Chapter 12

INCREASING RETURNS AND IMPERFECT COMPETITION II: MONOPOLISTIC COMPETITION

9.1 Trade and gains from trade through increased product diversity

The previous chapter introduced economies of scale and imperfect competition as a determinant of trade and gains from trade. We concentrated on a homogeneous-good industry (firms produce identical products) and the idea that scale economies limit the number of firms in the market. This limitation leads to imperfect competition in equilibrium, with products marked up above marginal costs. Firms interact strategically with one another and the consequence of this is that the effective enlargement of the market following the opening of trade leads to pro-competitive gains from trade. Firms move down their average cost curves and consumers benefit from lower prices for the same goods.

In this Chapter, we are again going to look at increasing return to scale and imperfect competition but from a different point of view. We are going to assume that firms produce differentiated products but that the market can support a relatively large number of firms such that there is minimal strategic interaction among the firms. Instead of trade resulting in pro-competitive gains, the same products at lower prices, trade will result in a greater variety of products at the same prices and this will raise consumers' welfare.

These assumptions, that products are different but that there are many firms and minimal strategic interaction, are typically referred to as monopolistic competition. Monopoly refers to the fact that each firm produces a somewhat different product and hence will have influence over its market price even if there are literally hundreds of firms. Competition refers to the fact that there are sufficiently many firms such that there is little strategic interdependence among them.

The following preferences (utility function) are typically used to introduce monopolistic competition and are generally referred to as Dixit-Stiglitz (1977) preferences. This approach is also sometimes referred to as "love of variety" for a reason that should become clear. Let X_i refer to one good in a set of n goods. Ignoring any other sectors for the moment, utility is given by

$$U = \left[\sum_i^n X_i^\alpha\right]^{\frac{1}{\alpha}} \qquad 0 < \alpha < 1 \qquad \frac{1}{\alpha} > 1 \qquad \sigma = \frac{1}{1-\alpha} > 1 \tag{12.1}$$

Many of you will recognize this as a CES function that we introduced earlier in the book, where σ gives the elasticity of substitution among the varieties. It is in fact a special case in which all goods carry the same weighting in producing utility, and in which the elasticity of substitution is restricted to values greater than one. The latter assumption means that indifference curves intersect the axis and so positive utility can be derived from a subset of goods. Indeed, the whole point of this approach is that only a subset of goods gets produced in equilibrium and that the number of goods available is endogenous.

The individual goods in (12.1) are said to be symmetric but imperfect substitutes. Symmetric means that they all have the same weight in producing utility as just noted and hence a consumer is indifferent between one apple and one orange. However, they are also imperfect substitutes meaning that variety is valuable: a consumer would rather have one apple and one orange than either two apples or two oranges. To see this last point, assume that each good that is produced is produced in the same amount, so the that the summation in (12.1) is just n, the number of goods, time X, which we will term the quantity of a "representative good".

$$U = \left[n X^{\alpha} \right]^{\frac{1}{\alpha}} = n^{\frac{1}{\alpha}} X \qquad (12.2)$$

We see from (12.2) that utility has constant returns in the amount of each good consumer: double each X and utility doubles. But we also see that utility has increasing returns to scale in the range of product variety (henceforth just "variety"). Let n^0 and X^0 denote the number of goods and the representative quantity produced initially, and U^0 the initial level of utility. Then suppose that the number of goods doubles to $2n^0$ but the quantity consumed of each falls in half to $X^0/2$. Utility is then given by

$$U = (2n^0)^{\frac{1}{\alpha}} (X^0/2) = 2^{\frac{1}{\alpha} - 1} (n^0)^{\frac{1}{\alpha}} X^0 = 2^{\frac{1-\alpha}{\alpha}} \left[(n^0)^{\frac{1}{\alpha}} X^0 \right] = 2^{\frac{1-\alpha}{\alpha}} U^0 > U^0 \qquad (12.3)$$

Equation (12.3) shown that welfare improves when the consumer has half as much of each of twice as many goods. Hence the term "love of variety".

Some intuition is provided in Figures 12.1 and 12.2, where we assume that each of two varieties are produced with increasing returns: a fixed cost and a constant marginal cost as in the previous Chapter. Consider first Figure 12.1, where the production frontier $\bar{X}_2 X_2^{\prime} X_1^{\prime} \bar{X}_1$. It may be that in autarky, a country may wish have variety even though it is expensive in terms of having to pay the fixed costs for both goods, and hence prefers the autarky outcome shown as $X^a = D^a$ in Figure 12.1. Trade with the second identical country can then allow each country to specialize in one of the goods, trading half of its output for half of the output of the other country's good. Then both countries can consumer at point D* in Figure 12.1.

Figure 12.1

On the other hand, the high fixed costs and the sacrifice of scale economies may mean that it is better to produce and consumer just one good in autarky, which is the situation shown in Figure 12.2. Here the country achieves utility level U^a in autarky by producing either X_1 or X_2 but not both. This is a higher level of welfare than producing both goods: the added variety is not worth the sacrifice of quantity. Now let the two identical countries get together, with each specializing in one of the goods. Now they can trade to the common consumption point D^* in Figure 12.2, achieving a gain from trade.

Figure 12.2

Note the difference in the source of gains from trade between Figure 12.1 and 12.2. In the former case, the consumer gets more of the same goods, the source of gains in the previous Chapter. In the case of Figure 12.2, the consumer actually gets less quantity of a given good, but enjoys more variety. This is in fact exactly the outcome explored in equation (12.3). It is crucial to note in Figure 12.2 that trade does not result in increased output of any good that is produced initially. No firm moves down it average cost curve, there is no increase in firm scale. Nevertheless it is indeed scale economies that are responsible for the welfare gains in that scale economies limit the number of goods produced initially.

To press this last point a little further, note that if there were no fixed costs and all goods were produced with constant returns to scale, that the optimum under Dixit-Stiglitz preferences would be to have infinitely many goods produced in infinitely small quantities. This is just a logical extension of the argument behind equation (12.2). Increasing returns to scale make diversity costly and hence limit the range of goods in equilibrium.

Figures 12.1 and 12.2 illustrate what can happen, but they by no means prove that this is what will happen in a market environment characterized by imperfect competition.

12.2 A more formal approach to Dixit-Stiglitz and love of variety

We will assume that there are two sectors: sector *X* is composited of firms producing differentiated goods as above, and sector *Y* produces a homogeneous good with constant returns to scale. We will assume a very simple factor market structure much the same as in the previous chapter. There is only one factor of production which we will call labor, and we will use this as numeraire assigning a value of one to the wage rate, $w = 1$. We will assume that the consumer has Cobb-Douglas preferences between Y and X, and CES preferences among the *X* varieties. Much of what follows is based in Krugman (1979, 1981) and Helpman and Krugman (1985).

Total income is given by *L* when the wage is chosen as numeraire. We are also going to assume that all potential *X* varieties have the same cost function. This is a common assumption that, when combined with symmetry in demand, gives us the result that any good that is produced is produced in the same amount and sells for the same price. Henceforth *X* and p_x will denote the price of a representative good which are the same for all goods actually produced. The utility function and the budget constraint for the economy are given by:

$$U = \left[\sum_i X_i^{\alpha}\right]^{\frac{\beta}{\alpha}} Y^{1-\beta} \qquad \sigma = \frac{1}{1-\alpha} \qquad L = np_x X + p_y Y \tag{12.4}$$

If you solve the optimization problem, the consumer's demands for *X* varieties and *Y* are

$$Y = (1-\beta)\frac{L}{p_y} \qquad X_i = p_{xi}^{-\sigma}\left[\sum_i p_{xi}^{1-\sigma}\right]^{-1}\beta L \qquad nX = \beta\frac{L}{p_x} \tag{12.5}$$

The demand response for a given variety in response to a change in its own price is a bit complex, since the variety's own price appears both as the first term on the right-hand side of the second equation of (12.5) but also appears in the summation term inside the square brackets. Thus the derivative of the demand for *X* with respect to its own prices must be found by using the differentiation of a product rule. However, it can be shown (and we will not do so here) that the effect of a change in a firm's price on the summation term in square brackets become extremely small as the number of varieties (firms) n becomes large. As a consequence, most work in this area assumes that an individual firm is too small to affect the summation term in (12.5), an assumption known as "large-group monopolistic competition. Assuming that the term in brackets in (12.5) is viewed as a constant by an individual firm, the price elasticity of demand for an individual goods is given simply by σ, the elasticity of substitution among the *X* goods. Referring back to our derivation of marginal revenue in the previous chapter, the markup takes on the very simple formula 1/σ. The elasticity of demand and marginal revenue are then given as follows.

$$-\frac{p_x}{X}\frac{\partial X}{\partial p_x} = \sigma \qquad mr_x = p_x(1 - 1/\sigma) = mc_x \tag{12.6}$$

Turning to production, marginal cost for *Y*, marginal cost for *X*, and fixed costs of an *X* variety are denoted by mc_y, mc_x, and fc_x respectively. The full general equilibrium model for a single economy is given by a set of inequalities with associated variables as described back in Chapter 4. First, there is are pricing equations for the *Y* industry and for each *X* variety. Second, there is a zero profit condition for each *X* variety, which is typically written as markup revenues equal fixed costs instead of the longer equation for revenues equal total costs. It is useful to think of fixed costs as a produced good, such as factor and equipment, hence there is a pricing equation for factories (fixed costs). These three pricing inequalities are given as follows:

Inequality	Definition	Complementary Variable	
$p_y \le mc_y$	pricing for *Y*	*Y*	(12.7)
$p_x(1 - 1/\sigma) \le mc_x$	pricing for *X*	*X*	(12.8)
$(p_x/\sigma)X \le fc_x$	pricing for *n* (free entry)	*n*	(12.9)

Then there are three market-clearing conditions, which require that supply equal demand (strictly speaking supply is greater than or equal to demand). There is demand and supply for Y, for total X production, and for labor. These equations follow from (12.5) and are as follows.

$(1 - \beta)L/p_y \le Y$	demand/supply *Y*	p_y	(12.10)
$\beta L/p_x \le nX$	demand/supply *X* varieties	p_x	(12.11)
$(mc_y)Y + n(mc_x)X + n(fc_x) = L$	demand/supply *L*	*w*	(12.12)

This model can be solved analytically due to the powerful advantages of the large-group monopolistic-competition assumption. Equations (12.7) and (12.8) can be solved for both *X* and p_x. Then these solution values can be used in (12.11) to get *n*. The solution values are:

$$X = (\sigma - 1)\frac{fc_x}{mc_x} \qquad n = \frac{\beta L}{\sigma fc_x} \qquad nX = \frac{(\sigma - 1)}{\sigma}\frac{\beta L}{mc_x} \tag{12.13}$$

Note from the first equation of (12.13) that the output of any good that is produced is a constant and that from the second equation that any expansion in the economy creates a proportional increases in variety n. Let *X/L*, the consumption of a representative variety per capita, be given by *C*. Then note from the last equation of (12.13) that *nC* is a constant:

$$nC = n\frac{X}{L} = \frac{(\sigma - 1)}{\sigma}\frac{\beta}{mc_x} \tag{12.14}$$

Figure 12.3 plots n against *C*, and equation (12.14) is shown as a negatively-sloped curve in this Figure. Next, note from the second equation in (12.13) that n depends only on L and fixed parameters. Thus we show a second curve which is just a horizontal line in Figure 12.3 which gives the fixed value of *n* for a given value of *L*. The intersection of these two curves gives the number of varieties and consumption of a representative variety per capita. For a single economy, the outcome could be shown by variety level n^0 and variety consumption per capita by C^0.

Figure 12.3

Now repeat our usual experiment: put two identical countries together in free trade. This is represented by simply letting *L* double. The *nC* curve in Figure 12.3 does not shift as shown by (12.14), but the n curve shift up in proportion to *L*, doubling in value. If *n* doubles, then from (12.14) *C* must be cut in half. The new values in the open economy are C^1 and n^1 in Figure 12.3. Note that this is exactly what is analyzed in (12.3) and suggested in Figure 12.2 above. Equations (12.7) and (12.10) will show that consumption per capita of Y remains unchanged after trade, and hence welfare increases in each country due to the variety effect.

12.3 Monopolistic competition in specialized intermediate inputs (basics idea, then algebra after (12.20) can be skipped without loss of continuity)

The basic idea behind love of variety has also been applied to intermediate inputs, starting with Ethier (1982). This has in turn been applied in a number of different context including endogenous growth models, technology transfer through trade, and trade in producer services. Here we will analyze a much simplified Ethier model, in particular as extended to consider trade in final goods only versus trade in intermediates in Markusen (1989).

Suppose that there are two final consumption goods, X and Y, which are homogeneous and produced with constant returns to scale by competitive firms. Utility of the representative consumer is given by

$$U = U(X, Y) \tag{12.15}$$

There is a factor of production labor, L, which is in fixed supply. In addition, we assume a sector-specific factor K in the Y sector. The purpose of K is to generate a concave (bowed out) production fontier as we will discuss shortly. Good X is assumed to be costlessly assembled from differentiated or specialized intermediate goods S_i in a Dixit-Stiglitz fashion. The two production functions are given as follows, where σ is the elasticity of substitution as derived earlier in the book.

$$Y = G(L_y, \bar{K}) \qquad X = \left[\sum_i^n S_i^\beta\right]^{1/\beta} \qquad \sigma = \frac{1}{1-\beta} \tag{12.16}$$

Each S_i is produced with increasing returns to scale, consisting of the constant marginal cost and fixed-cost technology that we have now used many times. To reduce notation, one unit of S requires a single unit of labor. Labor requirements in S goods and the total labor supply constraint are given as follows, where n is the (endogenous) number of intermediates.

$$L_{xi} = wS_i + wF \qquad \bar{L} = L_y + nL_{xi} \tag{12.17}$$

Since each S enters (12.16) symmetrically and each has an identical technology, we can anticipate the result that any S that is produced is produced in the same amount and sells for the same price as any other S. Let superscript “a” denote a situation in which only the final X and Y goods can be traded and n^a the number of and S^a the amount of each S good in the “a” equilibrium. The X technology reduces to

$$X^a = \left[\sum_i^{n^a} (S_i^a)^\beta\right]^{1/\beta} = \left(n^a\right)^{1/\beta} S^a \tag{12.18}$$

Now again do our standard experiment where we put two identical economies together in trade. In order to illustrate the main idea, hold the amount of each S good produced constant and assume the number produced in each country constant. The number of intermediate goods used in each country in free trade in intermediates is double the number in autarky *(n^* = na)*, with the total output of each shared evenly between the two countries. Output of X in each country is now given by

$$X^* = \left[\sum_i^{n^*} (S_i^a/2)^\beta\right]^{1/\beta} = \left(2n^a\right)^{1/\beta}(S_i^a/2) \tag{12.19}$$

Simplifying the right-hand side, we can compare *X* output with intermediates trade to output under trade in final goods only given in (12.18).

$$X^* = 2^{\frac{1-\beta}{\beta}} (n^a)^{1/\beta} S_i^a = 2^{\frac{1-\beta}{\beta}} X^a > X^a \tag{12.20}$$

Allowing trade in intermediates increases productivity in *X* production as *X* producers now have access to a larger range of specialized intermediates, a greater division of labor.

Results are shown in Figure 12.4, where we assume that the diminishing returns to the fixed factor *K* in the *Y* sector outweigh the increasing returns to scale in the *X* sector, so that the production frontier is concave. The frontier through A gives the frontier when only final goods can be traded. Both countries are identical by assumption, and so there are no gains from trade: countries could trade but there is no benefit from doing so. Point A could represent the equilibrium production and consumption point for each country under goods trade only (the non-tangency of the price ratio with the frontier will be treated shortly).

Figure 12.4

If we do allow trade in intermediate goods (which could include specialized services), then the production frontier shifts to the one passing though F in Figure 12.4. Each country exports half of each of its inputs for half of each of the other country's inputs. With free trade in intermediatates, production and consumption for each country could be at a point like F in Figure 12.4.

As noted earlier, this formulation of the monopolistic competition model in a manner reminiscent of Adam Smith's division of labor has been used in a number of contexts including endogneous growth theory and the liberalization of trade in producer services. We now look at the issue of optimality of the market outcomes in this model; this is a somewhat more esoteric issue and the remainder of the section may be skipped by some readers. Results of this are equally applicable to the more standard final-goods model treated in the previous two sections.

Suppose that the economy faces a fixed price of *X* relative to *Y*, denoted *p*. Then the optimal number of intermediates and the output level of each can be found by maximizing the value of final output of *X* minus input costs (representing the opportunity cost of labor in producing *Y*.

$$Max\ \pi^* = p\left[\sum S_i^{\beta}\right]^{1/B} - \sum (wS_i + wF) \quad \text{with respect to } n \text{ and } S_i\text{, for all i} \tag{12.21}$$

The first-order condition with respect to S_i is given by applying the chain rule.

$$\frac{\partial \pi^*}{\partial S_i} = (p/\beta)\left[nS_i^{\beta}\right]^{\alpha} \beta S_i^{\beta-1} - w = pn^{\alpha} - w = 0 \qquad \alpha \equiv (1-\beta)/\beta \tag{12.22}$$

The first-order condition for the number of goods is given by the effect of adding one more *n*.

$$\frac{\partial \pi^*}{\partial n} = (p/\beta)\left[nS_i^{\beta}\right]^{\alpha} S_i^{\beta} - wS_i - wF = (p/\beta)n^{\alpha}S_i - wS_i - wF = 0 \tag{12.23}$$

If we solve these two equations in two unknowns, we get the optimal output of any intermediate that is produced. Note that w is just the marginal product of labor in the *Y* sector, G_l. From (12.22), we can also get the optimality condition for a tangency between the price ratio and the marginal rate of transformation along the production frontier.

$$S_i = \left(\frac{\beta}{1 - \beta} \right) F \qquad p = \frac{w}{n^{\alpha}} = \frac{G_l}{n^{\alpha}} = MRT \tag{12.24}$$

Now turn to the outcome under a market solution. The price of an individual *S* is the value of its marginal product in producing *X*. This is in fact given in the first term on the right-hand side of the first equation in (12.22). Let *r* denote the price of an individual *S*.

$$r = (p/\beta)\left[nS_i^{\beta}\right]^{\alpha} \beta S_i^{\beta - 1} = qS_i^{\beta - 1} \qquad q \equiv p\left[nS_i^{\beta}\right]^{\alpha} \tag{12.25}$$

In the tradition of large-group monopolistic competition discussed above, assume that each individual *S* producer views q in (12.25) as fixed. Then each *S* producer maximizes the following expression with resepect to S_i viewing *q* as fixed.

$$Max\ \pi = (qS_i^{\beta - 1})S_i - wS_i - wF \tag{12.26}$$

The first-order condition is given by

$$\frac{\partial \pi}{\partial S_i} = q\beta S_i^{\beta - 1} - w = 0 \tag{12.27}$$

Secondly, the free entry condition to determine *n* is that each *S* producer's profits are zero.

$$qS_i^{\beta} - wS_i - wF = 0 \tag{12.28}$$

Solving (12.27) and (12.28), we get the market equilibrium amount of any *S* that is produced. Second, substituting the expression for *q* in (12.25) back into (12.27), we can get the relationship between the competitive price ratio and the marginal rate of transformation given in (12.24). These are given by

$$S_i = \left(\frac{\beta}{1 - \beta} \right) F \qquad p\beta = \frac{w}{n^{\alpha}} = MRT < p \tag{12.29}$$

Comparing the optimum in (12.24) to the market outcome in (12.29) we see that any *S* that is produced is produced in the optimal amount. But we also see that the market outcome involves a distortion between the price ratio and the marginal rate of transformation, much as in the case of a tax, or the external-economies case of Chapter 10, or the oligopoly outcomes in Chapter 12. This is the distortion between the price ratio and the slope of the frontier shown in Figure 12.5.

The conclusion of this exercise is that the market outcome is not first best: it produces the optimal output of any good that is produced, but too few goods are produced. The intuition behind this is essentially an externality argument. (12.28) gives the private profits of entry. But when one firm enters, it increases the productivity of every other firm holding prices constant. This is seen in (12.25): the value of the marginal product of an additional unit of *S* in producing *X* (*r*) is increasing in n, the division of labor. This effect is not considered by an individual firm in its entry decision and hence there is a positive externality in the *X* sector. Note that when (12.28) holds with equality (the private profits from entry are zero), the "social" marginal product of an addition *S* given in (12.23) is strictly positive. There is thus a close analogy here between the present result and that in Chapter 10 in the section on production externalities.

12.4 The ideal variety approach to product diversity

Gains from trade through product differentiation can be looked at in a second way as well. While consumers may prefer diversity as just noted, consumers themselves may also have different tastes. Consumers may, for example, buy only one automobile each, but they have different views as to what is the "ideal" automobile for their tastes and income level. This approach to product diversity is thus labelled the "ideal variety" approach (Lancaster, 1980). Due to scale economies, no country can afford to produce a unique automobile for each consumer. Germany produces Volkswagens and Mercedes, and France produces Renaults and Peugeots, all of which have somewhat different characteristics from consumers' points of view. Trade in automobiles then occurs between France and Germany due to the fact that some Germans prefer Renaults or Peugeots and some Frenchmen prefer Volkswagens or Mercedes.

This situation is shown in Figure 12.5. Suppose that automobiles have only two characteristics: C_1 and C_2 (e.g., size and speed). There is a trade-off between these two characteristics such that if one wants a bigger car he or she must sacrifice some speed if the two are going to cost the same (e.g., Mercedes versus Porsche). Figure 12.5 shows three possible combinations of C_1 and C_2, denoted X_1, X_2, and X_3, corresponding to three different types of cars. Suppose that all three models could be produced at the same average cost for the same volume of production and that at this common cost, the amounts of C_1 and C_2 embodied in the cars are given by points (X_2^0, X_3^0, X_1^0) in Figure 12.5. The straight line through these points represents a sort of iso-cost line at a common scale of production.

Figure 12.5

Now suppose that we have two groups of consumers with distinct tastes (identical within each group). Consumer type-1 has a relative preference for characteristic C_1 and consumer type-2 has a relative preference for characteristic C_2. On the isocost line shown in Figure 12.5, an indifference curve for consumer type-2 is tangent to the isocost line at point X_2^0 and an indfference curve for consumer type-1 is tangent at point X_1^0. X_2 is then referred to as consumer type-1's *ideal variety* and X_1 as consumer type-1's ideal variety.

Variety X_3 could be referred to as a *compromise variety*. Note from Figure 12.5 that for X_3 to be equally attractive to our two consumer types, more “stuff” (e.g., stereo, air conditioning) would have to be added to it: to make the consumers indifferent between the compromise variety and their ideal varieties, we would have to offer X_3^1, which lies outside the isocost line. The proportional difference $(X_3^1 - X_3^0)/X_3^0$ in Figure 12.5 is sometimes called the *compensation ratio*: the proportional amount of extra “stuff” needed to make the compromise variety as attractive as the ideal varieties.

If there were constant returns to scale, the problem would be trivial: each consumer would get there ideal variety, no one would even buy the compromise variety because it would cost more. However, the problem is far from trivial with increasing returns to scale. The production scale in producing the compromise variety is twice as large as in giving each consumer their ideal variety. Thus, adjusting for the benefits of larger production scale, variety , X_3^1, may in fact cost less than variety X_2^0 or X_1^0.

Suppose that there is only a single factor labor, L, with a wage one. We assume, as is typical in this approach, that each consumer wants only one unit of the good but of course prefers their ideal variety. There are fixed and marginal costs for each variety. mc0 is the labor needed for on unit of (X_2^0, X_3^0, X_1^0) and mc1 is the labor need for one unit of X_3^1. Labor requirements are then

$$L_1^0 = L_2^0 = L_3^0 = mc^0 + fc \qquad L_3^1 = mc^1 + fc \qquad mc^0 < mc^1 \tag{12.30}$$

The relevant question is whether or not two ideal varieties (two times the first equation of (12.29)) costs more than two units of the compromise variety. The compromise variety is preferred if

$$2mc_3^1 + fc < 2mc_i^0 + 2fc \quad or \quad 2(mc_3^1 - mc_3^0) < fc \quad i=1,2,3 \tag{12.31}$$

The important point is that the compromise variety means incurring the fixed costs only once versus twice if each group gets its ideal variety.

Suppose that the inequality in (12.30) "marginally" holds, meaning that the left-hand side is less than the right-hand side by a very small amount. Then the compromise variety is preferred. Consider then our now-familiar experiment of putting two identical countries together. To give every consumer one unit, the total world labor requirements for ideal varieties are now twice the marginal costs shown in (12.30) and similarly for the compromise variety: no additional resources are needed for fixed costs. Total requirements (costs) and the condition for the compromise variety to be preferred are now

$$4mc_3^1 + fc < 4mc_i^0 + 2fc \quad or \quad 4(mc_3^1 - mc_3^0) < fc \quad i=1,2,3 \tag{12.32}$$

If (12.30) holds marginally, the inequality in (12.31) will *not* hold. It will be reversed and the ideal varieties will be preferred.

As in the love-of-variety approach, we see here that the ideal-variety approach means that there are gains from trade through increased product variety. The nature of those gains are different here. The idea is that trade allows each consumer type to get a product closer to their ideal variety embodying their ideal characteristic combination. In more general models with many consumer types and goods, the welfare statement in these models is typically that "on average" consumers get varieties closer to their ideal type.

In a world of imperfect competition that accompanies scale economies, we have to be cautious and note that just because something is preferred doesn't mean it is going to happen. These ideal-variety models get quite mathematically complicated, much more so than the simple love-of-variety model. For example, there may be a continuum of consumers who preferences are "located" at difference points of a line or a circle. Such models are sometimes referred to as "location" or "address" models of product differentiation, but these embody the same basic ideas: a consumer's location or address on the circle is the consumer's ideal variety (see Helpman, 1981). These models have been perhaps more used in industrial-organization economics. The problem for international trade is that they quickly get very difficult in general-equilibrium settings.

12.5 Some useful algebra for Dixit-Stiglitz (may be skipped without loss of continuity but useful later for analyzing trade costs)

This section lays out the algebra required to solve for the demand function and price index (defined and discussed shortly) for the Dixit-Stiglitz utility function. In order to simplify it a bit, let's assume that there is just a single section producing differentiated *X* varieties. But there is a straightforward extension to a two-sector model in which the other sector, *Y*, has constant returns to scale and perfect competition, and there is Cobb-Douglas substitution between the *X* goods and good *Y*. In this situation, the consumer always devotes constant share of income to each sector, so we will start with an allocation of income M_x to the X sector.

Let X_c denote the utility derived from the X varieties; X_c is sometimes referred to as a composite commodity. X_c does have a price associated with it. This is a price index, the minimum expenditure necessary to buy one unit of composite good X_c. We will denote this price or rather price index as e_x. The value of X_c (the utility from X consumption) is defined as follows.

$$X_c = \left[\sum_i^n X_i^\beta\right]^{\frac{1}{\beta}} \qquad \sigma = \frac{1}{1 - \beta}, \quad \beta = \frac{\sigma - 1}{\sigma}$$

The consumer maximizes utility subject to a budget constraint. The maximization problem is written as a Lagrangean function and the first-order condition for an arbitrary good X_i are given as follows.

$$\max X_c = \left[\sum X_i^\beta\right]^{\frac{1}{\beta}} + \lambda(M_x - \sum p_i X_i) \quad => \quad \frac{1}{\beta}\left[\sum X_i^\beta\right]^{\frac{1}{\beta} - 1} \beta X_i^{\beta - 1} - \lambda p_i = 0 \qquad (12.33)$$

Divide the first-order condition in (12.33) for an arbitrary good i by the corresponding condition for good j.

$$\left[\frac{X_i}{X_j}\right]^{\beta - 1} = \frac{p_i}{p_j} \qquad \frac{X_i}{X_j} = \left[\frac{p_i}{p_j}\right]^{\frac{1}{\beta - 1}} = \left[\frac{p_i}{p_j}\right]^{-\sigma} \qquad since \quad \sigma = \frac{1}{1 - \beta} \qquad (12.34)$$

Now perform several steps. (1) write the second equation of (12.34) with X_j on the left. (2) multiply both sides of this equation by p_j. (3) sum this equation over all goods j. These three steps are

$$X_j = \left[\frac{p_i}{p_j}\right]^{\sigma} X_i \qquad p_j X_j = p_j p_j^{-\sigma} p_i^{\sigma} X_i \qquad \sum p_j X_j = M_x = \left[\sum p_j^{1 - \sigma}\right] p_i^{\sigma} X_i \qquad (12.35)$$

Inverting this last equation, we have the demand for an individual variety i:

$$X_i = p_i^{-\sigma}\left[\sum p_j^{1 - \sigma}\right]^{-1} M_x \qquad \sigma = \frac{1}{1 - \beta}, \quad \beta = \frac{\sigma - 1}{\sigma} \qquad (12.36)$$

Now we can use X_i to construct X_c and then solve for e_x, the price index, noting the relationship between α and σ. First, raise (12.36) to the power β.

$$X_i^\beta = X_i^{\frac{\sigma - 1}{\sigma}} = p_i^{1 - \sigma}\left[\sum p_j^{1 - \sigma}\right]^{\frac{1 - \sigma}{\sigma}} M_x^\beta \qquad (12.37)$$

Now sum over all of the i varieties of X.

$$\sum X_i^\beta = \left[\sum p_i^{1-\sigma}\right]\left[\sum p_j^{1-\sigma}\right]^{\frac{1-\sigma}{\sigma}} M_x^\beta = \left[\sum p_j^{1-\sigma}\right]^{\frac{1}{\sigma}} M_x^\beta \tag{12.38}$$

Now raise both sides of this equation to the power 1/β to get the composite commodity demand for X_c.

$$X_c = \left[\sum X_i^\beta\right]^{\frac{1}{\beta}} = \left[\sum X_i^\beta\right]^{\frac{\sigma}{\sigma-1}} = \left[\sum p_j^{1-\sigma}\right]^{\frac{1}{\sigma-1}} M_x = M_x/e_x \tag{12.39}$$

The demand for X_c must be the expenditure M_x on X_c, divided by the price index e_x. Examining the last equation of (12.39), this means that the price index is given by:

$$e_x = \left[\sum p_j^{1-\sigma}\right]^{\frac{1}{1-\sigma}} \tag{12.40}$$

Again, the price index e_x, also called the expenditure function, is the minimum cost or expenditure necessary to buy one unit of the composite commodity X_c. If all of the X varieties sold for the same price, the expenditure function (price index) in (12.40) simplifies as follows.

$$e_x = n^{\frac{1}{1-\sigma}} p \tag{12.41}$$

Note that the price index is homogeneous of degree one in the prices of the individual goods: if we double all prices we double to cost of buying one unit of X_c. However note that the price index is decreasing in the number of varieties available (1 - σ < 0). The is another way of thinking about the love-of-variety effect. The same utility derived from two apples or two oranges might be derived from (for example) 0.8 apples and 0.9 oranges. When more variety is available, the consumer can achieve the same utility (same value of X_c) by actually reducing total expenditure.

Finally, having derived e, we can then use equation (13) in (9) to get the demand for an individual variety.

$$X_i \equiv p_i^{-\sigma} e_x^{\sigma-1} M_x \qquad \textit{since} \qquad e_x^{\sigma-1} = \left[\sum p_j^{1-\sigma}\right]^{-1} \tag{12.42}$$

We now move onto a chapter introducing the existence of trade costs. The price index in (12.40) and the demand function in (12.42) will prove very useful in discussing trade costs in the context of monopolistic-competition models in the next chapter.

12.6 Summary: what you should know

This is the second of two chapters on trade with increasing returns to scale and imperfect competition. The first (Chapter 11) focused on a pure case in which firms produced identical goods but scale economies limited the number of firms such that individual firms took into account their strategic interactions with other firms in their output decisions. A principal result is that the larger economy created through trade offer welfare gains through pro-competitive and production-scale effects. This Chapter focused also on a pure case in which firms produce somewhat different goods and there are a large number of firms such that each views the decision of rival firms as exogenous. A principal result is that the larger economy created through trade offers welfare gains through increased product or intermediate-good diversity: more products at the same prices rather than the same products at lower prices in Chapter 11. Of course, the two approaches can be combined, but the analysis becomes more difficult and is left to more advanced treatments (e.g., Melitz and Ottaviano (2008)) and indeed the ideal variety approach does involve variable markups and pro-competitive effects.

The two main approaches to product diversity are considered. The "love of variety" approach assume an endogenous set of symmetric but imperfectly substitutable products. Consumers are rewarded by a more diverse consumption bundle through trade. The "ideal variety" approach works rather differently. Consumers are assumed to differ in their views as to the ideal product, a product being a bundle of characteristics. Often in this approach, the consumer is assumed to buy just a single unit or nothing. As in the love-of-variety approach, diversity, in this case giving each consumer their ideal product, is costly in the presence of scale economies and so consumers accept compromise varieties in small autarky markets. The rewards to trade are that consumers, on average, get products closer to their ideal varieties.

While much of the literature has focused on final goods, an important variation of these models considers the differentiated products to be intermediate goods used in producing homogeneous final goods. Allowing trade in intermediate goods offers final producers higher productivity by increasing the division of labor. This ideal is a cornerstone of what is referred to as endogenous growth theory (e.g., Romer 1987) and has also been applied to trade in components and in producer services.

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Figure 12.1

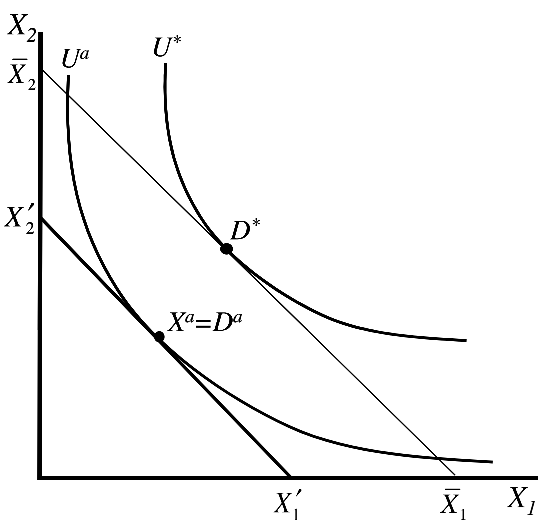

Figure 12.2

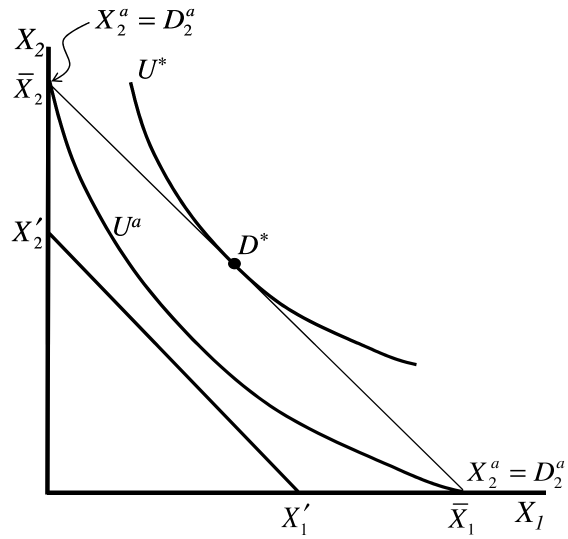

Figure 12.3

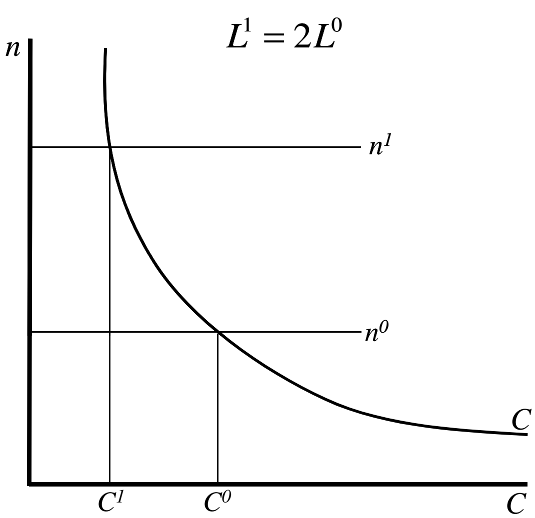

Figure 12.4

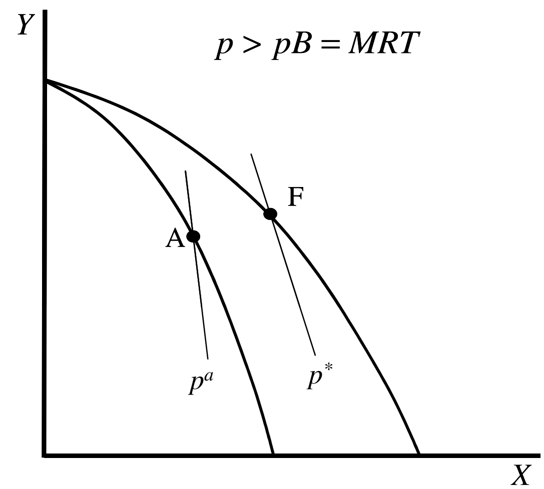

Figure 12.5