1. Suppose that $U$ is a continuous random variable that is uniformly distributed on the interval $(-1, 1)$. That is, $U \sim \text{unif}(-1, 1)$.

Let $\alpha > 0$ and let

$$Y = \left( \frac{2}{1-U} \right)^\alpha - 1.$$ 

Find the distribution of $Y$. (Name it!)

2. Suppose that $X_1, X_2 \overset{iid}{\sim} \mathcal{N}(0, 1)$.

Show that $Y_1 := X_1 + X_2$ and $Y_2 := X_1 - X_2$ are independent.

3. Suppose that $X_1 \sim \Gamma(\alpha, \beta)$ and $X_2 \sim \text{exp}(\text{rate} = \beta)$ are independent random variables.

Find the distribution of $Y = 1 - X_1/(X_1 + X_2)$. (Name it!)

4. Suppose that $X_1, X_2, \ldots, X_n$ is a random sample from the uniform distribution over the interval $(0, 1)$.

   (a) Find the distribution of $X_{(1)} = \min(X_1, X_2, \ldots, X_n)$. (Name it!)

   (b) Find the distribution of $X_{(n)} = \max(X_1, X_2, \ldots, X_n)$. (Name it!)

5. Let $a > 0$. Suppose that $X_1, X_2, \ldots, X_n$ is a random sample from the $\text{Beta}(a, 1)$ distribution and that $Y_1, Y_2, \ldots, Y_n$ is a random sample from the $\text{Beta}(1, a)$ distribution.

Find $E[X_{(n)} + Y_{(1)}]$.

(Here we are using the usual notation: $X_{(n)} = \max(X_1, X_2, \ldots, X_n)$ and $Y_{(1)} = \min(Y_1, Y_2, \ldots, Y_n)$.)

6. Let $X \sim \text{geom}_0(p)$. (This is the geometric distribution that starts from 0.) Find (from scratch) the moment generating function for $X$. Be sure to give and justify the domain of your mgf.

7. Let $X_1, X_2, \ldots, X_n \overset{iid}{\sim} \text{geom}_0(p)$.

   (a) Find the distribution of $Y = \sum_{i=1}^n X_i$. (Name it!)

   (b) Consider $m$ independent random samples, each of size $n$, from the $\text{geom}_0(p)$ distribution. Let $Y_j$ be the sum of the $n$ values in the $j$th sample.

   Find the distribution of $Z = \sum_{j=1}^m Y_j$. (Name it!)

8. [Required for 5520 Students Only]

   (a) Find the moment generating function for the $\text{binomial}(n, p)$ distribution.

   (b) Show that the moment generating function for the $\text{binomial}(n, \lambda/n)$ converges, as $n \to \infty$ to that of the $\text{Poisson}(\lambda)$ distribution.

9. [Required for 5520 Students Only] Suppose that $X_1, X_2, \ldots, X_n$ is a random sample from a distribution with pdf $f$ and cdf $F$. Derive a formula for the joint density of $X_{(1)}$ and $X_{(n)}$ (the min and max) in terms of $f$ and $F$. 
