1. (15 points) Let \( f(x) = \frac{1}{x} \)
   
   (a) Estimate the area under the graph of \( f(x) \) from \( x = 1 \) to \( x = 4 \) with 6 rectangles and left endpoints. \( \text{You don't have to simplify your answer.} \)
   
   (b) Sketch the curve and the approximating rectangles.
   
   (c) Is your estimate an underestimate or an overestimate? Why?

2. (15 points) The acceleration function (in m/s\(^2\)) and the initial velocity are given for a particle moving along a line.
   
   \( a(t) = 2t - 1 \quad v(0) = -6 \)
   
   (a) Find the velocity of the particle as a function of time.
   
   (b) Find the distance traveled by the particle during the time interval \( 1 \leq t \leq 4 \). \( \text{You don't have to simplify your answer. Just get down to some number, even if it looks ugly!} \)

3. (10 points) A biologist investigating colony collapse disorder studied a population of honeybees for 6 months. She collected the following data about \( r(t) \), the rate of growth of the population (measured in bees per week), with time \( t \) measured in weeks:

   \[
   \begin{array}{c|ccccccc}
   t & 0 & 4 & 8 & 12 & 16 & 20 & 24 \\
   r(t) & 100 & 2000 & 3000 & 11000 & 4000 & 1000 & 0 \\
   \end{array}
   \]

   (a) What does \( \int_0^{24} r(t) \, dt \) represent? Answer with a sentence and include units.
   
   (b) Using the given data, write down an overestimate and an underestimate to the quantity in part (a). \( \text{You don't have to simplify your answer.} \)

4. (15 points) Please answer the following:

   (a) Evaluate \( \int_0^{\pi/4} \frac{5 + \cos^2(\theta)}{\cos^2(\theta)} \, d\theta \)
   
   (b) Find \( \frac{d}{dx} \left[ \int_{\frac{t}{2}}^{\pi/2} \sin \left( \frac{t}{3} \right) \cos \left( \frac{t}{3} \right) \, dt \right] \)
   
   (c) Say \( g \) is a continuous function.

   If \( \int_0^5 g(x) \, dx = 10 \) and \( \int_0^3 g(x) \, dx = 7 \) find \( \int_3^5 2g(x) \, dx. \)

OVER
5. (23 points)

(a) Determine a region whose area is equal to 

\[ \lim_{n \to \infty} \frac{3}{n} \sum_{k=1}^{n} \sqrt{1 + \frac{3k}{n}} \]

Your answer should include the relevant function, an interval and a labeled sketch of the region. *You don’t have to evaluate the limit.*

(b) Write down a definite integral that equals the given limit from part (a). *You don’t have to evaluate the integral.*

(c) What is the average value of the function you found in part (a) over the interval you found in part (a)? Simplify your answer.

(d) Apply the Mean Value Theorem for Integrals to your function over your interval, finding the \( x \)-value(s) that your book calls \( c \).

(e) On top of your graph from part (a), sketch a rectangle whose area is the same as the area of the region from part (a).

6. (22 points) Let \( G(x) = \int_{-3}^{x} f(t) \, dt \) for \(-3 \leq x \leq 3\), where the graph of \( y = f(t) \) is shown below.

![Graph of y = f(t)](image)

Please answer the following questions, briefly justifying your answers:

(a) What is \( G(-3) \)?
(b) What is \( G(-1) \)?
(c) What is \( G(0) \)?
(d) What is \( G(2) \)?
(e) What is \( G'(-1) \)?
(f) What is \( G'(2) \)?
(g) What is \( G''(1) \)?
(h) Where does \( G \) have a relative minimum? Why?
(i) Does \( G \) have any inflection points? Where?