11. Assignment 11
Due Wednesday, April 27

Gregory Beyikin, ECOT 323

(1) Solve the Poisson’s equation
\[ \Delta u = f \]
on the square \((x, y) : 0 \leq x, y \leq 1\) with the homogeneous Dirichlet boundary conditions. Assume that the function \(f\) is well approximated by
\[ f(x, y) = \sum_{m=1}^{N} \sum_{n=1}^{N} a_{mn} \sin(\pi mx) \sin(\pi ny). \]
Choose appropriate discretization for \(f(x, y)\) and organize computation to use the Fast Fourier Transform. Verify your results on several examples.

(2) Let \(\partial D\) be the ellipse \(x^2/a^2 + y^2/b^2 = 1\). Consider the boundary value problem
\[ \Delta u = 1, \quad (x, y) \in D, \]
\[ u = x^4 + y^4, \quad (x, y) \in \partial D. \]
(a) Reduce the problem to that with homogeneous boundary conditions
(b) Reduce the problem to the Dirichlet problem for the Laplace equation.