AT THE TOP OF THE PAGE write your name and your section number. Textbooks, class notes and electronic devices of any kind are NOT permitted.

**Do all your work on this exam.**

Part I. Show Your Work, 100 points
Part II. MC/TF/SA/Matching, 50 points
Total: 150 points

**Part I. Show Your Work.** Fully simplify all solutions. For these problems you must show a complete and valid solution method for full credit. Leave your answers in terms of $\pi$ as necessary.

1. [15 points] Find all solutions of $e^{2x} - 2e^x - 24 = 0$.

Answer: ______________________________________________________

2. [15 points] Find all solutions of $2 \cos 4x \cos x + 2 \sin 4x \sin x = -\sqrt{2}$ in the interval $[0, 2\pi)$.

Answer: ______________________________________________________
3. [15 points] Find the error in the verification of the identity below. Circle the letter of the line where the error occurs, then correct it and finish the verification.

\[ 4 \tan x \sec x = \frac{1 + \sin x}{1 - \sin x} - \frac{1 - \sin x}{1 + \sin x} \]

A. \[ 4 \tan x \sec x = \frac{1 + \sin x}{1 - \sin x} \cdot \frac{1 + \sin x}{1 + \sin x} - \frac{1 - \sin x}{1 + \sin x} \cdot \frac{1 - \sin x}{1 - \sin x} \]

B. \[ 4 \tan x \sec x = \frac{(1 + \sin x)^2}{1 - \sin^2 x} - \frac{(1 - \sin x)^2}{1 - \sin^2 x} \]

C. \[ 4 \tan x \sec x = \frac{1 + \sin^2 x}{\cos^2 x} - \frac{1 + \sin^2 x}{\cos^2 x} \]

D. \[ 4 \tan x \sec x = \frac{2 \sin^2 x}{\cos^2 x} \]

E. \[ 4 \tan x \sec x = 2 \tan^2 x \]

---

4. [15 points] Find the exact value of \( \cos^2 \left( \frac{1}{2} \tan^{-1} \frac{12}{5} \right) \).

Answer: ____________________________
5. [25 points] A colony of bacteria begins growing with 4 cells at time \( t = 0 \) hours. Twelve hours later there are 64 cells.

(a) Assuming the colony grows linearly, find the equation of a line that represents the number of bacteria as a function of time. Call this function \( L(t) \).

(b) Assuming the colony grows exponentially, find an exponential function of base 2 that represents the number of bacteria as a function of time. Call this function \( E(t) \).

(c) Assuming the colony grows according to a quadratic equation, find the quadratic equation that represents the number of bacteria as a function of time. Assume that the vertex is the point \((0, 4)\). Call this function \( Q(t) \).

(d) Fill in the table by computing the function values at each value of \( t \):

<table>
<thead>
<tr>
<th></th>
<th>( L(t) )</th>
<th>( E(t) )</th>
<th>( Q(t) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( t = 0 )</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( t = 12 )</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( t = 24 )</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(e) Which (if any) of the three functions predicts the greatest average rate of change (ARC) from \( t = 12 \) hours to \( t = 24 \) hours? Give the value of the greatest ARC.
6. [15 points] An angry bird is launched at a pig with an initial velocity \( v_0 \) and launch angle \( \theta \) as shown in the diagram. The horizontal distance \( x \) that the bird travels is given by

\[
x = v_0 t \cos \theta
\]

The vertical distance (height) \( y \) of the bird along its trajectory is given by

\[
y = v_0 t \sin \theta - 5t^2
\]

where \( t \) is the time in seconds. The bird is launched with a velocity \( v_0 = 100 \text{ m/s} \) at angle of \( \theta = 30^\circ \).

(a) Use the equation for \( y \) to determine the time that the bird hits the ground after launch.

(b) What horizontal distance \( x \) did the bird travel in that time?

(c) Assuming the launch angle stays the same \((30^\circ)\), would the bird’s initial velocity have to be increased or decreased to hit a pig 400 meters away?

---

Part II. Multiple Choice, True/False, Short Answer, Matching. [3 points each MC, TF, SA] These problems will be graded on your answers only.

7. True or false: \( \cot(\tan x) = x \)  
7. T F

8. Find all solutions of \( \frac{x^2 - x - 2}{x + 1} = 3x + 4 \).  
8. ______________
9. Find all solutions of \( \frac{1}{2}x^{-1/2}(3x + 4)^{1/2} - \frac{3}{2}x^{1/2}(3x + 4)^{-1/2} = 0 \). Choose all the correct answers.

A. \( x = 0 \)  
B. \( x = -2/3 \)  
C. \( x = -4/3 \)  
D. \( x = 2 \)  
E. There are no solutions.

10. Find all real and imaginary solutions of \( e^x \cdot \ln x \cdot (x^2 + 4) = 0 \). Circle all the correct answers.

A. \( x = 0 \)  
B. \( x = 1 \)  
C. \( x = \pm 2 \)  
D. \( x = \pm 2i \)

11. The function \( y = \frac{x - 4}{x^2 - 5x + 6} \) has a vertical asymptote at \( x = 2 \). Which of the following describes the behavior of the function as \( x \) approaches 2 from the left and right sides?

A. As \( x \) approaches 2 from the left, \( y \to +\infty \). As \( x \) approaches 2 from the right, \( y \to +\infty \).  
B. As \( x \) approaches 2 from the left, \( y \to +\infty \). As \( x \) approaches 2 from the right, \( y \to -\infty \).  
C. As \( x \) approaches 2 from the left, \( y \to -\infty \). As \( x \) approaches 2 from the right, \( y \to +\infty \).  
D. As \( x \) approaches 2 from the left, \( y \to -\infty \). As \( x \) approaches 2 from the right, \( y \to -\infty \).

12. The graph of a polynomial function is shown below. Which one of the following could be that function?

![Graph of a polynomial function]

A. \( y = -x^4 + 2x^2 - 3x + 10 \)  
B. \( y = x^4 + 4x^2 - 3x - 10 \)  
C. \( y = -2x^4 + 3x^3 - x^2 - 20 \)  
D. \( y = -x^3 + 4x^2 - x + 10 \)

13. 100 grams of a radioactive substance decays according to the equation \( A(t) = 100e^{-t} \) where \( t \) is time in years. It takes 1 year for a radioactive substance to decay to about 40% of the original amount. How much remains after 2 years?

A. 80%  
B. 10%  
C. 4%  
D. 16%  
E. 20%

14. The difference quotient \( \frac{f(x+h)-f(x)}{h} \) evaluates to which of the following for \( f(x) = x^2 - 4 \): 

A. \( \frac{h^2 - 8}{h} \)  
B. \( h \)  
C. \( 2x \)  
D. \( 1 \)  
E. \( 2x + h \)
15. Which function has a domain of \((-\infty, \infty)\)?

A. \(y = \tan x\)  
B. \(y = |x|\)  
C. \(y = \frac{1}{x}\)  
D. \(y = \ln x\)  
E. \(y = \sqrt{x}\)

16. Evaluate \(f(g(-2))\) for \(f(x) = x^2\) and \(g(x) = \sqrt{x}\).

A. \(-2\)  
B. \(2\)  
C. \(2i\)  
D. \([-2]\)  
E. Does not exist.

17. If \(f(x) = f(-x)\), then which is necessarily true about the function \(f(x)\)?

A. \(f(x)\) has an inverse.  
B. \(f(x)\) is symmetric about the \(x\)-axis.  
C. \(f(x)\) is an even function.  
D. The domain of \(f(x)\) is all real numbers.

18. If \(\sin \theta = x\) for all \(\theta\) in the interval \(\frac{\pi}{2} < \theta < \pi\) then \(\sin 2\theta =\)

A. \(\frac{x}{\sqrt{1-x^2}}\)  
B. \(2x\)  
C. \(2x\sqrt{1-x^2}\)  
D. \(-2x\sqrt{1-x^2}\)  
E. \(-\frac{2x}{\sqrt{1-x^2}}\)

19. [15 points] Match each function to its graph. Write the letter for the function in the box next to its graph. Asymptotes are shown as dotted lines. The graphs are not necessarily to the same scale. One graph is not used.

A. \(y = \log_{\frac{1}{3}}(x-1)\)  
B. \(y = -3^x + 1\)  
C. \(y = \left(\frac{1}{3}\right)^x + 1\)  
D. \(y = \log_3(x-1)\)

\[\text{Graphs of functions}\]