1. (24 points) Consider the differential equation \( \frac{dy}{dt} = (y - 1)(y + 2) \).

(a) Find all equilibrium solutions.

(b) Classify the stability of the equilibrium solutions you found in part (a).

(c) Find the general solution.

2. (18 points) Consider the following three direction fields, labeled (a), (b) and (c):

Now consider the following five differential equations, labeled I, II, III, IV, and V:

| I. \( \frac{dy}{dt} = \frac{y}{t} \) | II. \( y' = t - ty \) | III. \( \frac{dy}{dt} = y^2 - 2y \) | IV. \( \frac{dy}{dt} = \sin(t + y) \) | V. \( y' = -t + y \) |

Lastly, consider the following word bank:

| autonomous | linear | homogeneous | separable | nonlinear |

For each direction field (a), (b) and (c), match it to the correct differential equation from I – V, then write all of the words from the word bank that correctly describe the differential equation you chose. You don’t have to show work for this problem and no partial credit will be given for your work.

OVER FOR MORE PROBLEMS!
3. (40 points) Two unrelated problems:

(a) Solve the IVP \( \frac{dy}{dt} - \frac{3}{t} y = t^3, \ y(1) = 4, \) using the method of integrating factors. Show all of the details – don’t just plug stuff into a formula!

(b) Consider the differential equation \((1 + e^t) \frac{dy}{dt} + e^t y = 3t^2.\)

   (i) Verify that \( y_h(t) = \frac{C}{1 + e^t}, \ C \in \mathbb{R}, \) is the general solution to the homogeneous version of the given differential equation.

   (ii) Use \( y_h(t) \) and the method of variation of parameters to find the general solution of the given differential equation. Show all of the details – don’t just plug stuff into a formula!

4. (18 points) Please provide a short answer to each of the following questions. You don’t need to justify your answers in this problem and no partial credit will be given for your work.

(a) The number of bacteria in a jar increases at a rate proportional to the population in the jar. If the number of bacteria doubles in 12 hours, exactly how long will it take for the population to triple in size?

(b) Given the IVP \( y' = t + y, \ y(0) = 0, \) approximate \( y(1) \) using Euler’s method and a step size of \( h = 0.5. \)

(c) For what initial points \((t_0, y_0)\) does Picard’s theorem guarantee that the IVP \( y' = \sqrt[3]{y - t}, \ y(t_0) = y_0 \) has a unique solution?