1. Consider the function $y = \frac{x - a^2}{x^2 - 5}$ and its graph.

(a) (3 points) If $a = 5$ then what is the domain of $y$?

(b) (5 points) If $a = 0$ then demonstrate why $y$ is a decreasing function.

(c) (6 points) If $a = \sqrt{5}$, then describe any domain restrictions as asymptotes, holes, or jump discontinuities.

(d) (5 points) If $y$ has a $y$-intercept at 2, then what must the value of $a$ be?

2. Consider the function $f(x) = x^4 - 4x^3 + 10$

(a) (6 points) Name the relative extrema of $f(x)$.

(b) (6 points) Name any points of inflection for $f(x)$.

3. Each of the following problems are not related:

(a) (6 points) Name the critical points of $\sin(\cos x)$ on $[0, 2\pi]$

(b) (4 points) Name the general anti-derivative of $y = x \cdot x^3$

(c) (9 points) If $p$ and $q$ are integers and $f(x) = (x - 1)^p(x + 1)^q$ for $p \geq 2$ and $q \geq 2$, then $f$ has three critical numbers; name them.

4. (8 points) Use Newton’s Method to estimate a root of $x^3 - 4x + 4 = 0$. Start with an initial guess of $x_0 = 1$. Find $x_2$ and $x_{101}$.
5. (8 points) Near the surface of a planet the acceleration of gravity is $32 \frac{ft}{sec^2}$. A 2-pound ball is thrown upward from a point 24 ft. above the ground with an initial velocity of $40 \frac{ft}{sec}$. Assuming no air resistance, when will the ball reach its maximum height?

6. The following problems are not necessarily all related

   (a) (3 points) Set up (but do not solve for the optimum) the equation $L(x)$, that can be used to find two positive real numbers $x$ and $y$ such that their sum is 50 and their product is as large as possible.

   (b) (4 points) Set up (but do not solve for the optimum) the equation $A(x)$, that can be used to find the maximum possible area of a rectangle of perimeter 200 ft. with dimensions $x$ and $y$.

   (c) (5 points) Set up (but do not solve for the optimum) the optimization equation $V(x)$, that can be used for finding the maximum volume of a rectangular box with a square cross section such that the sum of its length and its girth (the circumference of its cross section) is at most 100 inches.

   (d) (6 points) List the dimensions of the box with maximum volume mentioned in part (c). Show all steps and verification in optimizing.

7. (16 points) Are the following functions Even, Odd, or Neither (Use definitions):

   (a) $y = \tan x$  \hspace{1cm} (b) $y = x^3$  \hspace{1cm} (c) $y = |\tan x|$  \hspace{1cm} (d) $y = |x^3|$