1. (15 points) Please evaluate the following limits or prove that they don’t exist. You must actually show all of your work and state any relevant theorems to receive credit.

(a) \( \lim_{x \to -\infty} \frac{\sqrt{4x^2 + 1}}{x - 5} \)  
(b) \( \lim_{x \to 0} x^4 \cos \left( \frac{1}{x} \right) \)  
(c) \( \lim_{t \to 0} \frac{t}{|t|} \)

2. (15 points) Please provide the requested information. State any derivative rules you use at the appropriate step.

(a) Find \( f'(x) \) if \( f(x) = x^3 \tan(2x) \)  
(b) Find \( h'(x) \) if \( h(x) = \frac{x}{\sqrt{4 - x^2}} \)  
(c) Find \( \frac{dy}{dx} \) given \( x \sin(xy) + x^2 = 0 \)

3. (20 points) Show that the equation \( x^5 + 5x = -1 \) has exactly one real solution. Please state any theorems you use (at the appropriate step), and explain why they are relevant.

4. (20 points) Find the absolute maximum and minimum values of the function \( g(x) = \sin(x) + \cos^2(x) \) on the interval \([0, \pi]\). Please also say where these values occur, and state any theorems you use and why they are relevant.

5. (15 points)

(a) Use linear approximation to estimate \( \frac{1}{\sqrt{17}} \). To receive credit you must find the appropriate linearization function along the way, providing any relevant details. Please write your answer as a single fraction.
(b) Does your answer from part (a) seem reasonable? Why or why not?

OVER
6. (15 POINTS) Consider the function

\[ f(x) = \begin{cases} 
\frac{8 - 2x^2}{3} & \text{if } x \leq 1 \\
\frac{c}{x} & \text{if } x > 1 
\end{cases} \]

where \( c \) is constant.

(a) What is the domain of \( f \)?
(b) What value of the constant \( c \) makes the function continuous on \( (-\infty, \infty) \)? You must use the definition of continuity in your solution to receive any credit.

7. (15 POINTS) Use the definition of derivative to find \( f'(x) \) given \( f(x) = \sqrt{x} + 3 \).

8. (20 POINTS) An airplane flies in level flight at constant velocity, eight miles above the ground (you may assume the earth is flat in this problem). The plane will pass directly over a point \( P \) on the ground. The distance from the plane to \( P \) is decreasing at a rate of 4 miles per minute at the instant when this distance is 10 miles. Compute the speed of the plane in miles per hour. Include a labeled picture in your solution.

9. (15 POINTS) The following graph shows the velocity \( v(t) \) of a particle moving along a line, \( 0 \leq t \leq G \).

![Graph of velocity v(t)]

Please answer the following questions. You don’t have to show any work or explain anything! Just write your answers down – but please use interval notation where appropriate.

(a) When does the particle move in the positive direction?
(b) When does the particle move in the negative direction?
(c) When is the particle instantaneously at rest?
(d) When is the particle’s acceleration positive?
(e) When does the particle move with the greatest speed?