1. (20 points) For each of the following unrelated questions, answer either ALWAYS TRUE or NOT ALWAYS TRUE. For this problem, no justification is necessary.
   (a) $\mathbf{u} \times \mathbf{v} = |\mathbf{u}| |\mathbf{v}| \sin \theta$, where $\theta$ is the angle between $\mathbf{u}$ and $\mathbf{v}$.
   (b) If $\mathbf{u} \perp \mathbf{v}$ and $\mathbf{v} \perp \mathbf{w}$, then $\mathbf{u} \perp \mathbf{w}$.
   (c) $\text{proj}_\mathbf{u}(\mathbf{u} \times \mathbf{v}) = 0$
   (d) If the speed of a particle is constant, then its acceleration is orthogonal to its velocity.

2. (25 points) Consider the two points $P_1(2, 0, 3)$ and $P_2(-3, 2, 1)$ and the plane $M$ described by the equation $2x - 6y + 3z = 10$.
   (a) Determine the value of $c$ such that the point $P_3(1/2, 1/4, c)$ lies in the plane $M$.
   (b) Find the standard equation for a new plane $N$ containing the points $P_1$ and $P_2$, that is perpendicular to the plane $M$.
   (c) Find the parameterization for the line orthogonal to $N$, that lies in the plane $M$, and also passes through $P_3$.

3. (30 points) Pimm the Pirate is sailing the seas in search of a secret treasure. His path during this search is given by $\mathbf{r}(t) = t\hat{i} + \cos(t)\hat{j} + \sin(t)\hat{k}$
   (a) Calculate $\mathbf{T}$ and $\mathbf{N}$ (the unit tangent and unit normal) of the ship’s path.
   (b) Calculate the ship’s speed and the curvature of its path.
   (c) At $t = \frac{9\pi}{4}$, the winds shift direction. At this time, the wind is given by the vector $\mathbf{w} = 4\sqrt{2}\hat{i} + 2\hat{j} + 2\hat{k}$. Pimm knows that in order to continue sailing along the same path, there needs to be at least 3 units of wind in the direction that his ship is traveling. Will Pimm be able to continue along his current path?

4. (25 points) As part of an engineering project, you are trying to weld two steel objects together. The surfaces of the two objects are given by
   \[ \frac{(x-1)^2}{50} + \frac{y^2}{100} = 1 \quad \text{and} \quad z = \frac{x^2}{2} + \frac{y^2}{4} \]
   where distances/points are measured in feet. The objects are joined where these surfaces intersect. You know that to weld the objects together, you will need 0.05 pounds of welding wire per foot of weld.
   (a) Find a parameterization of the curve of intersection.
   (b) If you have 3 pounds of welding wire available, will you be able to complete the weld?

OVER
Projections, and distances from a point to a line and a plane:

\[
\text{comp}_a \mathbf{b} = \frac{\mathbf{a} \cdot \mathbf{b}}{||\mathbf{a}||} \quad \text{proj}_a \mathbf{b} = \frac{\mathbf{a} \cdot \mathbf{b}}{||\mathbf{a}||^2} \mathbf{a} \quad d = \frac{||\vec{PS} \times \mathbf{v}||}{||\mathbf{v}||} \quad d = \frac{||\vec{PS} \cdot \mathbf{n}||}{||\mathbf{n}||}
\]

Arc length, frenet formulas, and tangential and normal acceleration components:

\[
\begin{align*}
    ds &= |\mathbf{v}| dt \\
    \mathbf{T} &= \frac{\mathbf{v}}{|\mathbf{v}|} \\
    \mathbf{N} &= \frac{d\mathbf{T}}{dS} \\
    \mathbf{B} &= \mathbf{T} \times \mathbf{N} \\
    \frac{d\mathbf{T}}{ds} &= \kappa \mathbf{N} \\
    \frac{d\mathbf{B}}{ds} &= -\tau \mathbf{N} \\
    \kappa &= \frac{|\mathbf{T} \times \mathbf{a}|}{|\mathbf{v}|^3} \\
    \tau &= -\frac{d\mathbf{B}}{ds} \cdot \mathbf{N} \\
    \mathbf{a} &= a_T \mathbf{T} + a_N \mathbf{N} \\
    a_T &= \frac{d}{dt} |\mathbf{v}| \\
    a_N &= \kappa |\mathbf{v}|^2
\end{align*}
\]