• Textbooks, class notes and electronic devices of any kind are NOT permitted.
• If you leave the exam room, you will not be allowed back in and your exam will be concluded.

**Box your final answers for each question.**

* Begin each numbered problem on a blank, right-hand page of your bluebook. All problems should be clearly numbered and in order.

For problems #1 - #4, show your work. Fully simplify all solutions. You must show a complete and valid solution method for full credit.

1. [20 points] Consider the functions $f(x) = x^2 - 1$ and $g(x) = \sqrt{x-1}$.

   (a) Find the domains of functions $f$ and $g$. Give your answers in interval notation.
   (b) Find $h(x) = (f \circ g)(x)$. Then give the domain of $h$ in interval notation.
   (c) Which function, $f$ or $g$, has a greater average rate of change from $x = 2$ to $x = 5$? Justify your answer.
   (d) Consider a function $m(x) = 2f(x)$. Prove that from $x = a$ to $x = b$, the average rate of change of function $m$ is two times the average rate of change of function $f$.

2. [20 points] Consider the equation $f(x) = \frac{1}{2} x^{-1/2} (3x+4)^{1/2} - \frac{3}{2} x^{1/2} (3x+4)^{-1/2}$

   (a) Find the greatest common factor of $f(x)$.
   (b) Factor the equation completely.
   (c) Evaluate $f(2)$.
   (d) Solve $f(x) = 0$. If there are no solutions, state "none" and show why.

3. [25 points] Consider the triangle formed by the $x$-axis, the $y$-axis, and the line $x + 2y = 10$ as shown in the figure below. Answer the following questions to find the area of the shaded region of the triangle outside of the circle. The circle is given by the equation $x^2 + y^2 - 8x - 6y = -21$.

   (a) Find the equation of the circle in standard form $(x-h)^2 + (y-k)^2 = r^2$.
   (b) Identify the center and radius of the circle.
   (c) Show that the center of the circle lies on the line $x + 2y = 10$.
   (d) Find the area of the triangle.
   (e) Find the area of the circle.
   (f) Find the area of the shaded region.
4. [15 points] The volume of a sphere can be computed from its surface area $S$ using the function $V(S) = \frac{1}{6\pi^{1/2}} S^{3/2}$.

The surface area of a sphere in terms of its radius $r$ is $S(r) = 4\pi r^2$.

(a) Find the surface area of a sphere of radius $r = 2$ inches.
(b) Find the volume of a sphere of radius $r = 2$ inches.
(c) Find and simplify $(V \circ S)(r)$ to obtain an expression for the volume in terms of the radius $r$. You must show a complete solution method to this problem; you will not receive full credit for only giving an answer.

5. [20 points] These questions will be graded on your answers only.

For parts (a), (b) and (c) refer to the functions $f$ and $g$ in the graph below. Point $A$ is a point on the graph of function $g$.

(a) Find $(f \circ g)(-2)$.

(b) Find $g^{-1}(2)$.

(c) Give the new coordinates of point $A$ if $g$ is transformed to $y = g(-x + 1)$.

(d) Which of the following is a step in the process of solving $\frac{x+1}{x-3} \leq \frac{5x-3}{x-3}$?

   A. $x + 1 \leq 5$  
   B. $0 \leq \frac{4(x-1)}{x-3}$  
   C. $0 \leq 4(x-1)$  
   D. $0 \leq \frac{2(2x-1)}{x-3}$

(e) Which of the following is the inverse function $f^{-1}(x)$ for $f(x) = \frac{x-5}{3x+4}$?

   A. $f^{-1}(x) = \frac{5+4x}{1-3x}$  
   B. $f^{-1}(x) = \frac{3x+4}{x-5}$  
   C. $f^{-1}(x) = \frac{4x-5}{1+3x}$  
   D. $f^{-1}(x) = \frac{-x+5}{-3x-4}$

(f) Which of the following is the function $y = |2x - 4| - 3$ written as a piecewise function?

   A. $y = \begin{cases} 2x - 7 & x \geq 4 \\ -2x + 1 & x < 4 \end{cases}$  
   B. $y = \begin{cases} 2x + 1 & x \geq 2 \\ -2x - 1 & x < 2 \end{cases}$  
   C. $y = \begin{cases} 2x - 7 & x \geq 2 \\ -2x + 1 & x < 2 \end{cases}$  
   D. $y = \begin{cases} 2x + 1 & x \geq \frac{1}{2} \\ -2x - 1 & x < \frac{1}{2} \end{cases}$

(g) True or false: $\sqrt{4 - x^2} = 2 - x$

END OF EXAM

Make sure your name is at the top of the page on the front side, then put this exam page inside your bluebook and make sure it is returned with your bluebook.