1. (15pts) For parts (a)-(c) answer either ALWAYS TRUE or NOT ALWAYS TRUE. No justification is required.
   
   (a) If \( \delta (r, z) \) is continuous then \( \int_{0}^{2\pi} \int_{0}^{R} \int_{0}^{H} \delta (r, z) \sin \theta \, r \, dz \, dr \, d\theta = 0 \).
   
   (b) If \( \delta(\rho, \phi) \) is continuous then \( \int_{0}^{2\pi} \int_{0}^{\pi} \int_{0}^{R} \delta(\rho, \phi) \rho^2 \sin \phi \, d\rho \, d\phi \, d\theta = 2 \int_{0}^{\pi/2} \int_{0}^{\pi} \int_{0}^{R} \delta(\rho, \phi) \rho^2 \sin \phi \, d\rho \, d\phi \, d\theta \).
   
   (c) Let \( \mathbf{F} \) be a continuous vector field. If \( C_1 \) is described by \( r(t) = (\cos t, \sin t), \) \( 0 \leq t \leq 2\pi \) and \( C_2 \) is described by \( r(t) = (\sin(2t), \cos(2t)), \) \( 0 \leq t \leq \pi \) then \( \int_{C_1} \mathbf{F} \cdot dr = \int_{C_2} \mathbf{F} \cdot dr \).

2. (30pts) Brendan’s favorite type of (leftover) Halloween candy is Mike and Ike. A particular piece of candy has a shape bounded on the outside by the cylinder \( x^2 + y^2 = 1 \), and on the top and bottom by the sphere \( x^2 + y^2 + z^2 = 4 \).
   
   (a) Sketch the cross-section of the candy in the \( rz \)-plane (that is, a plane of constant \( \theta \) in cylindrical coordinates) clearly labeling the object boundaries and their points of intersection.

   **Note:** For parts (b)-(d) you’re computing volume, so you’re free to simplify your integrals using symmetry.
   
   (b) Set up, but do not evaluate, a triple integral (or set of integrals) for the volume of the candy in Cartesian coordinates in the order \( dz \, dy \, dx \).
   
   (c) Set up, but do not evaluate, a triple integral (or set of integrals) for the volume of the candy in cylindrical coordinates in the order \( dr \, dz \, d\theta \).
   
   (d) Set up, but do not evaluate, a triple integral (or set of integrals) for the volume of the candy in spherical coordinates in the order \( d\rho \, d\phi \, d\theta \).
   
   (e) Evaluate one of the integral expressions from (b)-(d) to find the volume.

3. (30pts) A rectangular metal plate described by the region \( R \) in the \( xy \)-plane is bounded by the curves \( y = x, y = x - 2, y = -x, \) and \( y = 4 - x \). The plate’s mass is given by the integral \( M = \int \int_{R} (x + y) e^{x^2 - y^2} \, dy \, dx \).
   
   (a) Make a clear sketch of the region \( R \) in the \( xy \)-plane. Make sure to label all boundaries and points of intersection.
   
   (b) Find a transformation into \( uv \)-coordinates of the form \( u = u(x, y) \) and \( v = v(x, y) \) such that the new region of integration, \( S \), is a rectangle with sides parallel to the coordinate axes in the \( uv \)-plane. Then make a clear sketch of the new region of integration in the \( uv \)-plane. Be sure to check this because the rest of the problem depends on this result!
   
   (c) Solve the transformation you found in part (b) for \( x \) and \( y \) in terms of \( u \) and \( v \).
   
   (d) Rewrite the integral \( M \) over the new region \( S \) in the \( uv \)-plane in terms of \( u \) and \( v \).
   
   (e) Evaluate \( M \) using your integral from (c).

4. (25pts) A struggling math- and physics-based amusement park has a ride that allows visitors to experience alternate gravitational fields. You can ride the Super Fun Dizzy Slide, where you travel along a helical path described by \( r(t) = (\cos t, \sin t, -3t) \) from \((1, 0, 0)\) down to \((-1, 0, -3\pi)\), in a chamber that alters the gravitational field to be \( \mathbf{F} = (-4y, 4z, 5y) \). Among the many other reasons the park is struggling is the fact that it has a flying bug infestation. In particular, the density of bugs (in bugs per unit length) in the chamber is given by \( f(x, y, z) = x^2 + y^2 - 2z \).
   
   (a) Calculate the work done by \( \mathbf{F} \) on a rider as they go down the slide.
   
   (b) Given your answer in part (a), would you rather ride the slide in the alternative gravitational field \( \mathbf{F} \) or Earth’s standard gravitational field? Briefly, but clearly, justify your response.
   
   (c) Calculate the number of bugs that a rider collides with on one trip down the slide.
Projections and distances

\[ \text{proj}_A B = \left( \frac{A \cdot B}{A \cdot A} \right) A \quad \text{and} \quad d = \frac{|P \vec{S} \times \vec{V}|}{|V|} \quad \text{and} \quad d = \frac{|P \vec{S} \cdot \vec{n}|}{|\vec{n}|} \]

Arc length, frenet formulas, and tangential and normal acceleration components

\[ ds = |v| \, dt \quad \mathbf{T} = \frac{\mathbf{v}}{|v|} \quad \mathbf{N} = \frac{\frac{d\mathbf{T}}{ds}}{|\frac{d\mathbf{T}}{ds}|} \quad \mathbf{B} = \mathbf{T} \times \mathbf{N} \]

\[ \frac{d\mathbf{T}}{ds} = \kappa \mathbf{N} \quad \frac{d\mathbf{B}}{ds} = -\mathbf{T} \quad \kappa = -\frac{|f''(x)|}{|f'(x)|^3/2} \quad \tau = -\frac{\mathbf{B} \cdot \mathbf{N}}{|\mathbf{B}|} \]

\[ \mathbf{a} = a_N \mathbf{N} + a_T \mathbf{T} \quad a_T = \frac{d|v|}{dt} \quad a_N = \kappa |v|^2 = \sqrt{|a|^2 - a_T^2} \]

Directional derivative, discriminant, and Lagrange multipliers

\[ \frac{df}{ds} = (\nabla f) \cdot \mathbf{u} \quad f_{xx} f_{yy} - (f_{xy})^2 \quad \nabla f = \lambda \nabla g, \quad g = 0 \]

Taylor’s formula (at the point \((x_0, y_0)\))

\[ f(x, y) = f(x_0, y_0) + \left[ (x - x_0)f_x(x_0, y_0) + (y - y_0)f_y(x_0, y_0) \right] \]

\[ + \frac{1}{2!} \left[ (x - x_0)^2 f_{xx}(x_0, y_0) + 2(x - x_0)(y - y_0)f_{xy}(x_0, y_0) + (y - y_0)^2 f_{yy}(x_0, y_0) \right] \]

\[ + \frac{1}{3!} \left[ (x - x_0)^3 f_{xxx}(x_0, y_0) + 3(x - x_0)^2(y - y_0)f_{xxy}(x_0, y_0) \right. \]

\[ \left. + 3(x - x_0)(y - y_0)^2 f_{xyy}(x_0, y_0) + (y - y_0)^3 f_{yyy}(x_0, y_0) \right] + \cdots \]

Linear approximation error

\[ |E(x, y)| \leq \frac{M}{2} \left( |x - x_0| + |y - y_0| \right)^2, \quad \text{where} \max \{ |f_{xx}|, |f_{xy}|, |f_{yy}| \} \leq M \]

Polar coordinates \( x = r \cos \theta \quad y = r \sin \theta \quad r^2 = x^2 + y^2 \quad dA = dx \, dy = r \, dr \, d\theta \)

Cylindrical and spherical coordinates

\begin{center}
\begin{tabular}{|l|l|l|}
\hline
Cylindrical to Rectangular & Spherical to Cylindrical & Spherical to Rectangular \\
\hline
\( x = r \cos \theta \) & \( r = \rho \sin \phi \) & \( x = \rho \sin \phi \cos \theta \) \\
\( y = r \sin \theta \) & \( z = \rho \cos \phi \) & \( y = \rho \sin \phi \sin \theta \) \\
\( z = z \) & \( \theta = \theta \) & \( z = \rho \cos \phi \) \\
\hline
\end{tabular}
\end{center}

\[ dV = dx \, dy \, dz = r \, dr \, d\theta \, dz = \rho^2 \sin \phi \, d\rho \, d\phi \, d\theta \]

Substitutions in multiple integrals

\[ \int \int_R f(x, y) \, dx \, dy = \int \int_S f(x(u, v), y(u, v)) \left| \frac{\partial (x, y)}{\partial (u, v)} \right| \, du \, dv \quad \text{where} \quad \frac{\partial (x, y)}{\partial (u, v)} = \frac{\partial x}{\partial u} \frac{\partial y}{\partial v} - \frac{\partial x}{\partial v} \frac{\partial y}{\partial u} \]

Mass, moments, and center of mass

\[ M = \int \int_R \delta \, dA \]

\[ M_x = \int \int_R y \, \delta \, dA \quad M_y = \int \int_R x \, \delta \, dA \quad \text{Center of mass} \quad \bar{x} = M_y/M \quad \bar{y} = M_x/M \]

Work/Flow and flux

\[ \text{Work/Flow} = \int_C \mathbf{F} \cdot T \, ds = \int_C \mathbf{F} \cdot \mathbf{v} \, dt = \int_C \mathbf{F} \cdot d\mathbf{r} = \int_C P \, dx + Q \, dy + R \, dz \]

\[ \text{Flux} = \int_C \mathbf{F} \cdot \mathbf{n} \, ds = \int_C P \, dy - Q \, dx \]