Solutions

- Textbooks, class notes and electronic devices of any kind are NOT permitted.
- If you leave the exam room, you will not be allowed back in and your exam will be concluded.

**Box your final answers for each question.**
- **Start each numbered problem on a new page.** All problems should be clearly numbered and in order.

For problems #1 - #5, show your work. Fully simplify all solutions. You must show a complete and valid solution method for full credit.

1. [5 points] Factor and simplify: \( x(2x - 9)(1 - 2x)^{1/2} - 2x^2(1 - 2x)^{1/2} \). Leave no negative exponents in the resulting expression.

   \[
   x(1 - 2x)^{-1/2}[(2x - 9) - 2x(1 - 2x)]
   \]

   \[
   x(1 - 2x)^{-1/2}[4x^2 - 9]
   \]

   \[
   x \frac{[(2x - 3)(2x + 3)]}{(1 - 2x)^{1/2}} \quad \text{or} \quad x \frac{[(2x - 3)(2x + 3)]}{\sqrt{1 - 2x}}
   \]

2. [10 points] Answer the following concerning the function \( f(x) = x(1 - 2x)^{1/2}(2x - 3)(2x + 3) \).

   (a) Find the domain of \( f(x) \). Give your answer in interval notation.

   \[
   x \frac{[(2x - 3)(2x + 3)]}{(1 - 2x)^{1/2}}
   \]

   **Part a) total is 5 points**

   \[
   1 - 2x > 0
   \]

   \[
   (-\infty, \frac{1}{2})
   \]

   (b) Find all real \( x \)-values, if any, for which \( f(x) = 0 \).

   **Part b) total is 5 points**

   \[
   x(2x - 3)(2x + 3) = 0
   \]

   \[
   x = 0, \frac{3}{2}
   \]

   Note that \( x = \frac{3}{2} \) is not a solution since it is not in the domain.
3. [20 points] On a distant planet an alien tosses a rock straight up from the top of a cliff and it falls to the ground below the cliff. The height of the rock in feet above ground is given by \( s(t) = -4t^2 + 12t + 40 \) where \( t \) is the time in seconds.

(a) What is the practical domain of the function (that is, the domain in the context of the problem)?

\[ s(t) = -4(t - 5)(t + 2) = 0 \]

The x-intercepts are at \( t = 5 \) and \( t = -2 \). Time must be positive so the practical domain is \([0,5] \).

(b) How long until the rock hits the ground at the bottom of the cliff? Give your answer with the correct units.

\[ s(t) = 0 \text{ or } -4t^2 + 12t + 40 = 0 \text{ or } (t - 5)(t + 2) = 0 \]

The rock hits the ground in 5 seconds.

(c) What is the average rate of change of the height of the rock from \( t = 0 \) to \( t = 2 \)? Give your answer with the correct units.

\[ ARC = \frac{s(2) - s(0)}{2 - 0} = \frac{48 - 40}{2} = 4 \text{ ft/sec} \]

Part c) total is 5 points

d) Find the difference quotient \( \frac{s(a + h) - s(a)}{h} \).

Part d) total is 5 points

\[ \frac{[-4(a + h)^2 + 12(a + h) + 40] - [-4a^2 + 12a + 40]}{h} = \frac{-8ah - 4h^2 + 12h}{h} = -8a - 4h + 12 \]

4. [15 points] These questions are not related.

(a) Write the function \( y = |2x| \) as a piecewise function.

\[ f(x) = \begin{cases} \pi - 2x, & x \leq \frac{\pi}{2} \\ 2x - \pi, & x > \frac{\pi}{2} \end{cases} \]

Part a) total is 5 points

The \( \frac{\pi}{2} \) can be included in either set.
(b) Solve \( x^{\frac{2}{3}} + 2 \ x^{\frac{2}{3}} \). Give your answer in interval notation.

**Part b) total is 5 points**

\[
0 \geq x^\frac{2}{3} - x^\frac{1}{3} - 2 \text{ or } x^\frac{2}{3} - x^\frac{1}{3} - 2 \leq 0 \\
(x^\frac{1}{3} - 2)(x^\frac{1}{3} + 1) \leq 0 \\

x^\frac{1}{3} - 2 = 0 \quad x = 8 \\
x^\frac{1}{3} + 1 = 0 \quad x = -1
\]

Test around the boundaries -1 and 8. The solution is \([-1,8]\).

(c) For what values of \( C \) does the equation \( 3x^2 + 3y^2 + 6x - C = 0 \) represent a circle?

**Part c) total is 5 points**

\[
3(x + 1)^2 + 3y^2 = C + 3 \\
(x + 1)^2 + y^2 = \frac{C + 3}{3} \\
\frac{C + 3}{3} > 0 \quad C + 3 > 0 \quad C > -3 \\
(-3, \infty)
\]

5. [20 points] The volume of a right circular cone is \( V = \frac{1}{3} \pi r^2 h \). The lateral surface area of a right circular cone is \( A = \pi \sqrt{r^2 + h^2} \). Answer the following questions concerning a right circular cone whose height \( h \) is equal to 3 times the radius \( r \). Give all answers without negative exponents.

(a) Find an expression for the volume \( V \) of the cone in terms of its radius \( r \).

**Part a) total is 5 points**

\[
V = \pi r^3
\]

(b) Using your expression from part (a), evaluate \( V^{-1}(8\pi) \).

**Part b) total is 5 points**

\[
V^{-1}(8\pi) = \frac{3\sqrt{8\pi}}{\pi} = 2
\]
(c) Find an expression for the radius \( r \) in terms of its volume \( V \).

**Part c) total is 5 points**

\[ r = \sqrt[3]{\frac{V}{\pi}} \]

(d) Which of the following is the correct expression for the surface area \( A \) of the cone in terms of the volume \( V \)?

**Part c) total is 5 points**

The correct answer is C.

A. \( A = \sqrt{10} \frac{V^{2/3}}{2^{1/3}} \)

B. \( A = 4V^{2/3} 1^{1/3} \)

C. \( A = \sqrt{10}V^{2/3} 1^{1/3} \)

D. \( A = 4V^{2/3} \)

E. \( A = 10V^{2/3} 1^{1/3} \)

[30 points] Problem #6 will be graded on your answers only.

6. All of the following (equations (a), (b), (c) and (d)) are FALSE. Provide the correct expression for the right side of the equation.

(a) \[ \frac{1}{\sqrt{2x+1} + \sqrt{2x+1}} = \frac{1}{2\sqrt{2x+1}} \] (5 points)

(b) \[ \sqrt[3]{x^2} = x^{3/2} \] (4 points)

(c) \( (3x+3)^2 = 3(x+1)^2 \) (4 points)

(d) \[ \frac{3}{2x^2} = 3(2x)^2 \] or \[ \frac{3x^2}{2} \] (5 points)

The graph of some function \( g(x) \) is shown in the graph below. Answer questions (e), (f) and (g) concerning this function.

Each of the following is worth 4 points.

(e) What are the coordinates of the point at (3,5) if \( g(x) \) is transformed to \( 2g(x+1) \)? \( (2,10) \)

(f) What are the coordinates of the point at (3,5) if \( g(x) \) is transformed to \( g(x-2) \)? \( (5,-5) \)

(g) The graph of \( g(x) \) shown is defined for \( x \geq 0 \). Complete the graph for \( x < 0 \) to make \( g(x) \) an even function.