1. (5 points each, 40 points total) (a) The graph of a 5th-degree polynomial function is shown below. The polynomial has real coefficients and has a zero at 2i. Find the equation of the polynomial. Leave your equation in factored form.

(b) Find all of the solutions to the equation \( x^2 \ln x - 3x \ln x - 10 \ln x = 0. \)

(c) Find all of the solutions to the equation \( \frac{50}{1 + e^x} = 4. \)

(d) True or false: \( \log_3 A = \frac{\ln A}{\ln 3} \)

(e) True or false: \( \ln e^{\ln x} = x \)

For equations (f) through (h), find all asymptotes for the functions given below. You must label each asymptote as horizontal, vertical or slant.

(f) \( f(x) = \log(x + 2) \)  
(g) \( g(x) = 2 - e^x \)  
(h) \( h(x) = \frac{x^3 + 5x - 3}{x^2 - x - 2} \)

2. (15 points) Consider the function \( f(x) = \frac{-x^2 + 4x - 4}{x^2 - 1}. \)

(a) What are the horizontal and vertical asymptotes? Label your asymptotes as vertical or horizontal.

(b) What are the \( x \)- and \( y \)-intercepts?

(c) Sketch the function.
3. (15 points) (a) Solve for $x$: $\ln(x^2 - 4) - 3 = \ln(x + 2)$

(b) Write the following as a single logarithm: $5 \log_2 x + \log_2 4 - \log_2 (x^2 + 1) - 3$

(c) Evaluate the following and fully simplify. [Hint: Your answer should be an integer.]

$$\log_{25} 5 + \log_4 36 - \frac{1}{2} \log_4 16 - 2 \log_4 3 + \log_9 27$$

4. (15 points) The 1986 explosion at the Chernobyl nuclear power plant in the former Soviet Union sent several hundred kilograms of radioactive Cesium-137 into the atmosphere. The area will not be considered safe until there is 100 kg or less of radioactive material remaining. The function

$$f(t) = 1000 \left(\frac{1}{2}\right)^{t/30}$$

describes the amount $f(t)$ in kilograms of Cesium-137 remaining in Chernobyl $t$ years after 1986.

(a) What is the half-life of Cesium-137?

(b) How much Cesium-137 will remain in the year 2046?

(c) When will it be safe for humans in Chernobyl? Use the fact that $\log\left(\frac{1}{2}\right) = -0.30$ to get an approximate value for the answer. [Hint: Your answer should be an integer.]

5. (15 points) The figure below shows a rectangle inscribed in Quadrant I of the coordinate plane. The left side of the rectangle is on the $y$-axis and the bottom side is on the $x$-axis. The upper right corner of the rectangle is the point $(x, y)$ on the line $y = -\frac{1}{2} x + 2$. Answer the following questions to find the value of $x$ such that the area $A$ of the rectangle is a maximum.

(a) Find an equation for area $A$ as a quadratic function of $x$.

(b) Find the value of $x$ that maximizes the area $A$.

(c) Find the maximum area $A$, and give the length and width of the rectangle with the maximum area.