Introduction

Micro-electro-mechanical Systems (MEMS) based pressure sensors have developed in several primary classes distinguished by mode of transduction: piezoresistive, capacitive, and optical [1]. A more recent class of pressure sensors uses changes in resonant frequency to detect pressure changes [2,3,4]. In this paper, we present the design and analysis of a novel MEMS device for resonant based wireless sensing of pressure. More specifically, analysis of the two operational components of the device, the diaphragm and resonator, is presented, showing the relationship between the component design parameters and device performance. This device, a Tympanic Pressure Transducer (TPT), is designed to be micro-machined from single crystal silicon, to operate without on-board electronics or power supply, and to function via acoustic interrogation and ultrasonic measurement.

Synthesis - Sensor Design and Operation

The TPT consists of two main operational components (Figure 1A): a resonator that receives the external acoustic excitation and a thin diaphragm that provides the acoustic output for pressure measurement. The resonator of the TPT operates in a vacuum to eliminate fluid damping effects and the first mode of vibration is in the vertical direction with respect to figure 1a. During interrogation of the sensor, the TPT is externally excited with acoustic waves and the resonator behaves similar to a base driven oscillator (Figure 1B). Exciting the TPT with acoustic waves at a frequency equal to the first mode resonant frequency of the resonator produces an oscillating displacement of increasing amplitude in the resonator. The magnitude of the resonator displacement will increase until the resonator collides with the diaphragm, causing the diaphragm to vibrate (Figure 1C). The central point of collision on the diaphragm surface maximizes the coefficient of restitution, thus maximizing the transfer of momentum from the resonator. A cross sectional view of a fully assembled device is shown in Figure 2.

The natural frequencies of the diaphragm are a function of the diaphragm geometry, material properties, and tension in the diaphragm. This relationship between diaphragm tension and natural frequencies enables the transduction of pressure into an acoustic signal. The inner
surface of the diaphragm is in vacuum, and the outer surface of the diaphragm is exposed to hydrostatic pressure external to the sensor. Thus, a change in external pressure causes a change in tension in the diaphragm, resulting in a change in the natural frequencies of the diaphragm. The vibrations of the diaphragm caused by the momentum transfer from the resonator are transmitted through a medium as acoustic waves and the first mode natural frequency can be received by an external transducer to provide a measurement of pressure. The feasibility of using ultrasonic waves to transmit data through a medium has been demonstrated by the recent work of Zhang et al [5]. By designing the TPT diaphragm with a first mode frequency significantly higher than the input frequency of the resonator, band pass filtering enables the acquisition of the diaphragm’s first mode frequency while attenuating the input frequency to the resonator and any higher mode frequencies of the diaphragm.

**Analysis – Finite Element Modeling**

Finite element analysis of the TPT resonator and diaphragm was performed using Coventorware. To determine the first mode natural frequency of the resonator, a modal analysis of a finite element model was performed. Multiple iterations were performed with different beam thicknesses to investigate the change of first mode resonant frequency with cantilever beam stiffness as a design parameter. The meshed model (Fig. 3) utilizes half symmetry and includes the thin beam of the resonator and the proof mass meshed as three dimensional (3-D) parabolic tetrahedral elements. The end of the beam is cantilevered with constraints in all six degrees of freedom and the model material is single crystalline silicon.

To determine the first mode natural frequency of the diaphragm as a function of pressure, a static and modal analysis was performed for each simulated pressure. First, a linear static analysis was performed with pressure applied to the external faces of the diaphragm to calculate the pressure induced stress in the diaphragm and supporting structure. Next, a modal analysis was performed on the diaphragm and supporting structure incorporating the pre-stressed condition from the linear static analysis of the pressure. The range of pressures analyzed yielded small displacements, allowing the modal analysis to consider pre-stressed conditions without geometrical deformation. The meshed model (Fig. 4) utilizes quarter symmetry includes the diaphragm and supporting structure, meshed as 3-D parabolic tetrahedral elements. The vertical faces of the supporting structure are cantilevered with constraints in all six degrees of freedom.
and the model material is single crystalline silicon. The simulated range of pressure was from 15 psi (atmospheric) to 75 psi.

**Analysis - Discussion of Results**

The simulated mode shapes of the resonator and diaphragm were in agreement with expected results. The first mode of vibration of the resonator is determined to only be in the vertical direction, as shown in Figure 5. The relationship between cantilever beam thickness and resonator first mode frequency is shown in Figure 6. In Figure 7, the first mode of vibration of the diaphragm is shown. The relationships between diaphragm thickness and first mode frequency, and pressure and first mode frequency are shown in Figure 8.

To provide validity to the simulated results, analytical models of the diaphragm and resonator were studied as well. To develop an analytical model of the cantilever beam/proof mass resonator, the system was treated as a spring mass system with a natural frequency determined by the mass and equivalent spring stiffness. Refer to appendix A for the derivation of the equivalent spring stiffness. The derivation yields the following expression for the first mode natural frequency of the resonator:

$$\omega_n = \sqrt{\frac{1}{m\left(\frac{dL^2}{2EI} + \frac{L}{3EI}\right)}}$$

Where \(m\) is the mass of the proof mass, \(d\) is the distance of the proof mass centroid from the cantilever beam end, \(L\) is the length of the cantilever beam, \(E\) is the elastic modulus of the cantilever beam, and \(I\) is the bending moment of inertia of the cantilever beam.

Due to the geometry of the resonator, this analytical solution does not agree well with the simulated results. The short cantilever beam and long proof mass are not well suited for a spring mass approximation. The results from the analytical solution differed from the simulation results by two orders of magnitude.

An analytical model of the diaphragm was approximated by modeling the diaphragm as a plate and using the following relationship [6]:

$$\omega_n = \frac{35.99}{2\Pi} \sqrt{\frac{D}{\rho \alpha^4}} \quad \text{and} \quad D = \frac{E h^3}{12(1 - \mu^2)}$$

Where \(\rho\) is the density of the diaphragm, \(h\) is the thickness, \(\alpha\) is the edge length, \(E\) is the elastic modulus, and \(\mu\) is Poisson’s ratio.
Unlike the resonator model, this analytical model agreed well with the simulation results. For the 3, 4, and 5 µm diaphragm thicknesses, the analytical model frequencies were 411019, 552026, and 690032, respectively.

**Conclusions**

Based on the results of the simulations, the components are feasible as designed and the operation of the device can be altered effectively by altering the design parameters of component thickness. However, the design is reliant on the impact of the resonator and diaphragm, and this requires further investigation.
References


Figure 1: TPT schematic showing principal device components and their functions: (a) device components before interrogation; (b) device operation under hydrostatic pressure with acoustic interrogation of the resonator; and (c) ultrasonic measurement of the diaphragm resonance following actuation with resonator.

Figure 2: Cross sectional view of the TPT
Figure 3 – Resonator finite element mesh

Figure 4 – Diaphragm finite element mesh

Figure 5 – Resonator first mode displacement
Figure 6 – Resonator frequency vs. beam thickness

Figure 6 – Diaphragm first mode displacement
Figure 7 – Diaphragm frequency vs. thickness
Figure 8 – Resonator first mode displacement
Appendix A

The deflection in the beam is caused by a force and moment exerted on the beam end. By superposition, the following expression is developed for beam end displacement.

\[ \Delta y = \frac{ML^2}{2EI} + \frac{FL^3}{3EI} = \frac{Fdl^2}{2EI} + \frac{FL^3}{3EI} \]

From this, one may obtain

\[ F = \left( \frac{dL^2}{2EI} + \frac{L^3}{3EI} \right)^{-1} y \]

The coefficient of \( y \) represents the equivalent spring stiffness of the beam, so the resonant frequency of the resonator becomes:

\[ \omega_n = \frac{1}{\sqrt{m \left( \frac{dL^2}{2EI} + \frac{L}{3EI} \right)}} \]