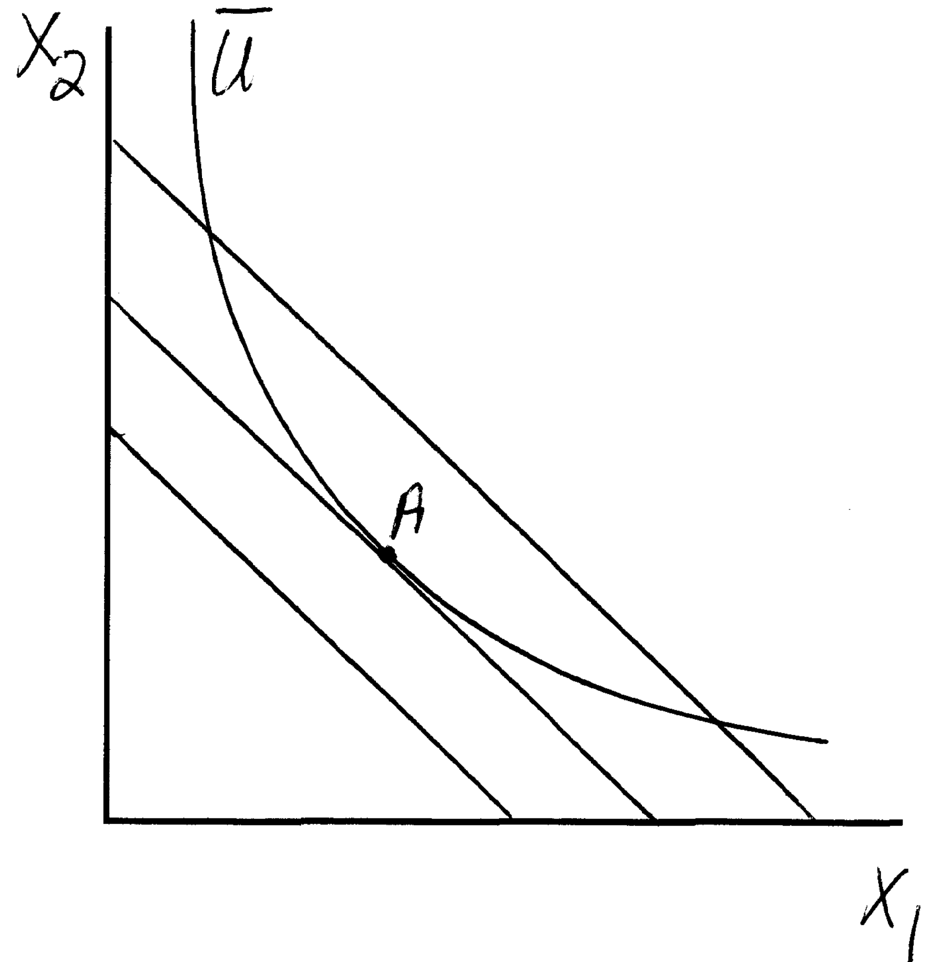

Figure 3.2

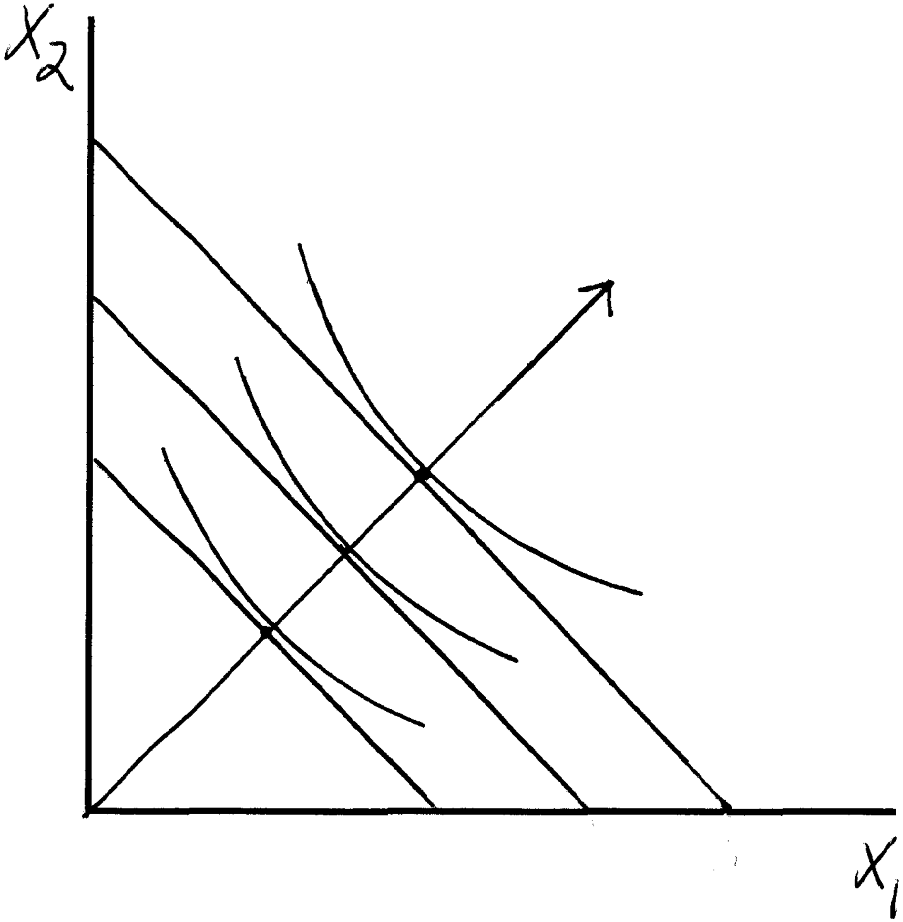

Figure 3.3

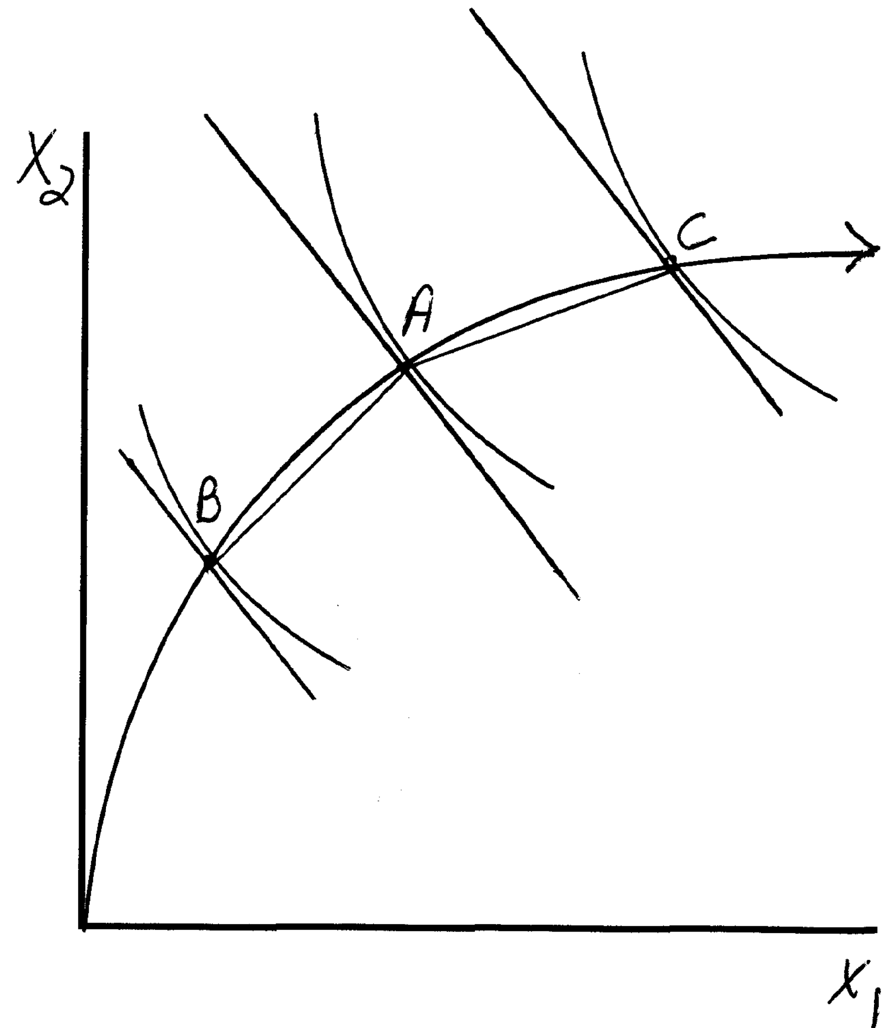

Figure 3.4

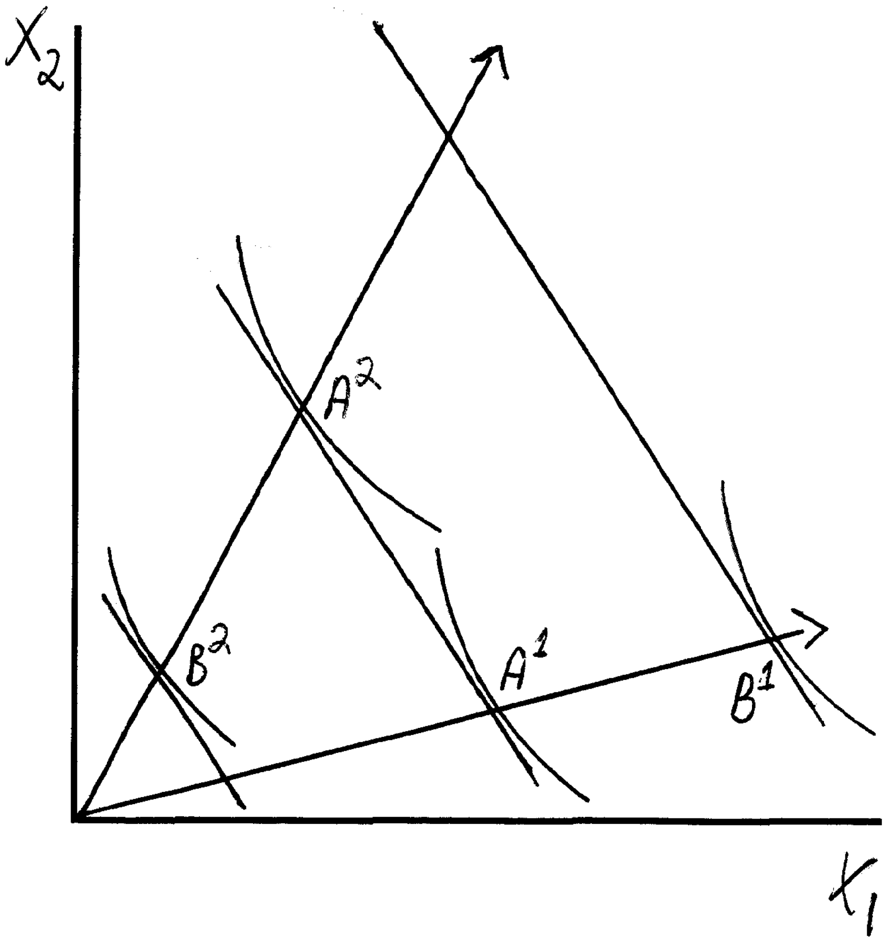

Figure 3.5

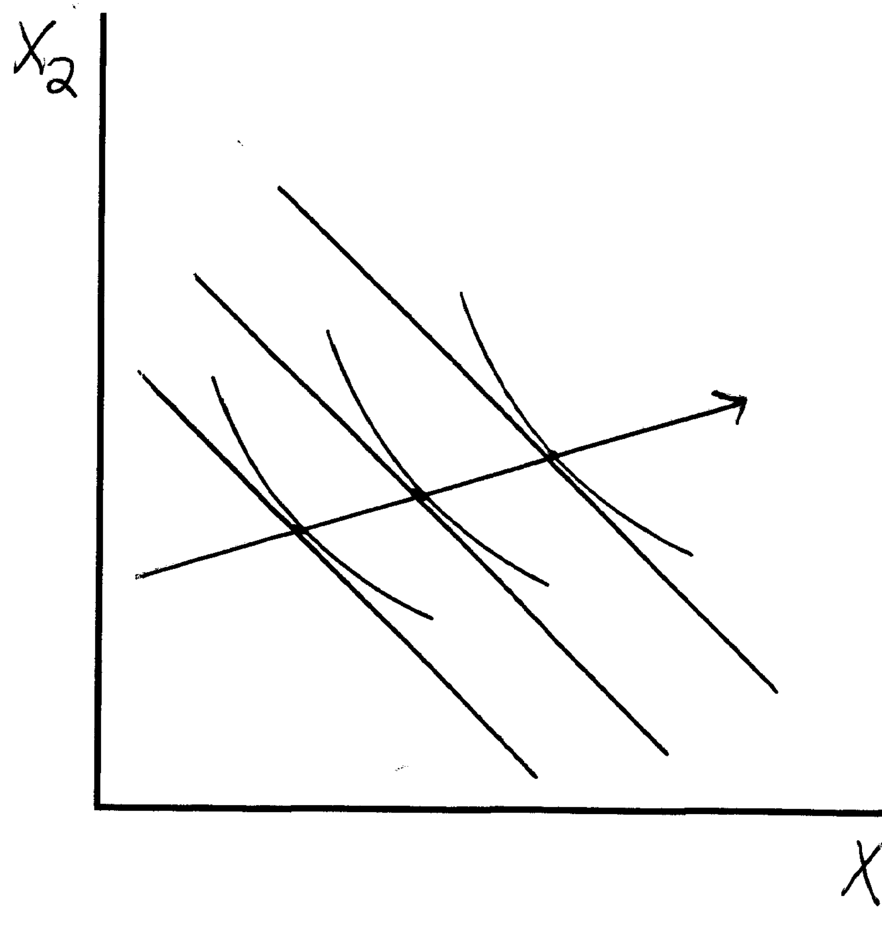

Figure 3.6

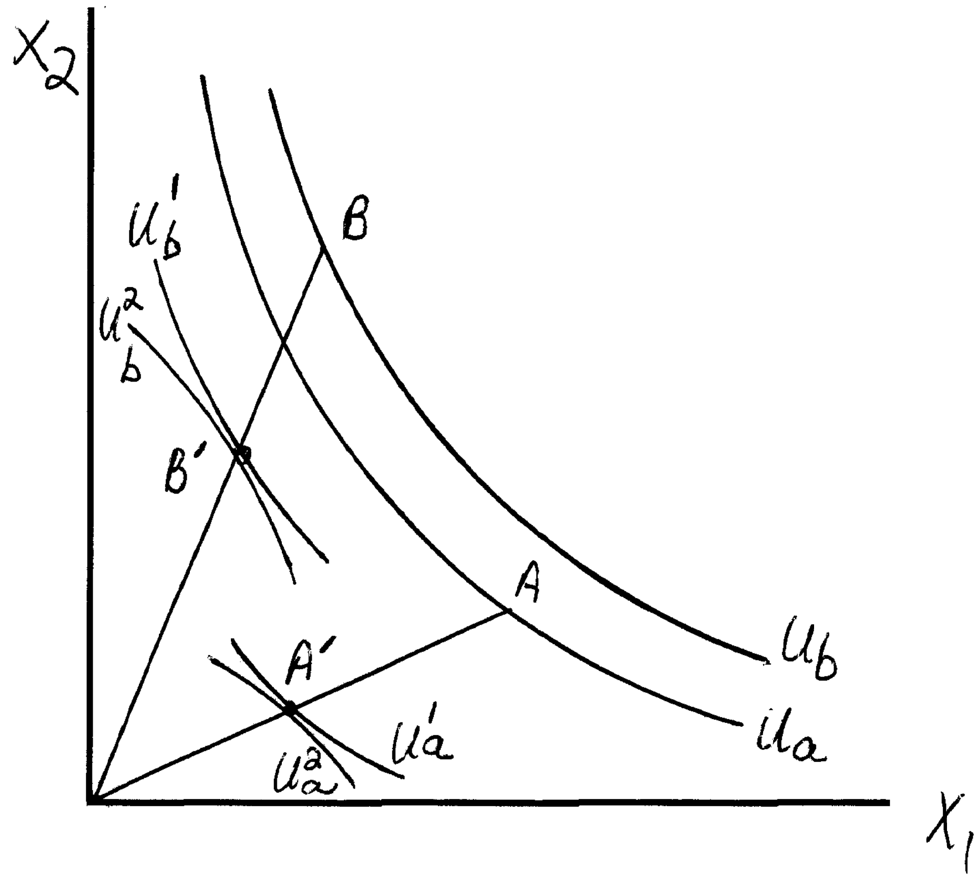

Figure 3.7