1. (20 points) Let \( f(x) = 1 - \frac{1}{2} - \frac{1}{3 - x} \) and \( g(x) = \frac{1}{x} + 2. \)

(a) Simplify \( f \).

(b) What is the domain of \( f \)? Express your answer in interval notation.

(c) For what value(s) of \( x \) does \( f(x) = \frac{1}{f(x)} \)?

(d) Simplify \( h(x) = (f \circ g)(x) \).

(e) Is \( h \) even, odd or neither?

2. (20 points) A 15-ft ladder leaning against a wall forms an angle of \( \theta \) with the ground, \( 0 < \theta < \pi/2 \). The base of the ladder is \( b \) feet from the wall. The top of the ladder is \( h \) feet above the ground. Answer the following questions which are not related.

(a) For what value of \( h \) will \( h \) be twice as long as \( b \)?

(b) For what value of \( h \) will \( \sec \theta = 3/2 \)?

(c) For what values of \( \theta \) will \( h/b < \sqrt{3} \)?

(d) For what value(s) of \( \theta \) will \( \frac{4 \cos \theta}{\tan \theta} = 3 \csc \theta \)?

3. (16 points) A monthly data plan has a basic charge of $29 for the first 500 megabytes (mb) of usage. For each additional 100 mb used (rounded up to the nearest 100 mb), the charge is $6. For example, if the monthly usage is 500 mb, the charge is $29, and if the usage is 501 mb, the charge is $35.

(a) Sketch a graph of the monthly cost \( C \) as a function of the number \( m \) of mb used, up to 800 mb. Label tick marks clearly.

(b) Evaluate the following limits.

\[
\lim_{m \to 550^+} C(m) = \lim_{m \to 600} C(m) = \lim_{m \to 700^+} C(m) = \lim_{m \to 700^-} C(m) =
\]
4. (10 points) Rolf and Ralph decide to race their bikes to the top of Buff Mountain. Their race route is flat for the first half of the course, then uphill for the second half. Rolf quickly takes the lead on the flat stretch but Ralph, who is a better hill climber, passes Rolf halfway up the mountain and reaches the top first.

Let the functions \( r(t) \) and \( R(t) \) represent the distances traveled by Rolf and Ralph, respectively, over time \( t \). Sketch a graph of the two functions on a single set of axes.

5. (10 points) An entomologist owns a collection of ants, crickets, and spiders. There are 114 ants and spiders combined, and 60 crickets and spiders combined. How many of each does the entomologist have if there are 876 legs in total? (Assume that the ants and crickets have six legs each and the spiders have eight legs each.)

6. (10 points) A conical tank with a base diameter of 84 meters is draining water at a rate of \( 20\pi \) m\(^3\)/min. It will take 3 minutes and 20 seconds to empty the tank, which currently holds water 2 meters deep. How high is the tank?

7. (14 points) Rapunzel’s hair increases in length year after year. At birth her hair was 2 inches long and grew at a constant rate until it reached 40 ft in length on her 12th birthday. Then her hair continued to grow but at a faster constant rate, reaching 70 ft on her 18th birthday.

(a) Express Rapunzel’s hair length \( H \), in feet, as a function of her age \( t \) in years.

(b) Suppose at age 18, when her hair is 70 ft long, Rapunzel decides to cut her hair to a more manageable 20 ft in length. If she takes her cut hair and arranges the strands end to end along the Earth’s equator, her cut hair would cover about \( 1/20 \) of the equator. Given that the Earth’s radius is approximately 4000 miles, about how many hairs does Rapunzel have on her head? Express your answer in terms of \( \pi \). (Hint: 1 mile = 5280 ft.)