

**Economics 4413
International Trade
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Problem Set #1

Keep in mind that problem sets will not be graded. I will circulate answers to problem sets in advance of the examinations.

Marginal Products

1. A metal products company has been producing metal spools, all of a uniform type, for many years. The monthly production schedule for labor (for a given capital stock K) is:

<u>Labor Employed</u>	<u>Spools per Month</u>
1	200
2	370
3	500
4	600
5	675
6	740
7	800
8	855
9	900
10	940
11	970

- a. Graph the total product of labor and marginal product of labor curves.
- b. The production of metal spools has constant returns to scale. What happens to the total product and the marginal product of labor when the capital stock doubles?
2. The monthly production schedule for capital at the same company (for a given amount of labor, L) is as follows:

<u>Capital Employed</u>	<u>Spools per month</u>
1	300
2	430
3	500
4	545
5	570
6	586
7	600
8	612
9	622
10	630
11	635

- a. Graph the total product of capital and marginal product of capital curves.

- b. What happens to the total and marginal products of capital when the labor force is cut in half?
- c. Discuss why the marginal products of labor and capital depend on the capital-to-labor ratio (k).

Cost Minimization and Isoquants

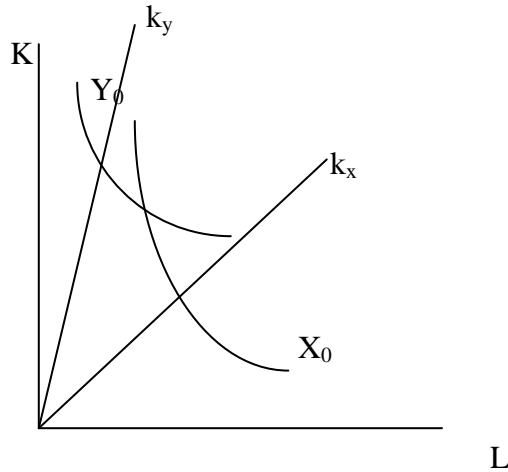
3. Show that each of the following production functions exhibits constant returns to scale. Also, develop an isoquant for each case. (Hint: solve the production function for K . Determine the slope of the isoquant. Is the slope constant or diminishing?)

a. Linear: $X = L + 2K$

$Y = L + K$

b. Cobb-Douglas: $X = K^{1/3}L^{2/3}$
 $Y = K^{1/2}L^{1/2}$

4. The isoquants associated with the Cobb-Douglas production functions (4b) are convex, as drawn below.



- a. How do we know from the expressions in 3b that Y is the capital-intensive sector and X is the labor-intensive sector?
 - b. Develop expressions for the slopes of the isoquants for X and Y from 3b.
 - c. Write down equations for an isocost line in each sector, where we set total costs equal to $\$C$ in each sector.
 - d. Show graphically where the minimum cost of producing X and Y occurs, where we choose outputs so they share the same cost level.
 - e. Show that the costs of producing X and Y are minimized where $\omega = w/r = MPL_x/MPK_x = MPL_y/MPK_y$.
5. Draw a set of isoquants for (a) a Leontief production function for Y (labor and capital are used in fixed proportions) and (b) a Cobb-Douglas production function for X . In your diagram, indicate k_x and k_y . Discuss how these capital-labor ratios change as the wage-rental ratio ω changes.

Edgeworth Boxes and Production Possibility Frontiers

6. Let factor endowments in an economy be $\bar{K} = 40$ and $\bar{L} = 100$.
- Draw an Edgeworth Box with these dimensions.
 - Suppose that $k_x = k_y$ for all wage-rental ratios (factor intensities in the two industries are identical). What must be the values of k_x and k_y ?

7. Suppose that both X and Y have CRS and that outputs follow this schedule.

	<u>X</u>	<u>Y</u>
$(L_x = 0, K_x = 0)$	0	1000
$(L_x = 25, K_x = 10)$	100	750
$(L_x = 50, K_x = 20)$	200	500
$(L_x = 75, K_x = 30)$	300	250
$(L_x = 100, K_x = 40)$	400	0

- Determine the values of L_y and K_y for each level of Y output.
- Determine the values of k_x and k_y .
- Draw the PPF.
- Provide an intuitive explanation of the concept of constant costs of production and its relation to the marginal rate of transformation.

8. Now suppose that X has IRS but Y has CRS, according to this schedule:

	<u>X</u>	<u>Y</u>
$(L_x = 0, K_x = 0)$	0	1000
$(L_x = 25, K_x = 10)$	50	750
$(L_x = 50, K_x = 20)$	150	500
$(L_x = 75, K_x = 30)$	350	250
$(L_x = 100, K_x = 40)$	650	0

- Determine the values of L_y and K_y for each level of Y output.
- Determine the values of k_x and k_y .
- Draw the PPF.
- Provide an intuitive explanation of the concept of decreasing costs of production and its relation to the marginal rate of transformation.

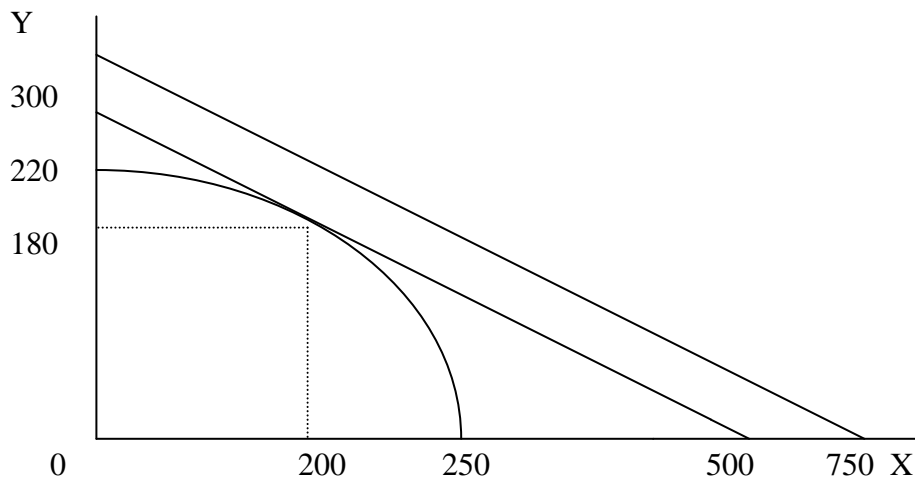
9. Using the same endowments of labor and capital as in questions 6 - 8, and assuming CRS in both X and Y, now let k_y be greater than k_x for any ω (Y is capital intensive).

- Develop the Edgeworth Box and draw the contract curve.
- Show that ω rises as production of X increases and production of Y decreases. (Hint: Remember that for a homogeneous production function, the capital-labor ratios are constant when ω is constant.)
- Draw the PPF.
- Provide an intuitive explanation of the concept of increasing costs of production and its relation to the marginal rate of transformation.

10. Note in problem 10b that as production of X increases and production of Y decreases, *both sectors become more capital-intensive* (that is, both k_y and k_x go up). How can this be possible with fixed endowments of labor and capital? Hint: consider the

following case, where $\bar{K} = 40$ and $\bar{L} = 100$. Suppose initially that $K_y = 30$ and $L_y = 40$. Compute K_x, L_x, k_y and k_x . Now transfer 5 units of capital and 10 laborers from Y to X. Compute K_y, L_y, K_x, L_x, k_y and k_x . Note that both capital-labor ratios are higher. What has happened to ω (higher or lower)? To outputs of X and Y? Provide an intuitive explanation for how these adjustments are possible.

11. Diagram a PPF with the following points on it: $(X,Y) = (100,0); (90,25); (70,50); (40,75);$ and $(0,100)$. Demonstrate that this PPF exhibits increasing opportunity costs of production.
12. Using your diagram from (11), estimate the marginal rate of transformation when 90 units of X are consumed and 25 units of Y are consumed. (Hint: draw the relative price line). What is the value of consumption in terms of units of Y? What is the value of national income in terms of units of X? If the price of X is \$3 billion per unit, find nominal national income.
13. Using the same PPF, estimate the relative price needed to shift production from $(90,25)$ to $(75,40)$.
14. Using the diagram below, if income rose to a value of 750 X with the same relative price of X, find the value of income in terms of Y.



Individual Utility Theory

15.
 - a. Draw a set of indifference curves for an individual consumer, who consumes goods X and Y. What is the slope of the indifference curve (in absolute value) and what does it indicate?
 - b. Now suppose this consumer has income level I_0 , while prices of the two goods are p_x and p_y . Develop an expression for the consumer's budget constraint and place it in your diagram. What is the slope (in absolute value) of the budget constraint? In which way does the budget constraint move if income rises? In

which way does the budget constraint move if the price of X rises? If the price of Y falls?

c. Show where utility is maximized for a particular budget constraint and discuss the meaning of the tangency between the constraint and the indifference curve.

16. Draw a consumer equilibrium with a given budget constraint and indifference curve between goods X and Y. Holding the price of Y fixed, suppose there is a fall in the price of X. Distinguish graphically between the *substitution effect* (which holds utility constant) and the *income effect* (which allows utility to change at the new set of prices). What is the meaning of normal and inferior goods?

Aggregation of Preferences

17. Suppose there are two people, 1 and 2. Define an aggregate demand curve for good X. Demonstrate graphically that it is possible to find such a curve when individuals do not begin with the same income, as long as their preferences are identical and homogeneous. Why is it not enough in this case to assume only that preferences are identical?

- *18. Here's a challenging problem for those of you who like to work things through mathematically. Don't spend much time on this one unless you like puzzles.

a. Suppose that individuals 1 and 2 have the following utility function:

$$U_1 = (X_1 - X^*)^a(Y_1 - Y^*)^{(1-a)}$$

$$U_2 = (X_2 - X^*)^a(Y_2 - Y^*)^{(1-a)} \quad 0 < a < 1$$

Here, X^* and Y^* are *minimum levels of consumption* of goods X and Y that must be available for the consumer to have positive utility (eg, a minimum amount of food or shelter). These are called "quasi-homogeneous utility functions". Suppose each person maximizes utility subject to a budget constraint. Can you derive the associated demand curves for each person for goods X and Y?

b. In this case, can you show that aggregate demand curves exist? (Hint: transfer income from person 1 to person 2.) This is the case depicted in Figure 3.6 in the text.

19. Write an essay indicating the problems with interpreting community indifference curves as valid indicators of social welfare when income distribution might change. What assumptions may be made to make them valid indicators?

General Equilibrium and Excess Demand Curves

20. Prove in a two-good model that if there is an equilibrium relative price where one country's exports of good X equal the rest of the world's imports of good X, then a matching equilibrium exists for good Y.
21. Draw a trade equilibrium shown by a production possibilities curve and a community indifference curve, such as the one in Figure 4.2. Identify on the graph the production point, the consumption point, the imported good, the exported good, the quantity of imports and exports, relative prices in autarky and free trade, and national income measured in terms of good X or Y in autarky and free trade (measured at free-trade relative prices).

22. Draw a graph like Figure 4.5 in the text and identify on it autarky prices in both countries, equilibrium quantities of imports and exports (of both X and Y) with trade, and the importing and exporting countries. Explain why welfare increases for either country as it moves along its excess demand curve away from autarky.
23. Define the concept "terms of trade". Using the diagram for problem 22, indicate what happens to the home country's terms of trade as its excess demand curve shifts to the left, and explain why.

Gains from Trade

24. Use graphical analysis to show that, starting from autarky, the gains from trade grow as the volume of trade grows and the terms of trade improve.
25. Using a two-sector PPF diagram, pick a production point in autarky (A) and with free trade (T). Draw the price lines through these points and select consumption points and community indifference curves for both autarky and free trade. Now use this graph to detail the breakdown between the gains from trade, gains from exchange and gains from specialization.
26. Suppose workers in the textile industry all sign a petition asking Congress to stop free trade. Explain what lies behind their position, and construct an argument against it.