1. Evaluate the following integrals. Show all work to justify your answer and make sure to simplify as much as possible.

(a) (6 pts) \( \int \frac{x + 2}{\sqrt{x^2 + 4}} \, dx \)

(b) (6 pts) \( \int \frac{\sinh x}{e^x} \, dx \)

(c) (6 pts) If \( f \) is continuous and \( \int_{0}^{9} f(x) \, dx = 4 \), find \( \int_{0}^{3} xf(x^2) \, dx \).

2. Find \( \frac{dy}{dx} \) for the following. Show all work to justify your answer and make sure to simplify as much as possible.

(a) (6 pts) \( y = (\sin x)^x \)

(b) (6 pts) \( ye^{x^2} = \cos^{-1}(e^y) \)

(c) (6 pts) \( y = \int_{e}^{e^x} t^{\ln t} \, dt \)

3. Answer the following.

Given \( f(x) = \frac{e^x}{x} \) with, \( f'(x) = \frac{e^x(x - 1)}{x^2} \) and, \( f''(x) = \frac{e^x(x^2 - 2x + 2)}{x^3} \), find the following for \( f \).

Make sure to state any rules or theorems you utilize.

(a) (3 pts) State the domain of \( f \).

(b) (8 pts) Find all asymptote(s) for \( f \). Justify your answer(s) using the appropriate limits.

(c) (5 pts) Find the intervals of increase and decrease for the function \( f \). Justify your answer(s).

(d) (5 pts) Find the local maximum and minimum values for the function \( f \). Justify your answer(s).

(e) (6 pts) Find the intervals of concavity and the inflection points for the function \( f \). Justify your answer(s).

(f) (7 pts) Use parts (a) - (e) to sketch the graph of \( f \). LABEL the asymptote(s), maximum(s), minimum(s), and inflection point(s) on your graph.

TWO MORE ON THE OTHER SIDE
4. (12 pts) Sketch a function \( y = f(x) \) that satisfies **all** of the following conditions. No explanation is necessary. Clearly label all important features of the graph.

(a) \( f(-x) = -f(x) \)  
(b) \( f(-1) = 1 \)  
(c) \( \lim_{h \to 0} \frac{f(2 + h) - f(2)}{h} > 0 \)  
(d) \( \lim_{x \to -\infty} f(x) = 2 \)  
(e) \( \lim_{x \to -1} f(x) = 3 \)  

5. Some unrelated questions:

(a) (6 pts) Find the linearization of \( f(x) = \sqrt{1 - x} \) at \( a = -3 \) and use the linearization to approximate \( \sqrt{5} \). Show all work to justify your answer and make sure to simplify as much as possible.

(b) (6 pts) Suppose a rectangle is entirely contained in the first quadrant of the \( xy \)-plane. The rectangle borders the \( x \)-axis and \( y \)-axis and its upper right corner touches the curve \( y = \frac{2}{x} \). What dimensions minimize the perimeter of the rectangle? Show all work to justify your answer and make sure to simplify as much as possible.

(c) (6 pts) **True** or **False**: \( \int_{-1}^{1} \frac{\sin x}{1 + x^2} \, dx = 0 \). Justify your answer for full credit.